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Theory of the Adiabatic RF Plugging in the Magnetic Cusp Field

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Synopsis

The necessary and sufficient condition is obtained for describing the guiding center motion of a particle by the quasi-potential. This analysis leads to a new concept of the "critical energy", the height of the adiabatic barrier, which is proportional to $E_0{}^{8/5}$, E_0 being the strength of the rf electric field in the plasma. The theoretical results are confirmed by numerical calculations.

§1. Introduction

We have long known the proposals for plugging the leakage of the open system by the rf electric field, and many people have investigated the practicability of this idea.[1)~4)] It is based upon the phenomena that charged particles in a plasma running along the magnetic field will receive the repelling force when they enter into the strong rf field region. The net increase of a particle energy takes place at the expense of the rf field energy after its successive transits through the region of strong rf field; irreversible transfer of energy occurs from the rf field to the plasma. If this energy transfer is so large, particles gain the energy quickly and go over the rf barrier, hence the life time of the plasma will be short.

The irreversible energy gain occurs only in the resonance region where rf frequency ω is nearly equal to the cyclotron frequency ω_c of the particle. In the off-resonance region, the energy transfer of rf field is reversible; the particles increase energy as they approach the resonance region, but this energy is returned to the rf field as the particle leaves there. If we choose the frequency ω slightly larger than the maximum cyclotron frequency in the plasma, rf energy dissipation in the plasma becomes negligible. In this case, it is shown that the motion of the guiding center is governed by a "quasi-potential" in the terminology of Watson[5)], who gives the explicit expression to this potential in the general weakly inhomogeneous magnetic and rf electric field. He denotes this stopping scheme as "adiabatic" one and says that the thermonuclear prospects of this approach is encouraging.

The adiabatic plug requires very strong rf field so as to repel particles before they reach the resonace region. The purpose of the present paper is to introduce a concept of the critical energy[6], W_c, for the adiabatic reflection of a particle in the inhomogeneous magnetic and rf electric field and to give the guarantee that the required strength of the rf field is within the limit of the present technology.

We deal with the plasma confined in the magnetic cusp field that is very convenient for the rf plugging. Adiabatic rf plug is intended by applying strong electrostatic rf field locally in the part of line and point cusp with the frequency slightly larger than the ion cyclotron frequency in the plasma. Sato and Miyake et al have proceeded the experiments under the above situation mainly in the line cusp, and have obtained some fruitful results.[7)~12)] Two merits of the cusp field for working the rf plugging are the followings. First, it is possible to make the width(radius) of the plasma in the line(point) cusp to the order of ion thermal Larmor radius,[13)] then it will be easy to make strong rf field because of the small distance of the plates of electrode; since there exist eigen modes[14)] in such a thin(slender) plasma near the ion cyclotron frequency, the enough penetration of the external rf field will occur. Secondly, the existence of B=0 region in the cusp has an effect of making the velocity distribution isotropic, resulting in the stabilization of the loss cone instability. Present theory is based upon the single particle theory, with the plausibly chosen form of the standing rf field under the electrode, although the true form of the field has to be determined by a selfconsistent theory. The theory of the eigen oscillation in the thin (slender) plasma is given based upon the linearized Vlasov equation;[14] the nonlinear theory is one of the future problems.

Consider a simple cusp field (See Fig. 1):

$$B_r = B_0 \frac{r}{R_0} , \qquad (1\text{-a})$$

$$B_z = B_0 \frac{-2z}{R_0} , \qquad (1\text{-b})$$

$$B_\theta = 0 . \qquad (1\text{-c})$$

The following analysis is restricted to the line cusp; we may say that there is no essential difference in the case of point cusp. The externally applied rf field is in the direction of z, the symmetry axis, like $E_{ex}\sin(\omega t + \phi_i)$. We designate the thin plasma in the line cusp as the "sheet plasma," whose width can be reduced to the order of the ion thermal Larmor radius. The external rf field resonates the eigen oscillation of the sheet plasma to excite the standing electrostatic wave there. The characteristic scale of z-dependence of the standing wave is of order the ion thermal Larmor radius ρ_i , while the scale of the r-dependence is of order R_0 much larger compared with ρ_i . So as not to overcomplicate the problem and to understand the essential point, the z-dependence of the rf field will be henceforth neglected, thus we assume the electrostatic field in the plasma as

$$\left.\begin{aligned} E_z &= E(r)\sin(\omega t + \phi_i), \\ E_r &= E'(r) z \sin(\omega t + \phi_i), \\ E_\theta &= 0. \end{aligned}\right\} \quad (2)$$

The form of the quasi-potential $\psi(r)$ provided by the rf field (2) will be given by (see §3)

$$\psi(r) = \frac{M}{4} \frac{(eE(r)/M)^2}{\omega^2 - \omega_c^2(r)} , \qquad (3)$$

where
$$\omega_c(r) = \frac{eB_r}{MC} = \frac{eB_0}{MC} \frac{r}{R_0} , \qquad (4)$$

and R_0 is the radius of the resonance point;

$$\omega = \omega_c(R_0) = \frac{eB_0}{MC} .$$

In §2, a simplified mechanical model is dealt with to elucidate the essential point of the physical mechanism of the quasi-potential. In §3 we derive the explicit form $\psi(r)$ in the cusp field, and give six required conditions. §4 is concerned with the derivation of the critical energy, W_c, and by virture of this the theory of the loss flux is given there. In §5, a reduced equation of motion in the resonance region is derived and a theoretical expression is given for the increase of the magnetic moment in the resonance region. §6 contains numerical computations which confirm the present theoretical results.

§2. Simple Model

We start with a simple mechanical equation

$$M \frac{d^2x}{dt^2} = F(x)\sin \omega t. \qquad (5)$$

We consider the case that the x(t) of this equation can be divided into two parts as

$$x(t) = X(t) + \xi(t). \qquad (6)$$

Here X(t) is the slowly varying part of the oscillating center and $\xi(t)$ is rapidly oscillating part with frequency ω. We first impose the condition that the oscillating part ξ is much less compared with the scale of the X-dependence of the amplitude F(X), i.e.,

$$\left|\xi \frac{dF(X)}{dX}\right| << F , \qquad (7)$$

and then F may be expanded as

$$F(X + \xi) = F(X) + \xi \frac{dF(X)}{dX} . \qquad (8)$$

Substituting the expansion (8) into the eq.(5), we obtain

$$M \frac{d^2X}{dt^2} = \frac{dF(X)}{dX} \quad < \xi(t) \sin \omega t > \quad , \qquad (9)$$

$$M \frac{d^2\xi}{dt^2} = F(X) \sin \omega t, \qquad (10)$$

where the average means the time average over the interval much larger than ω^{-1} but much less than the time scale of $X(t)$. We require the second condition that the dependence of X on the variable t is slow enough to be ignored while evaluating ξ:

$$\left|\frac{\dot{X}}{\omega} \frac{dF(X)}{dX}\right| \ll F , \qquad (11)$$

thus the eq.(10) has a solution

$$\xi = - \frac{1}{M\omega^2} F(X) \sin \omega t . \qquad (12)$$

By virtue of (12), the first condition (7) may be written as

$$\frac{1}{M\omega^2} \left|\frac{dF}{dX}\right| \ll 1 . \qquad (13)$$

After squaring both sides of (13) and using (9) and (12), we have

$$\left|\frac{\ddot{X}}{\omega^2} \frac{dF}{dX}\right| \ll F , \qquad (14)$$

which implies that the accereration $\ddot{X}$ is small enough for the condition (11) to break down in the period ω^{-1} of the rapid oscillation.

With use of (12) and (9), we obtain the equation for the coordinate X(t), the oscillating center,

$$M \frac{d^2X}{dt^2} = - \frac{d}{dX} \Psi_K(X) \quad , \tag{15}$$

where

$$\Psi_K(X) \equiv \frac{M}{4} \frac{(F(X)/M)^2}{\omega^2} = \frac{M}{2} \langle \dot{\xi}^2 \rangle \tag{16}$$

One sees that the particle motion is governed by the effective potential[15)] given by $\psi_K(X)$, which can be identified with the average kinetic energy of the rapidly oscillating motion. If should be noted that this simplified description by the effective potential is allowed only when the both conditions (11) and (14) are satisfied. We now have a good correspondence between the potentials $\psi(r)$ and $\psi_K(X)$ given by (3) and (16) respectively. The force of the simplified system originates from the spacial gradient of the amplitude F of the high frequency field, while in the case of the rf confinement such new effects are added as the spacial gradient of both the steady magnetic field and the rf electric field and also the resonance effect between ω and ω_c . In the next section we extend the above analysis to the case of the three dimensional motion in the cusp magnetic field and the rf electric field.

§3. Quasi-Potential and Required Conditions

In the cusp field given by (1) and in the rf electric field given by (2), the equations of motion for ions are

$$\ddot{z} = -\omega_c^2(r)z - \frac{p_\theta \omega}{MR_0} + \frac{e}{M} E(r)\sin(\omega t + \phi_i), \qquad (17)$$

$$\ddot{r} = \frac{(p_\theta/M)^2}{r^3} - \omega^2 \frac{z^2}{R_0{}^2} r + \frac{e}{M} E'(r) z \sin(\omega t + \phi_i), \qquad (18)$$

$$p_\theta = Mr(v_\theta - \omega\frac{rz}{R_0}) = \text{const.} \quad , \qquad (19)$$

where $\omega_c(r)$ and R_0 are already defined in §1, and p_θ is the canonical conjugate momentum to the angle variable θ around the symmetry z-axis. The p_θ is a constant of the motion in the line cusp field when $E_\theta = 0$.* By virtue of the linearity of the eq.(17) with respect to z, the solution z can be divided into two parts:

$$z = z_B + z_{RF} \quad , \qquad (20)$$

where z_B is identical with the solution in the case without rf field, and z_{RF} is the contribution of the rf field.

