

INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

An Effect of Non-uniform Electric Field
on Drift-Dissipative Instability

Yoshinosuke TERASHIMA

IPPJ-211

December 1974

Further communication about this report is to be sent
to the Research Information Center, Institute of Plasma
Physics, Nagoya University, Nagoya, Japan.

Abstract

A dispersion relation of drift-dissipative mode in the presence of a non-uniform static electric field is derived from a kinetic model with the BGK collision operator. A finite gyroradius correction to the ion drift due to the electric field, which has a de-stabilizing effect, is thus reasonably introduced.

§1. Introduction

The purpose of this note is to remark an effect of a non-uniform static electric field on drift-dissipative instability and to make certain of the dispersion relation derived by Dodo et al based on fluid equations.¹⁾ The present interest in drift-dissipative instability was aroused from the studies on plasma transport processes in a stellarator device.^{1~3)}

In a slab model, Dodo assumed zeroth order electron and ion velocity as

$$\begin{aligned}\vec{v}_e^{(0)} &= (0, -E_0/B_0 + T_e/eB_0, 0) , \\ \vec{v}_i^{(0)} &= (0, -(1-\delta)E_0/B_0 - T_i/eB_0, 0) ,\end{aligned}\tag{1}$$

where δ is the finite ion gyroradius correction, E_0 the static electric field in the x-direction along which the density gradient exists, B_0 the magnetic field in the z-direction and the other symbols have the usual meanings. Further calculations were carried out by using fluid equations. An important effect of inhomogeneity in electric field was then shown to be a new term in the growth rate of drift-dissipative mode, which is due to $\delta E_0/B_0$, the difference between $\vec{E}_0(x) \times \vec{B}_0$ motion for the electrons and that for the ions. The assumption (1) is, however, not self-evident and some arguments for particle motions are necessary to determine δ .

We here show that the dispersion relation of interest is obtained from a kinetic model with the BGK collision operator and the finite gyroradius correction to the ion drift can be reasonably introduced.

§2. Model and Procedure

We consider a slab model with the same geometry as in (1) and assume

$$E_0(x) = E_0 \left(1 - \frac{x^2}{a^2}\right) = - \frac{d\phi_0(x)}{dx}, \quad \phi_0(x) = E_0 \left(-x + \frac{x^3}{3a^2}\right), \quad (2)$$

where a is thought to be comparable to the mean ion gyroradius ρ_i , but much larger than the Debye length. Since the constants of motion for particles with mass m and charge q are $W = (1/2)mv^2 + q\phi_0(x)$ and $L = x + v_y/\Omega$ where $\Omega = qB_0/m$, the equilibrium distribution function is expressed as $f_0 = f_0(W, L)$. It will be reasonable to determine a form of $f_0(W, L)$ by the requirements that it must be close to a maxwellian distribution and, after integration over velocity space, satisfy the quasi-charge-neutrality condition in the region $|x| \lesssim \rho_i < a$, namely

$$n_{0e}|_{x=0} = n_{0i}|_{x=0} = n(0), \quad \frac{n_{0e}'}{n_{0e}|_{x=0}} = \frac{n_{0i}'}{n_{0i}|_{x=0}} = \epsilon, \quad n_{0e}''|_{x=0} = n_{0i}''|_{x=0}$$

$$\text{and} \quad \frac{n_{0e}'''}{n_{0e}|_{x=0}} = \frac{n_{0i}'''}{n_{0i}|_{x=0}}, \quad (3)$$

where the prime denotes the differentiation with respect to x and the densities $n_{0j}(x)$ are expanded up to third order in x in order to include the term of x^3 in $\phi_0(x)$. If we choose

$$f_{0j} = n(0) \left(\frac{m_j}{2\pi T_j}\right)^{3/2} \{1 + \alpha_j L_j + \beta_j L_j^2\} \exp(-W_j/T_j), \quad (4)$$

with

$$\alpha_e = \varepsilon, \quad \beta_e = 0,$$

$$\alpha_i = \varepsilon - \frac{eE_0}{T_i} \left(1 + \frac{\rho_i^2}{2a^2}\right), \quad \beta_i = \frac{eE_0}{T_i} \frac{1}{3a^2},$$

then the conditions (3) can be satisfied to sufficient approximation provided that

$$T_e > T_i, \quad v_E \ll \left| \frac{T_e}{m_e \Omega_e} \varepsilon \right|, \quad \frac{v_E}{v_{Ti}} \ll \frac{\rho_i^2}{a^2} < 1, \quad |\varepsilon| \rho_i \ll 1,$$

where $v_E = qE_0(x=0)/m\Omega$ and $v_{Ti} = (2T_i/m_i)^{1/2}$. Note that if quadratic terms L_j^2 are included into (4), their coefficients will vanish. Throughout this paper, the temperatures $T_{e,i}$ are thought to be constants.

