INSTITUTE OF PLASMA PHYSICS

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RESEARCH REPORT

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Enhancement of Damping of Large Amplitude Wave

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Abstract

Evaluating the exact population difference between resonant electrons with velocities smaller and larger than the phase velocity, a correction to Landau damping proportional to the square of the wave amplitude is obtained theoretically.

In a plasma, energy exchange between plasma waves and particles is, in many cases, made through Landau damping and inverse Landau damping. When we consider plasma heating by an intense longitudinal wave or by plasma turbulence, the absorption of wave energy by the particles may occur through Landau damping. Even in laser heating of plasma, Landau damping may play an important role because an intense transverse wave produces a longitudinal wave in a plasma through the so-called parametric effect and also because a strongly focused transverse wave yields a longitudinal component even in vacuum.[1]

It is well known that an infinitesimal amplitude electron wave damps exponentially.[2] When the initial amplitude is small but finite, O'Neil[3] predicted that the wave damps more slowly than Landau damping and that after one half bounce time $\pi\omega_B^{-1} \equiv \pi(m/ekE_o)^{1/2}$ the damping coefficient changes sign or the wave starts growing. This phenomenon is well understood in terms of particle trapping in the wave potential. Here, m and e are the mass and the charge of the electron and k and E_o denote the wave number and the initial electric field of the wave, respectively.

On the other hand, there is a prediction that at first the wave damps faster than Landau damping, and when the initial amplitude is larger than a critical value it again grows.[4] This is based on the following considerations. In estimating the difference between the number of resonant particles with velocities smaller and larger than the phase velocity λ, we have to take into account the exact shape of a distribution $f_o(\tilde{v})$ around λ. In the calculation of the usual Landau damping, however, the difference is assumed to be proportional to $\partial f_o(\lambda)/\partial\tilde{v}$. This differnece is smaller than the exact one; hence, the actual damping becomes larger than the Landau one, provided $f_o(\tilde{v})$ is Maxwellian and $\lambda >$ the thermal velocity v_T.

This phenomenon is seen in computer simulations by Knorr[5] and Armstrong[6]. Experimentally, Kawai and Kondo[7] show an enhanced damping of electron waves and Sato[8] of ion waves.

Here we present an analytical expression of the deviation from linear Landau damping. We consider a collisionless electron wave, and use the Lagrangian method to express the behavior of particles, and Ampére's law, that is,

$$\frac{d\tilde{v}}{dt} = -E(\tilde{x},t), \qquad \frac{d\tilde{x}}{dt} = \tilde{v} \tag{1}$$

$$\frac{\partial E(\tilde{x},t)}{\partial t} = \int \tilde{v}\ f(\tilde{x},\tilde{v},t)\ d\tilde{v}. \tag{2}$$

Now f is constant along the characteristics (1), that is, $f(\tilde{x},\tilde{v},t) = f[\tilde{x}(o),\tilde{v}(o),o]$, where $\tilde{x}(o)$ and $\tilde{v}(o)$ should be expressed in terms of $(\tilde{x},\tilde{v},t)$. The normalizations are as follows: t, $\tilde{x}$, and $\tilde{v}$ are scaled with the inverse of the plasma frequency, the Debye length L_D, and the thermal velocity $v_T = \sqrt{T/m}$, respectively, hence, E and f are measured with $4\pi e n_o L_D$ and v_T^{-1}. We assume that the field $E(\tilde{x},t)$ can be written as

$$E(\tilde{x},t) = E(t) \sin kx \tag{3}$$

in the wave frame which is defined by $x = \tilde{x} - \lambda t$. Substituting (3) into (2), multiplying the resultant expression by sin kx and integrating over x we have

$$\frac{dE(t)}{dt} = -\frac{1}{\pi}\int_{-\infty}^{\infty} dv_o \int_{-\pi/k}^{\pi/k} dx_o (v+\lambda) f[x_o+\lambda t,\ v_o+\lambda,\ o] \sin kx \tag{4}$$

where the conservation of phase space $dx\,dv = dx_o dv_o$ has been used and $v = \tilde{v} - \lambda$, $x_o = x(o)$ and $v_o = v(o)$[9]. Equations (1) have a formal solution $x = x_o + v_o t + \Delta x(t)$ where

$$\Delta x(t) = \int_o^t (t'-t)E(t')\sin k[x_o+v_o t'+\Delta x(t')]\ dt' \tag{5}$$

Assuming $k\Delta x$ is less than unity(which means $|k\Delta x| < kE_o t^2/2 = (\omega_B t)^2/2 < 1$) we obtain an explicit expression for Δx by iteration and substitute the resultant expression for x into Eq.(4) where sin kx is expanded up to $O((k\Delta x)^3)$.