We define z_B to satisfy the equation

$$\ddot{z}_B = -\omega_c^2(r)z_B - \frac{p_\theta \omega}{M\ R_0} \cdot \qquad (21)$$

Introducing a transformation defined by

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* For the case of the point cusp, p_θ is not conserved because of the non-zero E_θ, but the variance of p_θ is reversible just like that of the energy.

$$a = \sqrt{\frac{M}{2\omega_c(r)}} \, (\omega_c(r) z_B + i\dot{z}_B) \, , \tag{22}$$

or inversely

$$z_B = \frac{a + a^*}{\sqrt{2M\omega_c(r)}} \tag{23}$$

$$\dot{z}_B = (a - a^*) \frac{1}{i} \sqrt{\frac{\omega_c(r)}{2M}} \, , \tag{24}$$

We rewrite the eq.(21) as

$$\dot{a} = -i \, \omega_c(r) a - i \sqrt{\frac{M}{2\omega_c(r)}} \frac{p_\theta \omega}{MR_0} + \frac{1}{2} \frac{\dot{\omega}_c(r)}{\omega_c(r)} \quad a^* . \tag{25}$$

We impose an adiabatic condition,

$$|\dot{\omega}_c| \, / \omega_c{}^2 \quad << \quad 1, \tag{26}$$

implying that relative time derivative of cyclotron frequency is small in the time interval $\omega_c{}^{-1}$. The eq.(25) reduces to

$$\dot{a} = -i \, \omega_c \, a \, - i \sqrt{\frac{M}{2\omega_c}} \frac{p_\theta \omega}{MR_0} \, , \tag{27}$$

and has a solution by virtue of the condition (26)

$$a = \bar{a} \exp \{-i \int_{t_i}^{t} \omega_c(r(t')) dt'\} \; - \frac{1}{\omega_c} \sqrt{\frac{M}{2\omega_c}} \frac{P_\theta \omega}{MR_0} \, , \tag{28}$$

$\bar{a}$ being a complex constant. If $\bar{a}$ is written in the form

$$\bar{a} = \sqrt{\frac{Mc\mu}{e}}\; e^{-i\theta_i} \quad , \tag{29}$$

where θ_i means the initial phase of cyclotron gyration of a particle and μ represents the magnetic moment, the definition of which is given not by the ordinary formula $\frac{1}{2}Mv_\perp{}^2/B_r$ but by the modified one where $v_{B_\perp}{}^2 = (\vec{v}_\perp - \vec{v}_{RF_\perp})^2$ is used instead of $v_\perp{}^2$. Now we obtain various expressions for the adiabatic motion, from (28), (23), (24) and (19),

$$z_B = \sqrt{\frac{2c\mu}{e\omega_c(r)}}\ \cos\left(\int_{t_i}^{t}\omega_c(r(t'))dt' + \theta_i\right) - \frac{1}{\omega_c{}^2(r)}\frac{p_\theta\omega}{MR_0} , \tag{30}$$

$$v_{Bz}=\dot{z}_B=-\sqrt{\frac{2c\omega_c(r)\mu}{e}}\ \sin\left(\int_{t_i}^{t}\omega_c(r(t'))dt' + \theta_i\right), \tag{31}$$

$$v_{B\theta} = \frac{p_\theta}{Mr} + \omega\frac{r}{R_0}\ z_B = \sqrt{\frac{2c\ \omega_c(r)\mu}{e}}\ \cos\left(\int_{t_i}^{t}\ \omega_c(r(t'))dt'+\theta_i\right). \tag{32}$$

Note that the second term on the right-hand side of (30) is the z-coordinate of the guiding center.

We seek the contribution of the rf field, z_{RF}, which is described by the equation

$$\ddot{z}_{RF} = -\omega_c^2(r)z_{RF} + \frac{e}{M}E(r)\sin(\omega t + \phi_i), \tag{33}$$

and also we require the condition that z_{RF} vanishes as E tends to zero. As before, we introduce a new variable b defined by

$$b = \sqrt{\frac{M}{2\omega_c}}\,(\omega_c\, z_{RF} + i\, \dot{z}_{RF}) \quad , \tag{34}$$

or $$z_{RF} = (b + b^*)/\sqrt{2M\omega_c} \quad , \tag{35}$$

$$\dot{z}_{RF} = (b - b^*)\frac{1}{i}\sqrt{\frac{\omega_c}{2M}} \quad . \tag{36}$$

In terms of b, the eq.(33) is rewritten as

$$\dot{b} = -i\omega_c b + i\sqrt{\frac{M}{2\omega_c}}\,\frac{eE}{M}\,\sin(\omega t+\phi) + \frac{1}{2}\,\frac{\dot{\omega}_c}{\omega_c}\,b^* \quad . \tag{37}$$

The adiabatic condition (26) allows us to neglect the third term on the right-hand side of (37) compared with the first. By dividing b into positive and negative frequency parts,

$$b = b_+ e^{i\omega t} + b_- e^{-i\omega t} \quad , \tag{38}$$

the eq.(37) resolves into two equations,

$$\dot{b}_+ = -i(\omega + \omega_c)b_+ + \frac{1}{2}\sqrt{\frac{M}{2\omega_c}}\,\frac{eE}{M} \quad , \tag{39}$$

$$\dot{b}_- = i(\omega - \omega_c)b_- - \frac{1}{2}\sqrt{\frac{M}{2\omega_c}}\,\frac{eE}{M} \quad , \tag{40}$$

If we impose besides (26) the second condition,

$$\frac{\dot{r}}{\omega + \omega_c(r)}\,\frac{E'(r)}{E\,(r)} \ll 1 \quad , \tag{41}$$

concerning the r-dependence of E(r), the solution to eq.(39) can be obtained in a simple form,

$$b_+ = \frac{1}{i(\omega + \omega_c)} \frac{1}{2} \sqrt{\frac{M}{2\omega_c}} \frac{eE}{M} \quad . \tag{42}$$

The eq.(40) can be solved generally as

$$b_- = \frac{1}{i(\omega-\omega_c)} \frac{1}{2} \sqrt{\frac{M}{2\omega_c}} \frac{eE}{M} - e^{i\Phi(t)} \int_{t_i}^{t} dt' \frac{d}{dt'} \left\{ \frac{1}{i(\omega-\omega_c(r(t')))} \frac{1}{2} \sqrt{\frac{1}{2\omega_c(r(t'))}} \frac{eE(r(t'))}{M} \right\}$$

$$\times e^{-i\Phi(t')} \tag{43}$$

where $$\Phi(t) \equiv \int_{t_i}^{t} \{\omega - \omega_c(r(t'))\}dt' \quad . \tag{44}$$

If we require the third condition, the non-resonance condition,

$$|\dot{\omega}_c|/(\omega - \omega_c)^2 \quad << \quad 1 \quad , \tag{45}$$

the second term in (43) may be neglected, thus the solution to (40) reduces to

$$b_- = \frac{1}{i(\omega-\omega_c)} \frac{1}{2} \sqrt{\frac{M}{2\omega_c}} \frac{eE}{M} \quad . \tag{46}$$

When the conditions (26), (41) and (45) hold, we finally obtain the following expressions,

$$z_{RF} = - \frac{1}{\omega^2 - \omega_c^2(r)} \frac{eE(r)}{M} \sin(\omega t + \phi_i) \quad , \tag{47}$$

$$v_{RFz} = \dot{z}_{RF} = -\frac{\omega}{\omega^2-\omega_c^2(r)} \frac{eE(r)}{M} \cos(\omega t + \phi_i), \tag{48}$$

$$v_{RF\theta} = \omega_c(r)\, z_{RF} = -\frac{\omega_c(r)}{\omega^2-\omega_c^2(r)} \frac{eE(r)}{M} \sin(\omega t + \phi_i). \tag{49}$$

We summarize the above analysis as follows; three conditions (26), (41) and (45) concerning the time derivative $\dot{r}$ correspond to the condition (11) for the simple model in §2. Under these conditions, the rapidly varying variables such as z_B and z_{RF} can be expressed as (30) and (47) whose slow time dependences are represented only through the parameter $r(t)$. We say this case the adiabatic motion. We here give the explicit expression for the magnetic moment μ in the presence of the rf field, using (48) and (49):

$$\mu=\frac{M}{2B_r}[\{v_z+\frac{\omega}{\omega^2-\omega_c^2}\frac{eE}{M}\cos(\omega t+\phi_i)\}^2 + \{v_\theta+\frac{\omega_c}{\omega^2-\omega_c^2}\frac{eE}{M}\sin(\omega t+\phi_i)\}^2] . \tag{50}$$

We next investigate those conditions concerning the second time derivative, $\ddot{r}$, corresponding to (14). The three conditions (26), (41) and (45) require three inequalities, as was argued in §2,

$$|\ddot{\omega}_c|/\omega_c^{\,3} \ll 1 , \tag{51}$$

$$\frac{|\ddot{r}|}{(\omega + \omega_c)^2} \frac{E'(r)}{E(r)} \ll 1 , \tag{52}$$

$$\frac{|\ddot{\omega}_c|}{(\omega - \omega_c)^3} << 1 \quad . \tag{53}$$

It will be convenient to introduce dimensionless variables

$$\varepsilon(r) \equiv \rho_i(r)/R_0 \quad , \tag{54}$$

$$\bar{E}(r) \equiv \frac{eE(\bar{r})}{M\omega^2 R_0} \quad , \tag{55}$$

$$\bar{r} \equiv r/R_0 \quad , \tag{56}$$

where ρ_i is the ion Larmor radius, R_0 is the distance from the z-axis to the resonance point, as defined in §1, and $\bar{E}$ is the ratio of the fluctuation amplitude $eE/M\omega^2$ due to the rf field to the distance R_0. Then inequalities (51) through (53) are expressed as

$$\bar{\omega}^{-2}\ddot{\bar{r}}/\bar{r}^3 << 1 \quad , \tag{57}$$

$$\frac{\bar{\omega}^{-2}\ddot{\bar{r}}}{(1+\bar{r})^2}\,\frac{\bar{E}'(\bar{r})}{\bar{E}\;(\bar{r})} << 1 \quad , \tag{58}$$

$$\bar{\omega}^{-2}\ddot{\bar{r}}/(1 - \bar{r})^3 << 1 \quad . \tag{59}$$

Here, as will be shown below, the acceleration along the radial direction, $\bar{r}$, is given by

$$\bar{\omega}^{-2}\ddot{\bar{r}} = \bar{\mu} + \frac{1}{2}\,\bar{E}^2\,\frac{\bar{r}}{(1-\bar{r}^2)^2} + \frac{1}{2}\,\bar{E}'\;\bar{E}\,\frac{1}{1-\bar{r}^2} \quad , \tag{60}$$

where the normalized magnetic moment $\bar{\mu}$ is given by

$$\bar{\mu} = \frac{c\mu}{e\omega R_0{}^2} = \frac{1}{2}\,\varepsilon^2(r)\,\frac{1}{\bar{r}} \qquad (61)$$

As we may estimate $\bar{r} \sim 1 - \bar{r} \sim 1$ in the situation considered, the conditions (57) through (59) can be found to be equivalent to three infinitesimal dimensionless quantities,

$$\varepsilon(\bar{r}) \ll 1, \qquad (62)$$

$$\bar{E}(\bar{r}) \ll 1, \qquad (63)$$

$$\bar{E}'(\bar{r}) \ll 1. \qquad (64)$$

We can now derive the quasi-potential $\psi(r)$ defined by (3). We assume the form, $r = R + \xi$, where R represents the slowly varying coordinate of the guiding center and ξ the fluctuation satisfying $\dot{\xi} \sim \omega\xi \sim \omega_c\xi$. It can be proved that

$$\xi \sim \eta^2 R,$$

where η is a small constnat of the order of the largest one among the maximum values of ε, $\bar{E}$, and $\bar{E}'$, because the ordering, $\ddot{R} \sim \eta^2\omega^2 R$, which will be proved below, and because $\ddot{R}$ is of the same order as $\ddot{\xi}$ by inspection of the exact equation (18). Since R is much larger than $|\xi|$, we may replace the r in the eqs. (17) and (18) by R to sufficient approximation. In order to obtain the equation for R we average out the rapidly oscillating terms on the right-hand side of the eq. (18) with the substitution of the expressions (30) and (47), thus we obtain

$$M\frac{d^2R}{dt^2} = - \frac{d}{dR} (\mu B_r + \psi(R)) \ , \qquad (65)$$

where μ means the magnetic moment, the adiabatic constant defined by (50), and B_r is the radial component of the magnetic field. The function $\psi(R)$ represents the well-known "quasi-potential" defined by (3). We require six conditions (26), (41), (45), (62), (63) and (64) for the description of the guiding center motion by the potential force to be valid.

§4. "Critical Energy" for Adiabatic RF Plugging

In section 3 the required six conditions have been obtained for the adiabatic motion; on the basis of those results we introduce a concept or "critical energy" for the "adiabatic RF plugging". The (26) is identical with the condition for adiabatic motion in the absence of the rf field. We define the non-adiabatic region by

$$|\dot{\omega}_c|/\omega_c^2 \ > \ 1 \ . \qquad (66)$$

In the Fig.2 this region is denoted by NA in ($K_r \equiv \frac{M}{2}\dot{r}^2$, r) space. In the presence of the rf field, there exists the resonance region,

$$|\dot{\omega}_c| / (\omega - \omega_c)^2 > 1 \quad , \qquad (67)$$

which is denoted by R in Fig.2. We denote the other regions attributing to neither NA nor R by the adiabatic region shown as A in Fig.2. It is worth remarking that the concept of the adiabatic, non-adiabatic or resonance region means not the spatial region but the region in phase space.

We choose a point r_1 in the region A and consider the problem whether a particle starting from r_1 toward the region R will come back to r_1 adiabatically or not. Two typical cases are illustrated in Fig.2 ; the case (a) concerns the adiabatic reflection, where the particle remains always in the region A and its total energy W is nearly conserved;

$$W = K_r + \mu B_r + \frac{M}{4} \frac{(eE(r)/M)^2}{\omega^2 - \omega_c^2(r)} = \text{const.} \quad , (68)$$

μ being the magnetic moment. The case (b) is the non-adiabatic case where the particle with relatively high initial energy enters the region R and the increase of its energy amounts to a large value.

The boundary between the adiabatic and non-adiabatic reflection can be determined by the case where the trajectory (68) in the phase space touches the region R (67). This critical value W_0 of W can be determined by the condition that the equation

$$\frac{M}{2} \frac{R_0^2}{\omega^2} (\omega - \omega_c(r))^4 + \mu B_r(r) + \frac{M}{4} \frac{(eE(r)/M)^2}{\omega^2 - \omega_c^2(r)} = W_0 \qquad (69)$$

has the double root $r = r_0$. Provided that $E(r)$ is nearly constant in the vicinity of $r = r_0$, i.e., $E(r_0) = E_0$ and that the inequality

$$\omega - \omega_c(r_0) \quad << \quad \omega \tag{70}$$

holds, r_0 can be given by the relation

$$\omega - \omega_c(r_0) = \{\frac{2}{5M} \frac{\omega^2}{R_0^2} (W_0 - \mu B_0)\}^{1/4} . \tag{71}$$

Substitution of (71) into (69) yields

$$W_0 = W_c + \mu B_0 , \tag{72}$$

where W_c , the critical energy, is given by

$$W_c = \frac{5}{2^{21/5}} M\omega^2 R_0{}^2 (\frac{eE_0}{M\omega^2 R_0})^{8/5} , \tag{73}$$

or in the customary units,

$$W_c = 70 E_0{}^{8/5} B_0{}^{-6/5} R_0{}^{2/5} (M/M_p)^{3/5} , \tag{74}$$

where E_0 in V/cm, R_0 in cm, B_0 in G, W_c in eV and M_p is the proton mass. We note that E_0 is the value in the plasma not the externally applied value E_{ex}. The quantity W_c has an important meaning, namely, the "height of the adiabatic barrier". The adiabatic reflection condition, $W << W_0$, can be written explicitly for a particle starting from r_1 with the initial radial kinetic energy K_{r_1} and magnetic moment μ,

$$K_{r_1} \ll W_c + \mu(B_0 - B_{r_1}) , \quad (75)$$

provided

$$E(r_1) = 0. \quad (76)$$

The inequality (70) can be proved to hold, because this is expressed as, according to (71) and (72),

$$\frac{1}{2^{4/5}} \left(\frac{eE_0}{M\omega^2 R_0}\right)^{2/5} \ll 1 \quad (77)$$

which has already been satisfied by (63) and (55).

On the basis of the above analysis we show in Fig.3 the characteristic behaviours of a particle starting from r_1 toward the region R with the initial energies (K_{r_1} , $K_{\perp 1}$). The loss cone in the absence of the rf field is the above region of the line a) since the inequality

$$K_{r_1} + K_{\perp 1} > \mu B_{MAX} \quad (78)$$

is satisfied there. In the presence of the rf field the loss cone is shifted to the region above the broken line; it is noteworthy that the "bottom" of the modified loss cone is not zero but a finite value W_{MAX}. As will be shown in §6, it depends also on the phases of the gyration and the rf field in addition to the energies (K_{r_1} , $K_{\perp 1}$), whether particles reach the point of the maximum magnetic field or not; especially, the cases of finite $K_{\perp 1}$ have a relatively large dependences on these phases. The broken line in Fig.3 shows the value averaged over phases. The line b) in Fig.3 represents the relation obtained by changing the inequality sign in (75) into the equality one. The present theory predicts that a particle starting sufficiently below the line b) will be reflected adiabatically. The cross point of the K_{r_1} axis and the line b) corresponds to W_C , given by the expression (73), while the energy of such a point on the broken line is denoted by W_{MAX} and as it is difficult to treat analytically an empirical formula is obtained by numerical calculations (see Fig.9),

$$W_{MAX} = 0.8\ M\omega^2 R_0{}^2 \left(\frac{eE_0}{M\omega^2\ R_0}\right)^{1.4} \tag{79}$$

in the case $B_{MAX} = 1.5\ B_0$.

After the reflection by the rf barrier, a particle enters the non-adiabatic region, NA, and is scattered non-adiabatically. When it comes back to the initial point $r = r_1$, its radial and perpendicular kinetic energies, K_{r_1} and $K_{\perp 1}$, are in general essentially different from each initial values, but its total energy is conserved;

$$K_{r_1} + K_{\perp 1} = \text{const.} \quad , \tag{80}$$

Particles tend to jump into the loss cone due to this scattering. The particles in the dotted region in Fig.3 however remain sufficiently below the critical line (b) after some transits through the cusp center, hence particles in this region will have a long life time. We now introduce a normalized ion temperature by

$$\tilde{T}_i \equiv \frac{T_i}{W_c} \quad . \tag{81}$$

If $\tilde{T}_i \ll 1$, most of particles in the plasma belong to the dotted region in Fig.3, and then the adiabatic rf plugging of the plasma will be successfully achieved. We consider a deutron plasma of the reasonable data such as $R_0 = 1.4\text{m}$, $B_0 = 1.1 \times 10^5\text{G}$, $E_0 = 0.8 \times 10^6\text{V/cm}$, $T_i = 20\text{KeV}$, then we obtain the required inequality, $\tilde{T}_i = 10^{-2} \ll 1$. An estimation of the life time and a study about the feasibility of the thermonuclear reactor of this sheme will be given in a forthcoming paper.

We have to obtain the relation between E_0 and E_{ex} namely, the internal and the external strength of the rf field. The E_0 depends on E_{ex} and also on the normalized density

$$\tilde{n}_E = \omega_{pi}^2/\omega_{ci}^2 \quad \text{under the electrode,} \tag{82}$$

where ω_{pi} and ω_{ci} are the plasma frequency and the cyclotron frequency of ions. The relation between E_0 and E_{ex} is linear, provided the rf field is so weak as

$$\tilde{E}_0 \equiv \frac{eE_0}{M\omega^2\rho_i} = \frac{eE_0}{M\omega v_\perp} \ll 1. \tag{83}$$

The linearity condition (83) means that the oscillation amplitude $eE_0/M\omega^2$ due to the rf field is much less than the Larmor radius ρ_i or that the electric force eE_0 is much less than the Lorentz force $M\omega v_\perp$.

A series of experiments[7)~12)] have observed the loss flux Γ from the line cusp when the rf field is applied to the plasma steadily injected from one of the point cusps. The present theory gives an expression for the "decreasing factor" α of the loss flux, i.e.,

$$\alpha \equiv \frac{\Gamma(E_{ex} \neq 0)}{\Gamma(E_{ex} = 0)} \qquad (84)$$

which can be directly observed in the experiments. To calculate Γ, we assume the form for the ion distribution function,

$$f = \exp\{-\frac{M}{2T_i}(v_r{}^2 + v_z{}^2 + v_\theta^2)\} \quad g(p_\theta), \qquad (85)$$

at an arbitrary point where the rf field does not exist, for instance at $r = r_1$; p_θ is given by (19). The function $g(p_\theta)$ may be an arbitrary function if $g \in L^1(-\infty, +\infty)$. The loss flux Γ can be calculated by

$$\Gamma = \int_D d^3v \int dz \quad v_r \quad f \quad . \qquad (86)$$

If we assume the integration domain D as

$$v_r > 0, \quad K_r \geq W_c + (\frac{B_{MAX}}{B} - 1)K_{\perp} \quad , \tag{87}$$

then we readily obtain

$$\alpha = \exp(-\frac{W_c}{T_i}) = \exp(-\frac{1}{\tilde{T}_i}) \tag{88}$$

and we find

$$-\ell n\alpha \propto E_0{}^{8/5} ,$$

on the other hand the experimental results[16] shows

$$-\ell n\ \alpha \propto E_{ex}{}^{1.4} . \tag{89}$$

A Better fit will be obtained if we change the integration domain D in place of (87)

$$v_r > 0, \quad K_r \geq W_{MAX} + (\frac{B_{MAX}}{B} - 1)K_{\perp} \quad . \tag{90}$$

Then $\alpha = \exp(-\frac{W_{MAX}}{T_i})$, and $-\ell n\ \alpha \propto E_0{}^{1.4}$. This result is in good agreement with the experiment, if the linearity between E_0 and E_{ex} is justified. Using the experimental data, $E_{ex} = 50 \sim 100$ V/cm, B = 2700G, T_i = 10eV, then we have

$$\tilde{E}_{ex} = \frac{eE_{ex}}{M\omega^2\rho_i} \approx 1 \sim 2,$$

which does not satisfy the linearity condition (83) in the strict sense. The effects of nonlinearity and also of the z-dependence of the electric field have to be considered.

It should be noted that an estimation of a life time in the presence of the rf field by $1/\alpha$ times as that in the absence of the field will be an over-estimation, because particles under the nonadiabatic reflection escape rapidly and the use of the Maxwellian distribution is no longer justified. In the present experiments, however, steady supply of the Maxwellian plasma from the source is made, then the use of the Maxwellian distribution is justified and a good agreement of the experimental and the theoretical results is obtained.