We choose $f[\tilde{x}(o),\ \tilde{v}(o),\ o]$ as

$$f[\tilde{x}(o),\ \tilde{v}(o),\ o]=f_M(v(0)+\lambda)[1+\varepsilon \cos kx(0)]. \tag{6}$$

where f_M is a Maxwellian distribution and ε is kE_o, which provides the initial field $E(\tilde{x},0) = E_o \sin k\tilde{x}$.[6]

The calculation is rather tedious but straight-forward. Terms proportional to E correspond to linear Landau damping, those proportional to E^2 do not appear, and terms of $O((\omega_B t)^6)$ give the lowest order correction to Landau damping. In calculating the correction terms we employ the first order solution $E_o\exp(-\gamma_L t)$ as $E(t)$, γ_L being the Landau damping coefficient.

Finally we obtain an explicit form of the right hand side of (4) up to $O((\omega_B t)^6)$,

$$\frac{dE(\tau)}{d\tau} + E(\tau) = \{E(\tau)E_o k^2/64\gamma_L{}^4\}\ [(-\tfrac{1}{2}+\tau-\tau^2+\tfrac{2}{3}\tau^3+\tfrac{1}{2}e^{-2\tau}) + F(\lambda,\tau) + H(\lambda,\tau)] \tag{7}$$

$$F(\lambda,\tau) = A_0 + A_1\tau + A_2\tau^2 + A_3\tau^3 + A_4 e^{-2\tau}$$

$$H(\lambda,\tau) = B_o + B_1\tau + B_2\tau^2 + B_3\tau^3$$

where $\tau = \gamma_L t$. The function $H(\lambda,\tau)$ stems from the presence of the initial perturbation around $\tilde{v} = \lambda$ and depends on the shape of the perturbation. The A_i and B_i are functions of only λ and some typical values are given in Table 1. Expansion forms of F and H with respect to γ_L/k is given by

$$F(\lambda,\tau) = -(\gamma_L/k)^2\{\tfrac{1}{4}(\lambda^2+3-6\lambda^{-2})(2\tau-1+e^{-2\tau})+\tfrac{1}{2}\tau^2(3\lambda^2-7-2\lambda^{-2}) + (-\tfrac{1}{2}+\tau-\tau^2+\tfrac{2}{3}\tau^3+\tfrac{1}{2}e^{-2\tau})\} + O((\gamma_L/k)^4)$$

$$H(\lambda,\tau) = (\gamma_L/k)^2\lambda^{-4}\{3(\lambda^2+1)\tau^2 + (\lambda^2-1)\tau^3\} + O((\gamma_L/k)^4).$$

The first term in the bracket in Eq.(7) is always positive while F + H is negative and the absolute value becomes larger than the first term when τ is of the order of unity. Then the right hand side of Eq.(7) becomes negative and causes the enhanced damping.

Typical solutions to Eq.(7) are shown in Fig.1. The line (a) shows a solution with the initial distribution (6). This distribution deviates a little from the Maxwellian one over the entire velocity region including the phase velocity λ. The line (b) is for the case where initially no deviation around λ exists, that is, the line is the solution to Eq.(7) without $H(\lambda,\tau)$. We note that the damping in the presence of the deviation around λ is less than that in the absence of it. It may be concluded that generally a deviated initial distribution around λ seems to lessen the damping of waves; it has been shown elsewhere that sometimes it excites a wave[10].

Sugihara and Kamimura[11] have shown that when the quantity $q \equiv \gamma_L/\omega_B$ is smaller than about 0.8, there appears an amplitude oscillation. On the other hand the enhanced damping now becomes apparent for the same q value. In Fig.1,for example, the q for $E_0 = 0.2$ is 0.48. Hence, we remark that the temporal behavior of a finite amplitude wave can be correctly described only when both the effects of the enhanced damping and the particle trapping are included exactly.