§5. Non-Adiabatic Motion and Increase of Magnetic Moment

First we briefly treat a reduced equation of motion in the case where the non-resonance condition (45) breaks down but all other five conditions (26), (41), (62), (63), and (64) hold. We introduce a complex variable A defined by

$$A = \sqrt{\frac{Mc\mu}{e}}\exp[i\int_{t_i}^{t}\{\omega-\omega_c(r(t'))\}dt' + i(\phi_i - \theta_i)]$$
$$+ b_- - \frac{1}{i(\omega+\omega_c)}\frac{1}{2}\sqrt{\frac{M}{2\omega_c}}\frac{eE}{M}\ . \qquad (91)$$

By virtue of (40) and (41), we obtain the first order differential equation for A

$$\frac{d}{dt} A = i(\omega - \omega_c)A - \frac{\omega_c}{\omega + \omega_c} \sqrt{\frac{M}{2\omega_c}} \frac{eE}{M} . \tag{92}$$

To derive the equation for r, averaging out the high frequency terms in the right-hand side of eq.(18) and retaining terms with frequency $\omega - \omega_c$, we have

$$\frac{d^2}{dt^2} r = - \frac{\omega}{MR_0}|A|^2 + \frac{e}{M} E'(r) \frac{Im(A)}{\sqrt{2M \omega_c(r)}} . \tag{93}$$

If the non-resonance condition (45) holds, it is easy to derive the eq.(65) from (92) and (93). We give various expressions for the physical quantities of interest in terms of A:

$$\mu = \frac{e}{MC} \left| A - \frac{\omega_c(r)}{i(\omega^2-\omega_c^2(r))} \sqrt{\frac{M}{2\omega_c(r)}} \frac{eE(r)}{M} \right|^2, \tag{94}$$

$$<K_z> = \frac{\omega_c(r)}{2} \left| A + \frac{1}{i(\omega+\omega_c(r))} \frac{1}{2} \sqrt{\frac{M}{2\omega_c(r)}} \frac{eE(r)}{M} \right|^2, \tag{95}$$

$$<K_\theta> = \frac{\omega_c(r)}{2} |A|^2, \tag{96}$$

$$z = \sqrt{\frac{2}{M\omega_c(r)}} \{R_e(Ae^{-i\omega(t-t_i)-i\phi_i}) - \frac{1}{\omega_c(r)} \sqrt{\frac{M}{2\omega_c(r)}} \frac{p_\theta \omega}{MR_0} \} . \tag{97}$$

Next we consider the increase of the magnetic moment, $\Delta\mu$, when a particle starts at $t = t_i$ from $r = r_1$ with an initial magnetic moment μ and comes back at $t = t_f$ to the initial point after a reflection by the rf barrier. Assume $E(r_1) = 0$ as in §4, by virtue of (94),

$$\Delta\mu = \frac{e}{MC} |A(t_f)|^2 - \mu \tag{98}$$

and by partially integrating the solution (43) we have

$$b_-(t_f) = -e^{i\int_{t_i}^{t_f}(\omega-\omega_c)dt'} \int_{t_i}^{t_f} \frac{1}{2}\sqrt{\frac{M}{2\omega_c}} \frac{eE}{M} e^{-i\Phi(t')} dt' . \tag{99}$$

On account of (98) and (99) together with (91),

$$\Delta\mu = -2\sqrt{\frac{e\mu}{MC}} \int_{t_i}^{t_f} dt' \frac{1}{2}\sqrt{\frac{M}{2\omega_c}} \frac{eE}{M} R_e[\exp\{-i\int_{t_t}^{t'}(\omega-\omega_c)dt'' + i\Theta\}]$$
$$+ \frac{e}{MC}\Big| \int_{t_i}^{t_f} dt' \frac{1}{2}\sqrt{\frac{M}{2\omega_c}} \frac{eE}{M} \exp\{-i\int_{t_t}^{t'}(\omega-\omega_c)dt''\}\Big|^2 . \tag{100}$$

Here Θ is the phase difference of the gyration of a particle and the rf field at the time of turning point, t_t. The main contribution to the integral comes from around the turning point, hence we may tend t_i and t_f to $-\infty$ and $+\infty$, respectively. In terms of the dimensionless variables

$$\left.\begin{aligned} \bar{\mu} &= \frac{c\mu}{e\omega R_0{}^2} , \\ \bar{E}_t &= \frac{eE(r_t)}{M\omega^2 R_0} , \\ \bar{r}_t &= r_t/R_0 , \end{aligned}\right\} \tag{101}$$

suffix t indicating the turning point, we have

$$\Delta\bar{\mu} = -\sqrt{\frac{\bar{\mu}}{2\bar{r}_t}}\ \bar{E}_t\ I\cos\Theta + \left(\frac{1}{2}\frac{1}{\sqrt{2\bar{r}_t}}\ \bar{E}_t\ I\right)^2, \qquad (102)$$

where

$$I = \omega\int_{-\infty}^{\infty} dt'\ \exp\left\{-i\int_{t_t}^{t'}(\omega-\omega_c)dt''\right\}. \qquad (103)$$

The exponent in the integral may be expanded around the turning time:

$$\int_{t_t}^{t'}(\omega-\omega_c)dt'' = \frac{\omega}{R_0}(R_0-r_t)(t'-t_t) - \frac{\omega}{R_0}\frac{\ddot{r}_t}{6}(t'-t_t)^3 + \cdots. \qquad (104)$$

In the case under consideration, turning point is within the resonance point, i.e., $r_t < R_0$; after the transformation $t' - t_t = \tau\sqrt{(R_0 - r_t)/(-\ddot{r}_t/2)}$, (103) takes the form

$$I = \frac{\nu}{1-\bar{r}_r}\int_{-\infty}^{\infty} d\tau\ e^{-i\nu(\tau+\frac{\tau^3}{3})},$$

$$\nu \equiv \frac{\omega}{R_0}\sqrt{(R_0-r_t)^3/(-\ddot{r}_t/2)} \sim \frac{1}{\eta} >> 1, \qquad (105)$$

where η is a small positive constant defined in §3. By utilizing the assymptotic form,

$$\int_{-\infty}^{\infty} d\tau\ \exp\left\{-i\nu(\tau+\frac{\tau^3}{3})\right\} \sim \sqrt{\frac{\pi}{\nu}}\exp\left(-\frac{2}{3}\nu\right), \quad \text{for } \nu >> 1$$

we finally write down the expression for $\Delta\bar{\mu}$

$$\Delta\bar{\mu} = -\sqrt{\bar{\mu}} \frac{1}{\sqrt{2\bar{r}_t}} \bar{E}_t \frac{1}{1-\bar{r}_t} \sqrt{\pi\nu} \exp(-\frac{2}{3}\nu) \cos\theta$$

$$+ \{ \frac{1}{2} \frac{1}{\sqrt{2\bar{r}_t}} \bar{E}_t \frac{1}{1-\bar{r}_t} \sqrt{\pi\nu} \exp(-\frac{2}{3}\nu) \}^2 \quad . \tag{106}$$

In the specified case where

$$\frac{1}{8} \bar{E}_t{}^2 \quad << \quad W/M\omega^2 R_0^2 \quad - \bar{\mu} \quad << W_c/M\omega^2 R_0{}^2 = 0.27 \bar{E}_t{}^{8/5} \ ,$$

ν is given by

$$\nu = \frac{4}{5} \frac{W_c}{W - \frac{Mc\mu\omega}{e}} \ . \tag{107}$$

§6. Numerical Calculation

In this section we give the results of the numerical solution of the eqs. (17) through (19). Calculations are limited to the motions in the line cusp field when $p_\theta = 0$, namely gyration centers always stay on the r-axis. As in §4, the problem is to obtain the increase of the energies K_{r_1} and $K_{\perp 1}$, between the initial and the final states, when a particle starts from the initial point $r = r_1$, is then reflected by the rf barrier, and finally comes back to the initial point. In the case of the ideally adiabatic motion, the increase of energies is equal to zero.

The form of the rf electric field in the plasma has to be determined by a self-consistent theory, but in the present calculations we give a priori an reasonable form. The electrostatic potential of the field is assumed to take the form

$$\phi(r,z,t) = - E_0 \; F(r) \; G(r,z) \sin(\omega t + \phi_i), \tag{108}$$

where

$$F(r) \begin{cases} = 0 \, , & \frac{r}{R_0} < 0.5 \\ = 7.5(\frac{r}{R_0} - 0.5) - 250(\frac{r}{R_0} - 0.6)^3 - 0.25, & 0.5 < \frac{r}{R_0} < 0.7 \\ = 1 \, , & 0.7 < \frac{r}{R_0} \end{cases} \tag{109}$$

the function F being shown in Fig.(4-a), and

$$G(r, z) = \varepsilon_s z - \frac{2\rho_0 R_0}{r} (\varepsilon_s - 1) \tanh(\frac{zr}{2\rho_0 R_0}), \tag{110}$$

or the derivative $\frac{\partial G}{\partial z}$ (see Fig.(4-b)) is

$$\frac{\partial}{\partial z} G(r, z) = \varepsilon_s - (\varepsilon_s - 1) \operatorname{sech}^2 (\frac{zr}{2\rho_0 R_0}) \;, \tag{111}$$

ρ_0 being the Larmor radius $\sqrt{\frac{T}{M}}/\omega$. The form of the standing wave of the rf field in the "sheet plasma" is

$$E_z = - \frac{\partial \phi}{\partial z} = E_0 \; F(r) \frac{\partial G}{\partial z} \sin(\omega t + \phi_i),$$

$$E_r = - \frac{\partial \phi}{\partial r} = E_0 (\frac{\partial F}{\partial r} G + F \frac{\partial G}{\partial r}) \sin(\omega t + \phi_i), \tag{112}$$

$$E_\theta = 0 \; .$$

The average radius from the cusp center, r_E, of the electrode is chosen to be $0.7R_0$, implying that $\omega \sim 1.4\omega_{ci}(r_E)$. The strength E_0 of the rf field in the sheet plasma is related to the external strength E_{ex} as

$$E_{ex} = \varepsilon_s E_0 . \qquad (113)$$

In the numerical calculations given below, the "screening parameter" ε_s is chosen to be unity, i.e., $\frac{\partial E_z}{\partial z} = 0$ except in Fig.10. The increase of the perpendicular energy at $r = r_1$, $K_{\perp 1}$, is shown in Fig.5, where the initial value of $K_{\perp 1}$ is fixed to be zero ($\mu = 0$) and the initial value of K_{r_1} is varied. In Fig.6, the case of a finite initial $K_{\perp 1}$, $K_{\perp 1}/M\omega^2 R_0{}^2 = 10^{-4}$, is shown; in contrast with the case $K_{\perp 1}=0$, the results depend on the gyration phase θ_i. The maximum, minimum and averaged values of $\Delta K_{\perp 1}$ are plotted; these values are obtained by twenty trials of different ϕ_i's for each K_{r_1}. The theoretical predictions given by (106) are compared with the computational results. It is confirmed in Figs.5 and 6 that the increase of perpendicular energy, $\Delta K_{\perp 1}$, is certainly infinitesimal if $K_{r_1} << W_c$.

Fig.7 deals with the comparison of theoretical and computational results for the turning points. The theoretical results are in good agreement with the computational results if $K_{r_1} << W_c$. The bars illustrate the deviations due to phase differences.

The computational results are given in Fig.8 to confirm the theoretical prediction of various aspects of reflection by the rf barrier (see Fig.4). The marginal energy of the reflected particle within the maximum magnetic field depends on the phase;the maximum and the minimum marginal energies are shown and the averaged marginal energy is illustrated by the broken line. The increase of perpendicular energy averaged over the phase is plotted. The theoretical boundary line(b) between the adiabatic and the non-adiabatic reflection is very close to the contour line of $\langle \Delta K_{\perp 1} \rangle / (K_{r1} + K_{\perp 1}) = 1.0$.

To confirm the theoretical prediction $W_c \propto E_0{}^{8/5}$, the calculations are performed by varying E_0 with $K_{\perp 1}$ being fixed to zero. In Fig.9, the increase of the energy and the turning point are shown in $(E_0\ ,\ K_{r1})$ space. For $eE_0/M\omega^2 R_0 \lesssim 10^{-2}$, the line of W_c is almost parallel to the contour line of $\langle \Delta (K_{r1} + K_{\perp 1}) \rangle / (K_{r1} + K_{\perp 1}) = \text{const.}$; the line of $r_t/R_0 = 0.9$ $((\omega - \omega_c(r_t))/\omega = 0.1)$ is almost identical with a contour line of $\langle \Delta (K_{r1} + K_{\perp 1}) \rangle / (K_{r1} + K_{\perp 1}) = 1.0$.[17]

The theory and the numerical calculation have been so far performed under the assumption that the rf electric field is homogeneous in the direction of z, i.e., $\varepsilon_s = 1$. If $\varepsilon_s \neq 1$, large spatial gradient of the electric field,

$$\frac{1}{E_z}\frac{\partial E_z}{\partial z} \sim \frac{1 - \varepsilon_s}{\rho_i} ,$$

emerges along z-direction. As it is generally much larger than the gradient along the r-direction,

$$\frac{1}{E_z}\frac{\partial E_z}{\partial r} \sim \frac{1}{0.2\ R_0} ,$$

one may be afraid that the adiabatic confinement breaks down. In Fig.10, the cases of $\varepsilon_s = 0.5, 1, 2$ are compared with each other. Two typical cases are shown where the effect of the spatial gradient along the z direction is sensitive or not sensitive. The case $\varepsilon_s = 2$, $eE_0/M\omega^2R_0 = 10^{-2}$ is a sensitive case. The profile of the electric field is intermediate between the cases $\varepsilon_s = 1$, $eE_0/M\omega^2R_0 = 10^{-2}$ and $\varepsilon_s = 1$, $eE_0/M\omega^2R_0 = 2 \times 10^{-2}$, but the value of $<\Delta(K_{r1} + K_{\perp 1})>$ is much larger than those of the both cases. A non-sensitive case is the case $\varepsilon_s = 0.5$, $eE_0/M\omega^2R_0 = 10^{-2}$, where the results are not fundamentally different from the flat case $\varepsilon_s = 1$, $eE_0/M\omega^2R_0 = 10^{-2}$. In the real sheet plasma, the profile with ε_s of non-unity is inevitable, then we must choose such an insensitive case to realize the adiabatic rf plugging.

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Numerical calculations were carried out on the HITAC 8500 computer system of the computational center of the Institute of Plasma Physics.

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Figure Captions

Fig.1 A simple cusp field with the symmetry z-axis. The resonant point, $\omega=\omega_c$, exists outside the electrodes.

Fig.2 Resonance (R), Nonadiabatic (NA), and Adiabatic (A) regions.

Fig.3 Characteristic behaviours of a particle starting from $r=r_1$ toward the rf barrier with the initial energies $(K_{r_1}, K_{\perp 1})$. The line a), $K_{r_1}+K_{\perp 1}=\mu B_{MAX}$, is the loss cone in the absence of the rf field. In the presence of the rf field, the loss cone is shifted to the broken line. The line b), $K_{r_1}=W_c+\mu(B_0-B_{r_1})$, is the boundary between the nonadiabatic and adiabatic reflections. Particles in the dotted region sufficiently below the line c) will be reflected adiabatically indefinitely even after successive nonadiabatic scatterings at the cusp center.

Fig.4 The form of the rf field in the plasma is assumed to be $E_z(r,z) = E_0 F(r) \frac{\partial G(r,z)}{\partial z}$, where F and $\frac{\partial G}{\partial z}$ are shown in a) and b), respectively.

Fig.5 The increase of the perpendicular energy, $\Delta K_{\perp 1}$, after a reflection vs the initial value of K_{r_1}, where initial magnetic moment is zero, $\mu=0$. Theoretical values given by (106) are also shown. The bar means the deviation due to the phase difference.

Fig.6 The maximum, minimum and averaged values of the increase of the perpendicular energy vs. the initial value of K_{r_1}, where the initial magnetic moment is finite. The encircled marks mean the negative values. The initial phases of gyration of a particle take two typical values, $\theta_i=0$, $\pi/2$, while the initial phase of the rf field, ϕ_i, takes 20 different values for each θ_i. The theoretical results are also shown.

Fig.7 The numerically obtained turning points, r_t, are compared with the theoretical results given by (68).

Fig.8 Theoretical predictions for various aspects of reflections shown in Fig.3 are confirmed in this figure. The boundary between the loss and reflection of a particle is broadened due to the phase difference, and the averaged line are shown by the broken line. The phase averaged value of the increase of the perpendicular energy is plotted.

Fig.9 The turning points and the increase of the kinetic energy averaged over the phase are shown in (K_{r_1}, E_0) space. The theoretical prediction, $W_c \propto E_0{}^{8/5}$, is confirmed well for $eE_0/M\omega^2 R_0 \lesssim 10^{-2}$.

Fig.10 The increase of energy after a reflection by the rf barrier is studies under the non-uniform rf field along the z-direction, $\varepsilon_s \neq 1$.