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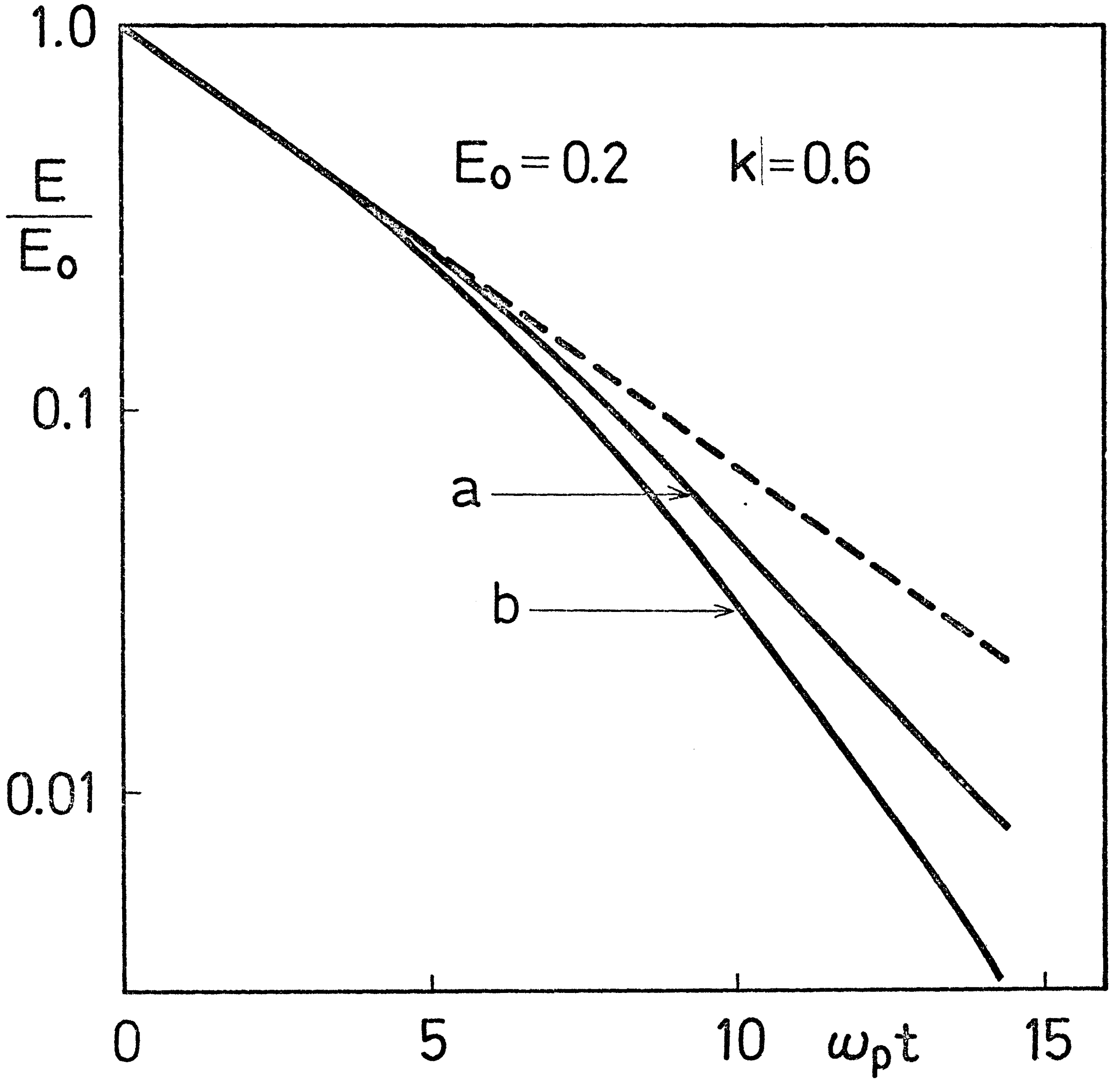
Figure Caption

Fig.1. Enhanced damping of the electron plasma wave. The lines (a) and (b) are the solutions to Eq.(7) with and without $H(\lambda,\tau)$, which represents the effect of a diviation of the initial distribution from the Maxwellian one around $\tilde{v} = \lambda$. The broken line is for the Landau damped wave. k and E_0 are the wave number and the intensity of the initial field, respectively.

Table I Some numerical values of the coefficients A_i and B_i in Eq.(7).

λ	A_0	A_1	A_2	A_3	A_4	B_0	B_1	B_2	B_3
			(	x 10^3	)				
3.0	126.7	-292.3	-543.1	-77.97	-168.0	31.94	-13.82	35.55	10.48
3.5	31.97	-69.19	-144.3	-24.20	-37.64	0.8351	-0.4209	4.416	1.284



1.0
0.1
0.01
E/Eo
Eo = 0.2
k|| = 0.6
a
b
0
5
10
ωpt
15