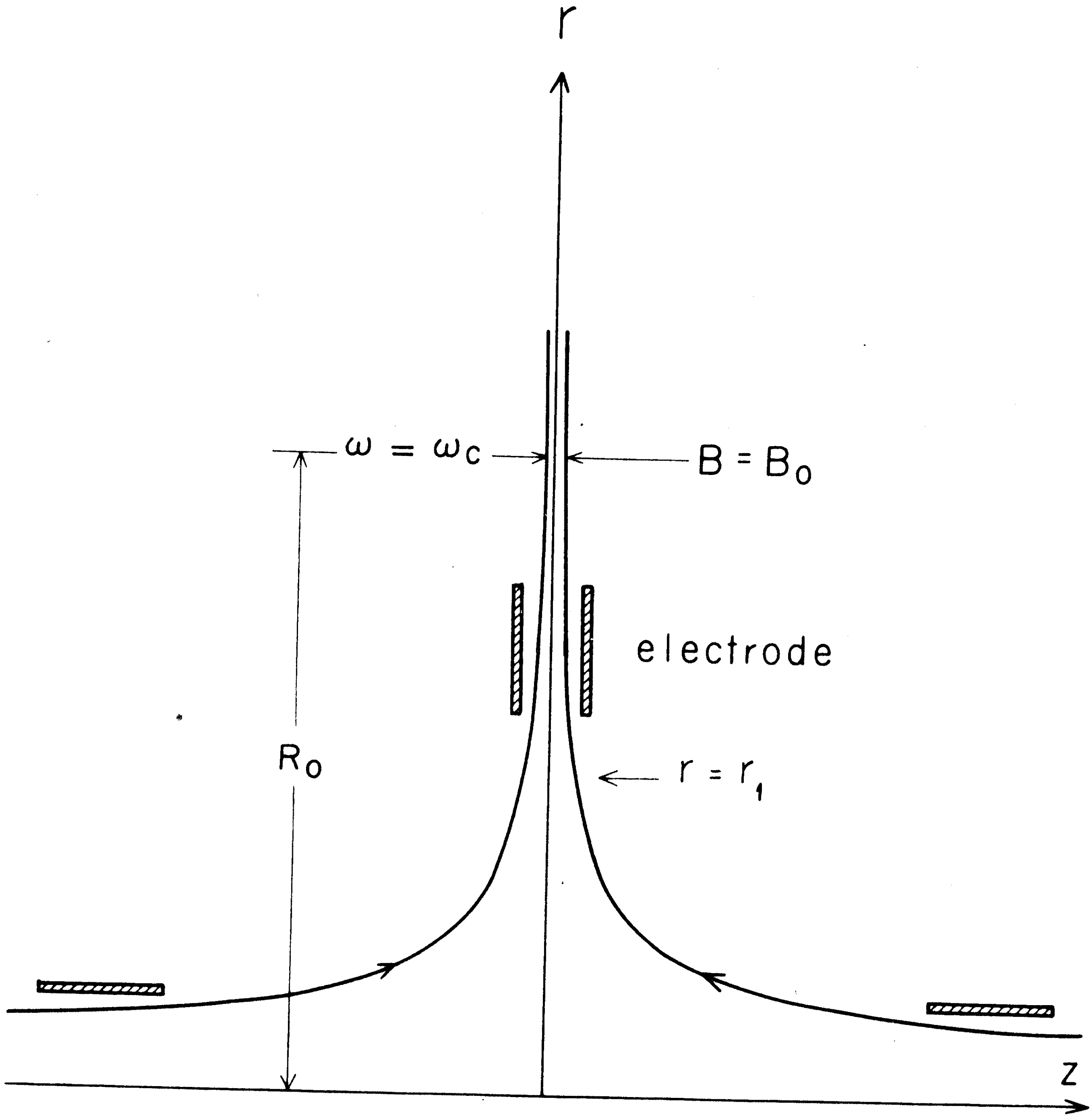


Fig. 1

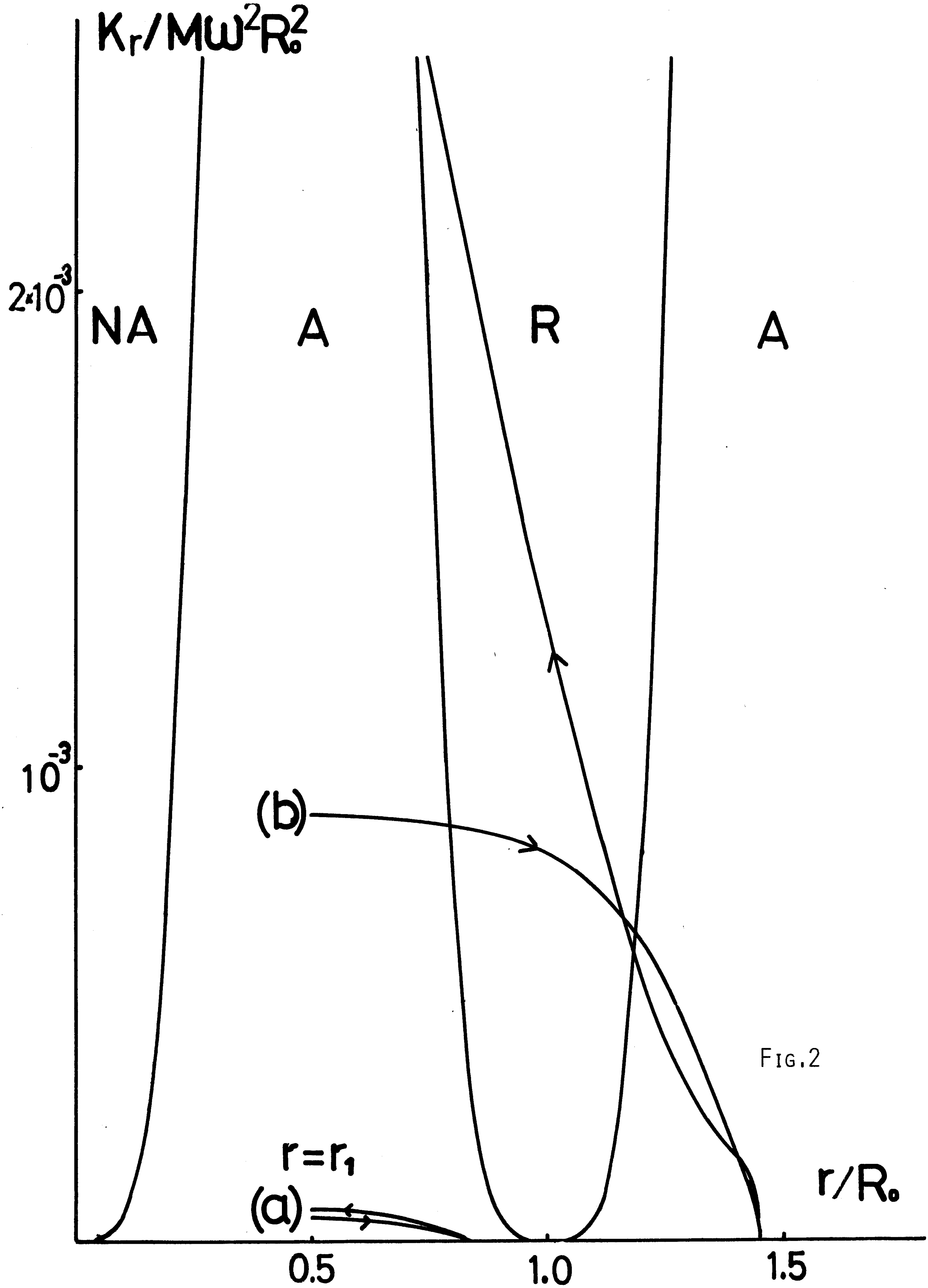


FIG.2

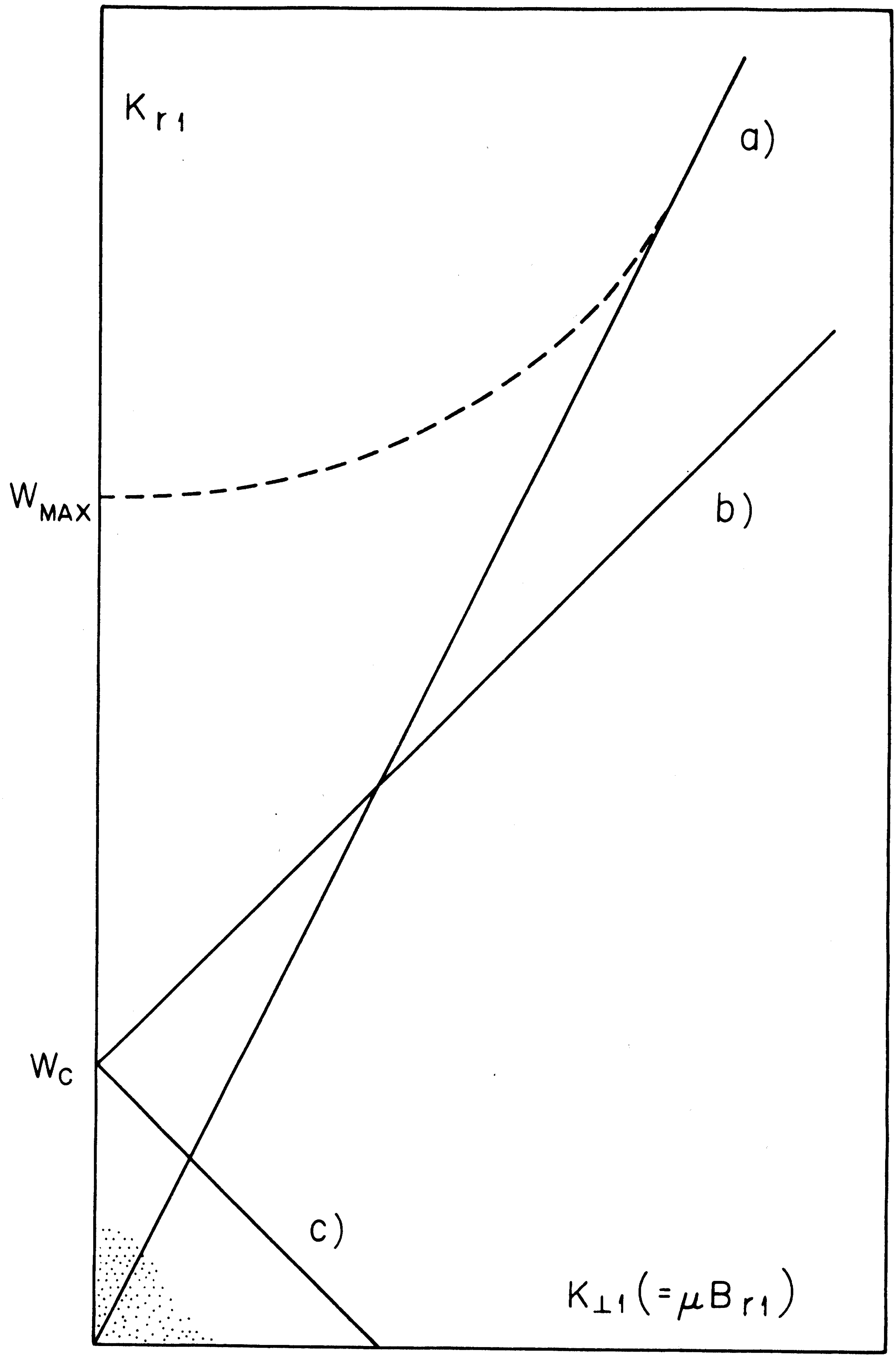


Fig. 3

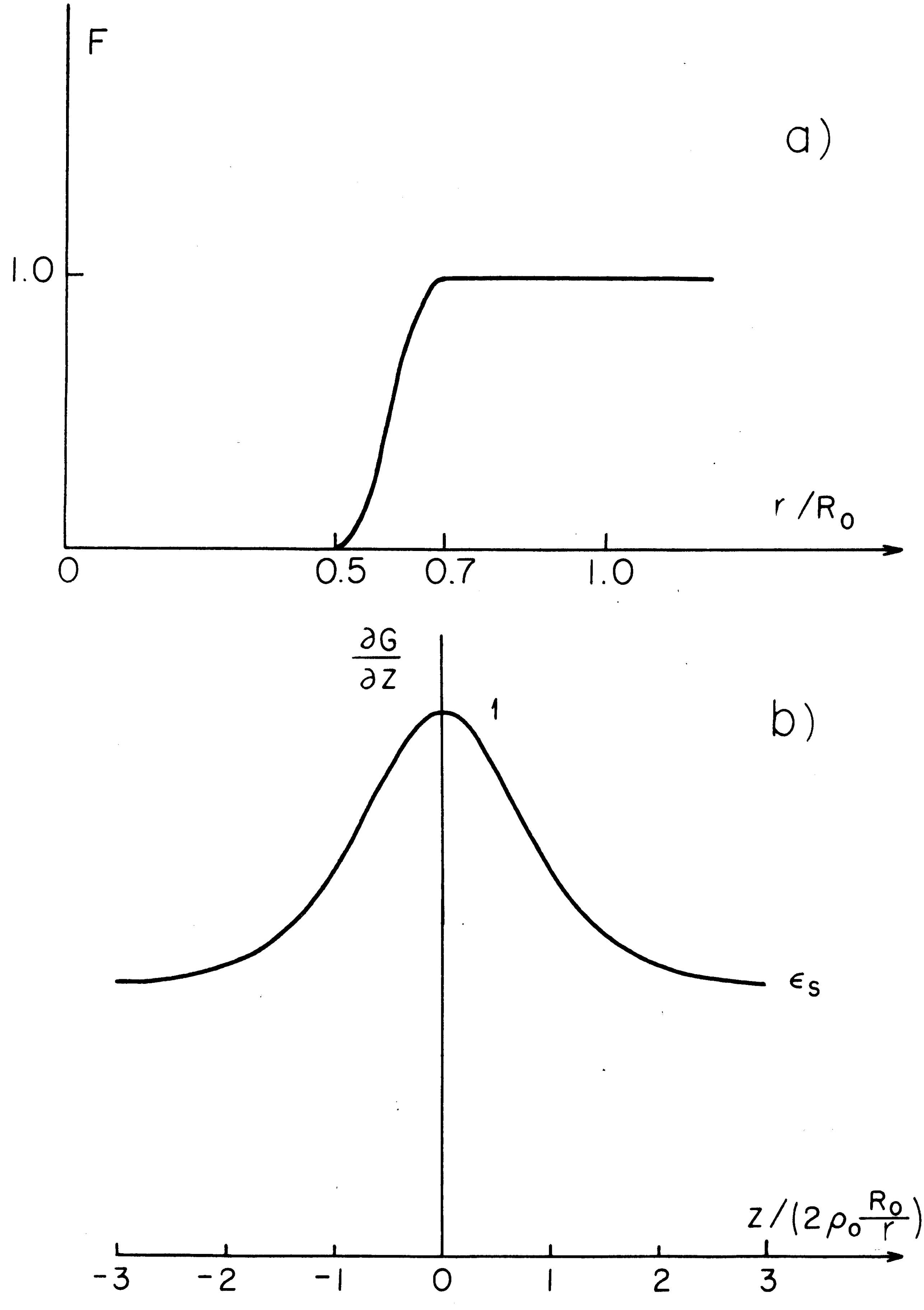
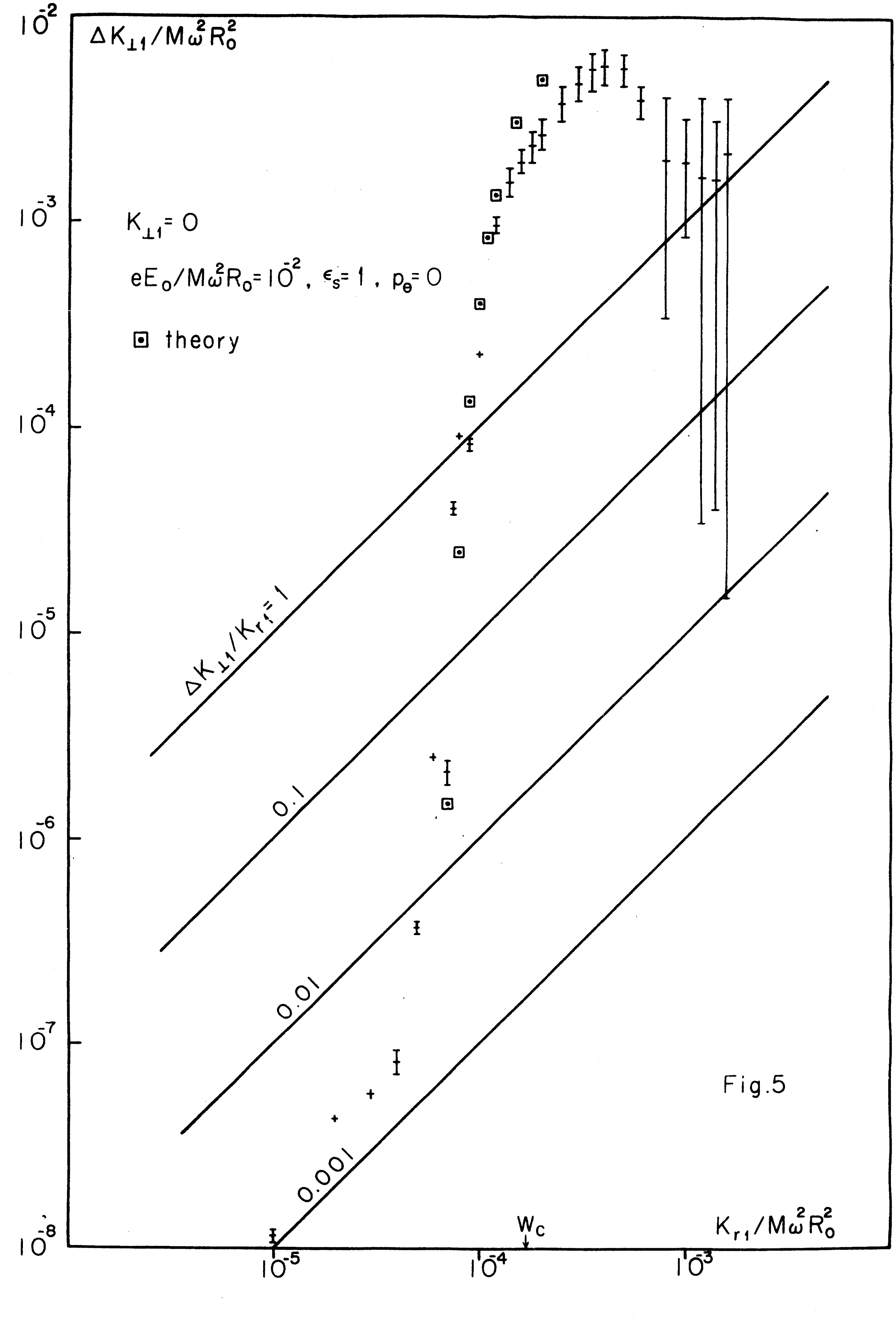


Fig. 4



10^{-2}
$\Delta K_{\perp 1}/M\omega^2 R_0^2$
10^{-3}
$K_{\perp 1} = 0$
$eE_0/M\omega^2 R_0 = 10^{-2}$, $\epsilon_s = 1$, $p_\theta = 0$
theory
10^{-4}
10^{-5}
$\Delta K_{\perp 1}/K_{r1} = 1$
0.1
10^{-6}
0.01
10^{-7}
0.001
10^{-8}
W_c
$K_{r1}/M\omega^2 R_0^2$
10^{-5}
10^{-4}
10^{-3}

Fig.5

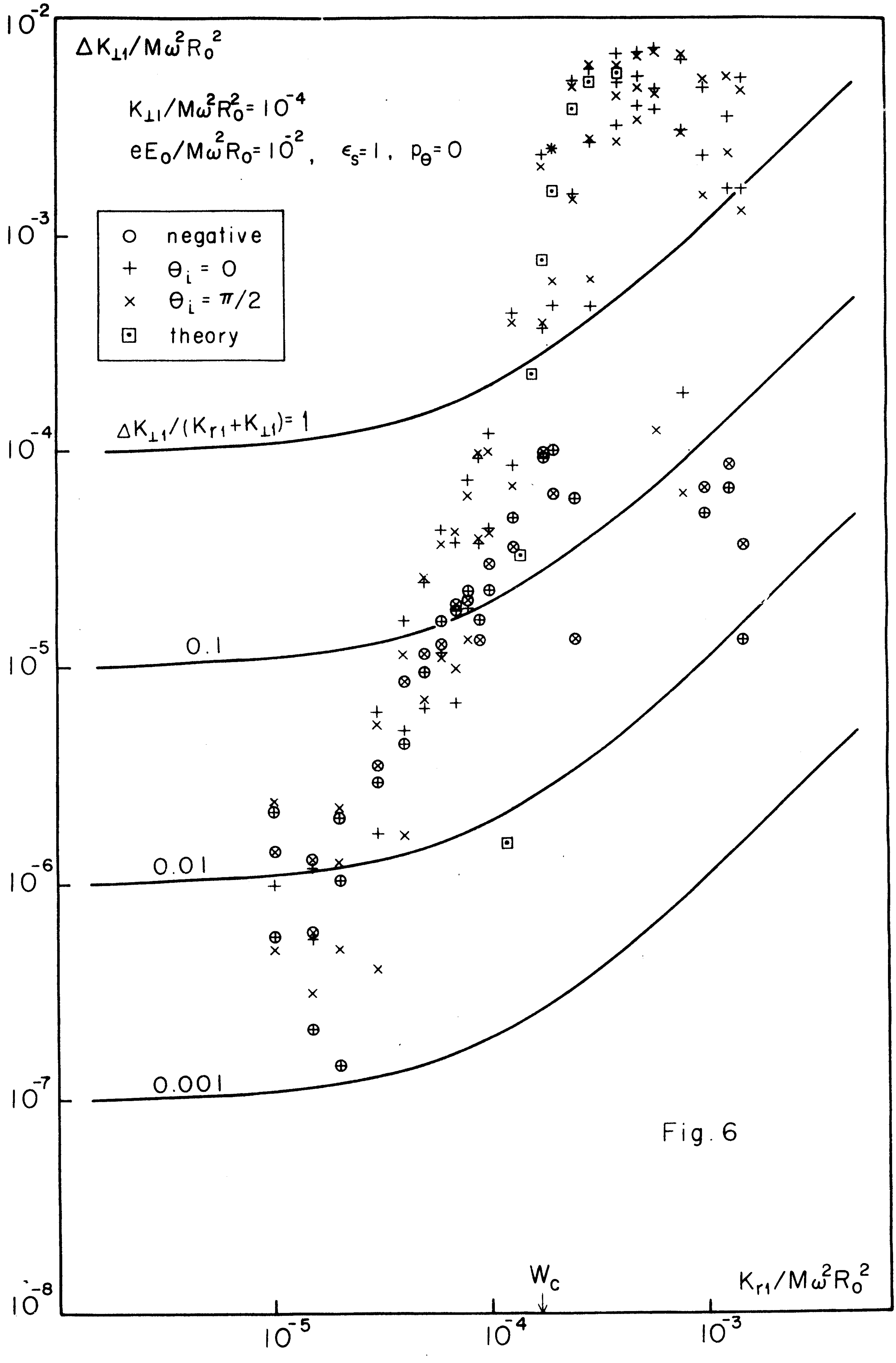

$\Delta K_{\perp 1}/M\omega^2 R_0^2$
$K_{\perp 1}/M\omega^2 R_0^2 = 10^{-4}$
$eE_0/M\omega^2 R_0 = 10^{-2}$, $\epsilon_s = 1$, $p_\theta = 0$
○ negative
+ $\theta_i = 0$
× $\theta_i = \pi/2$
⊡ theory
$\Delta K_{\perp 1}/(K_{r1}+K_{\perp 1}) = 1$
0.1
0.01
0.001
W_c
$K_{r1}/M\omega^2 R_0^2$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-5}
10^{-4}
10^{-3}

Fig. 6

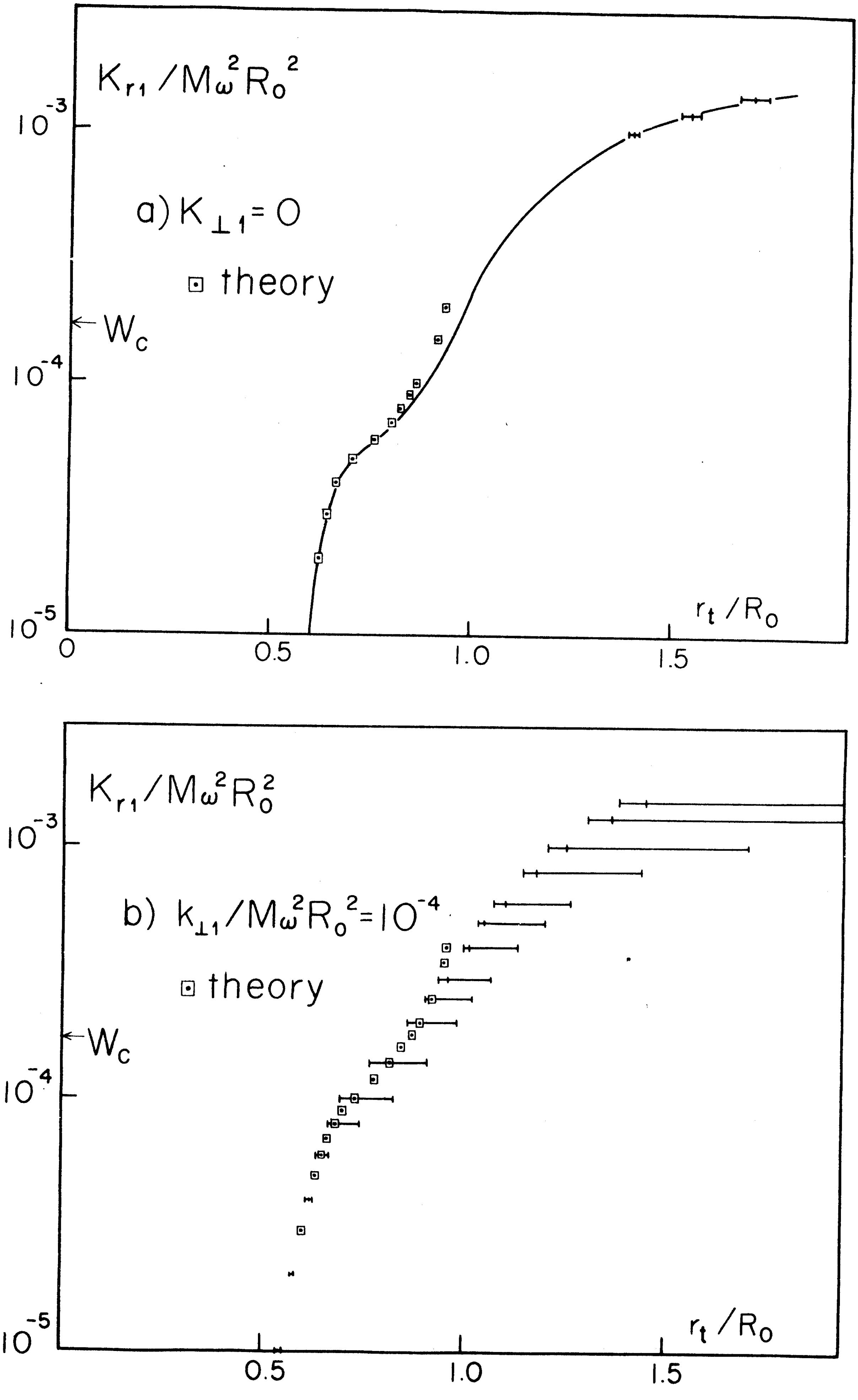


Fig. 7 $eE_0/M\omega^2 R_0=10^{-2}$, $\epsilon_s=1$, $p_\theta=0$

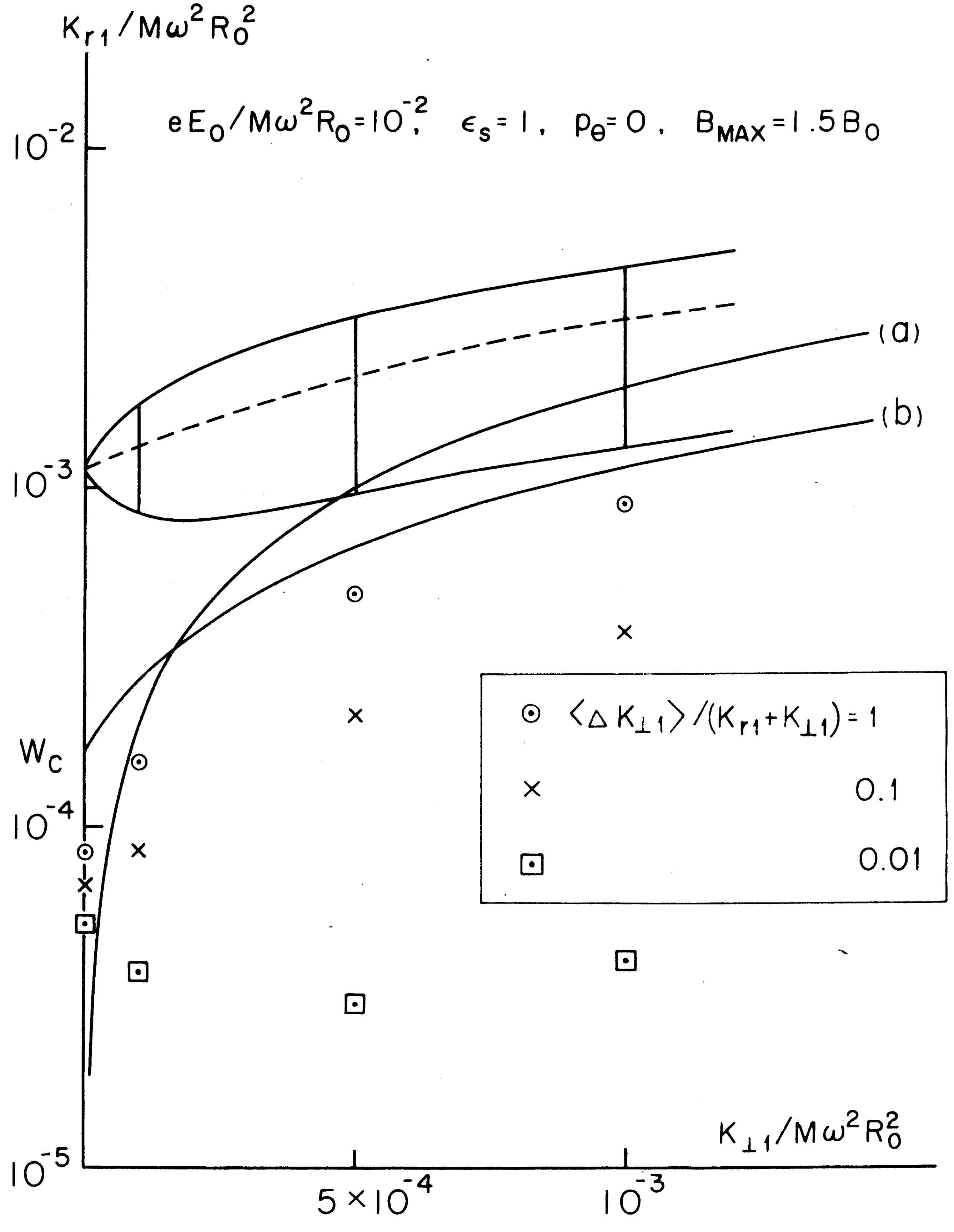
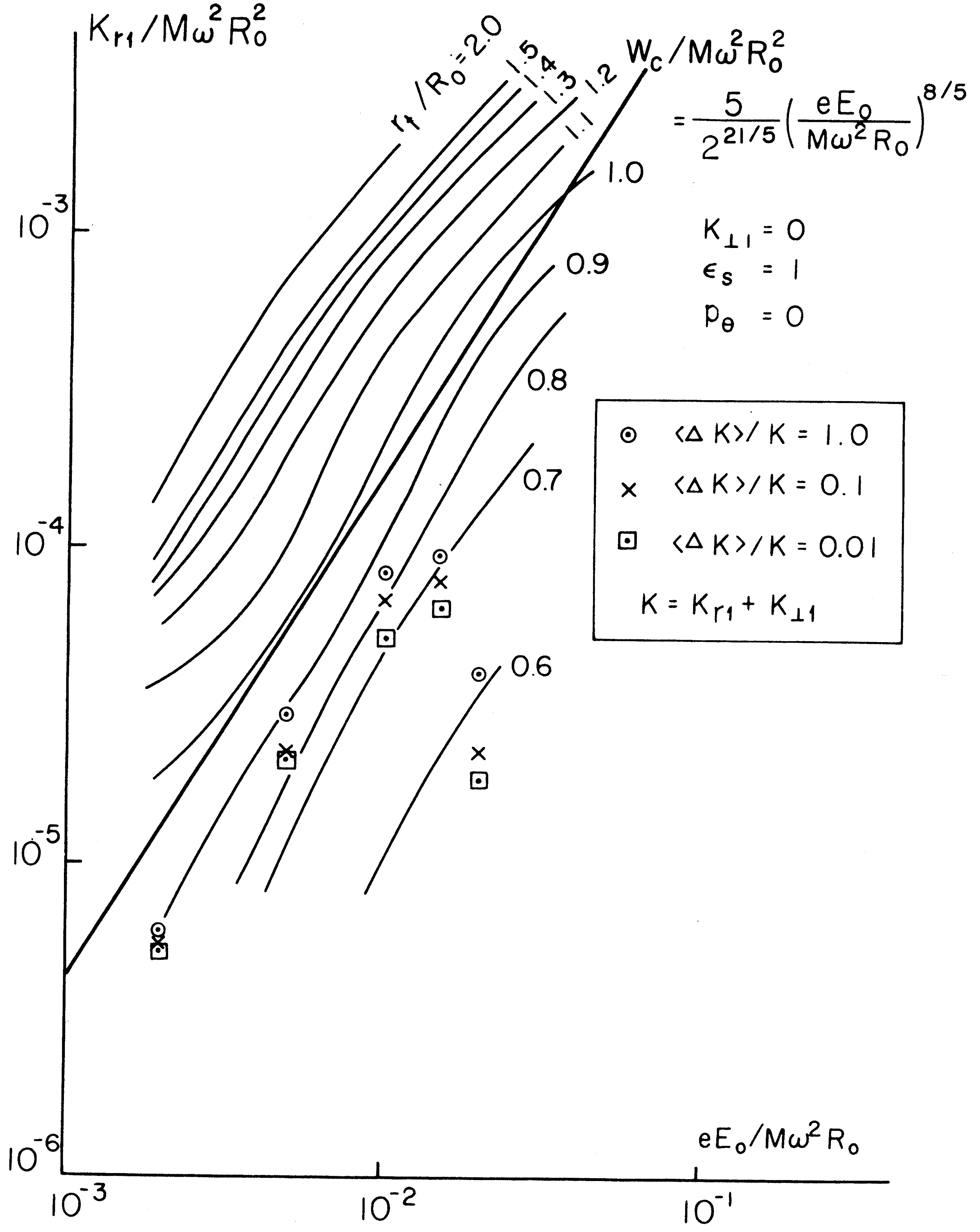
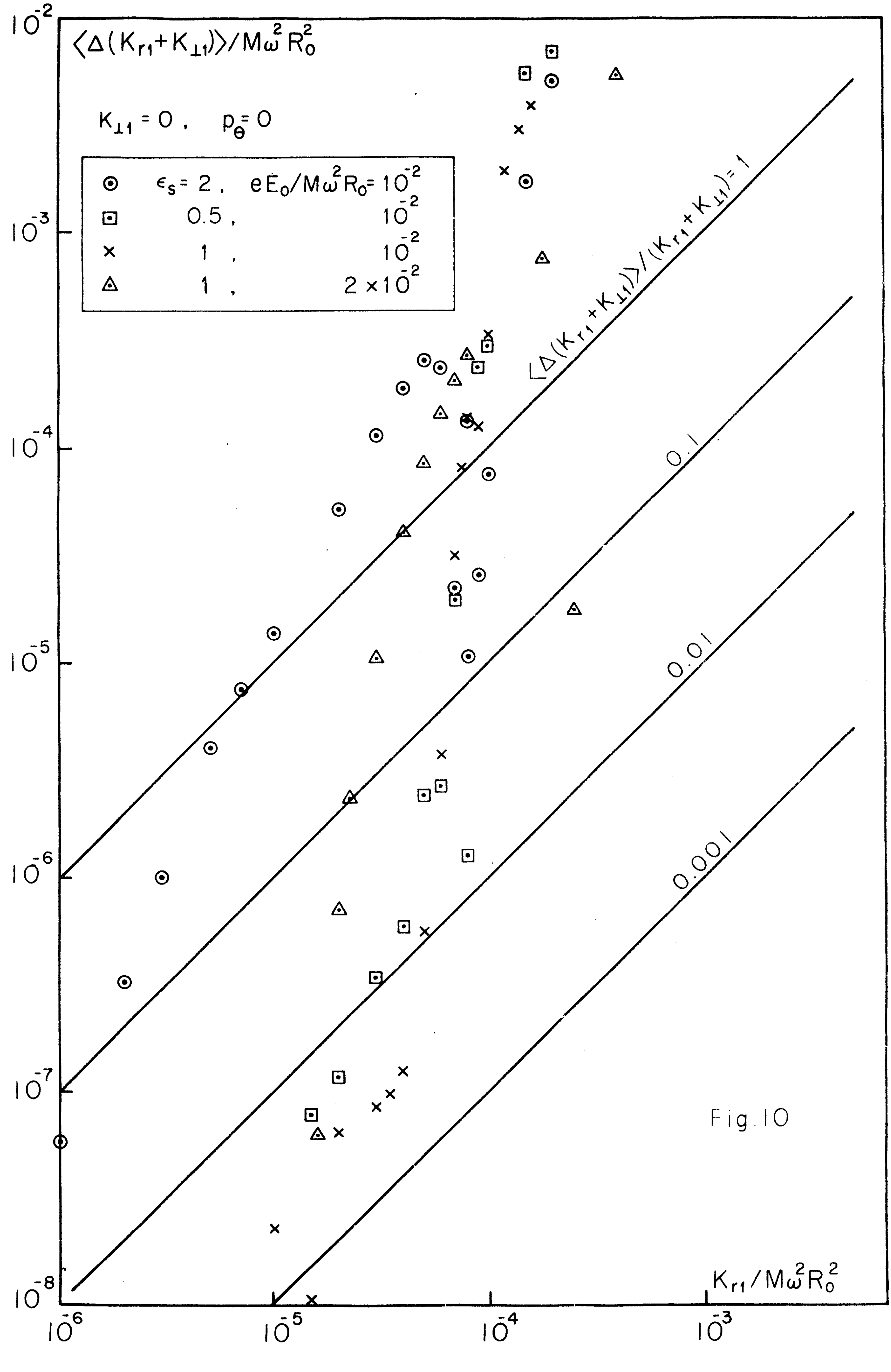

$K_{r1}/M\omega^2 R_0^2$
$eE_0/M\omega^2 R_0 = 10^{-2}$, $\epsilon_s = 1$, $p_\theta = 0$, $B_{MAX} = 1.5 B_0$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
W_c
(a)
(b)
$\langle \Delta K_{\perp 1} \rangle / (K_{r1} + K_{\perp 1}) = 1$
0.1
0.01
$K_{\perp 1}/M\omega^2 R_0^2$
5×10^{-4}
10^{-3}

Fig. 8



$K_{r1}/M\omega^2 R_0^2$
$r_1/R_0 = 2.0$
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
$W_c/M\omega^2 R_0^2$
$= \frac{5}{2^{21/5}}\left(\frac{eE_0}{M\omega^2 R_0}\right)^{8/5}$
$K_{\perp 1} = 0$
$\epsilon_s = 1$
$p_\theta = 0$
$\langle \Delta K \rangle / K = 1.0$
$\langle \Delta K \rangle / K = 0.1$
$\langle \Delta K \rangle / K = 0.01$
$K = K_{r1} + K_{\perp 1}$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$eE_0/M\omega^2 R_0$
10^{-3}
10^{-2}
10^{-1}

Fig. 9



$\langle \Delta(K_{r1}+K_{\perp 1})\rangle / M\omega^2 R_0^2$
$K_{\perp 1}=0$, $p_\theta=0$
$\epsilon_s = 2$, $eE_0/M\omega^2 R_0 = 10^{-2}$
0.5, 10^{-2}
1, 10^{-2}
1, 2×10^{-2}
$\langle \Delta(K_{r1}+K_{\perp 1})\rangle / (K_{r1}+K_{\perp 1}) = 1$
0.1
0.01
0.001
$K_{r1}/M\omega^2 R_0^2$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-6}
10^{-5}
10^{-4}
10^{-3}

Fig. 10