For the perturbed distribution and potential we assume

$$\begin{pmatrix} f_1 \\ \phi_1 \end{pmatrix} = \begin{pmatrix} \tilde{f}_1 \\ \tilde{\phi}_1 \end{pmatrix} \exp[i(k_\perp y + k_\parallel z - \omega t)]. \quad (5)$$

We shall work under the conditions that $k_\perp^2 \rho_i^2 < 1$, $|\omega| \ll \Omega_i$, $\omega_{*e} = \varepsilon k_\perp T_e / m_e \Omega_e$, ν_{ei} (the electron-ion collision frequency) and $k_\parallel^2 \ll k_\perp^2$. In a linearized form, f_1 is given by

$$f_1(r, v, t) = \int_{-\infty}^t dt' e^{\nu(t'-t)} \left(-\frac{q}{m} \vec{E}_1 \cdot \frac{\partial f_0}{\partial \vec{v}'} + \nu f_L \right)_{\vec{r}'=\vec{r}(t'), \vec{v}'=\vec{v}(t')} \quad (6)$$

where $\vec{E}_1 = -\vec{\nabla} \phi_1$, and for the ion component, ν is taken to be the ion-ion collision frequency and

$$f_{Li} = \left(\frac{n_{1i}}{n_0} + \frac{m_i}{T_i} \vec{v} \cdot \vec{u}_{1i} \right) f_{0i}$$

$$n_{1i} = \int f_{1i} d^3v, \quad \vec{u}_{1i} = \frac{1}{n_0} \int \vec{v} f_{1i} d^3v.$$

For the electron component, we have a similar expression for f_{1e} . The solution of the particle orbit in $\vec{E}_0(x)$ and \vec{B}_0 fields should be substituted into the integrand on the right-hand side of (6), which is obtained by the Bogoliubov - Mitropolskii method as

$$\begin{aligned} x(t') &= x(t) - \frac{v_{\perp}}{\Omega} \left(1 + \frac{1}{4} \frac{\tilde{E}_0'(x)}{\Omega^2} \right) [\sin(\theta - \Omega\tau) - \sin\theta], \\ y(t') &= y(t) + \Delta + \frac{v_{\perp}}{\Omega} \left(1 + \frac{3}{4} \frac{\tilde{E}_0'(x)}{\Omega^2} \right) [\cos(\theta - \Omega\tau) - \cos\theta], \\ z(t') &= z(t) + v_z \tau \end{aligned} \quad (7)$$

where θ is the phase of \vec{v}_{\perp} at $\tau = 0$, $\tau \equiv t' - t$ and

$$\tilde{E}_0(x) \equiv qE_0(x)/m,$$

$$\Delta = - \frac{\tilde{E}_0(x)}{\Omega} \left[1 + \frac{E_0''(x)}{E_0(x)} - \frac{1}{4} \left(\frac{v_{\perp}}{\Omega} \right)^2 + \dots \right] \tau. \quad (8)$$

It should be noted that $d\Delta/d\tau$ is the drift velocity and the second term in the parenthesis on the right-hand side of (8) is the finite gyroradius correction, which is just the term previously introduced by Steninger and Schmidt.⁴⁾

After some algebra, $f_{1i}(\vec{r}, \vec{v}, t)$ is found to be

$$\begin{aligned}
f_{1i}| = & -\frac{e}{T_i} \phi_1 f_{0i} + \frac{e}{T_i} \frac{1}{\omega + i\nu - \omega'_{Ei} - k_{\parallel} v_z} \left\{ \left(\omega - \frac{k_{\perp} T_i}{m_i \Omega_i} \alpha_i + i\nu \right) J_0^2(\mu) \right. \\
& - 3\beta_i \frac{k_{\perp} T_i}{m_i \Omega_i} \left(\frac{v_{\perp}}{\Omega_i} \right)^2 \frac{1}{\mu} J_0(\mu) J_1(\mu) \left. \right\} \phi_1 f_{0i} \\
& + \frac{i\nu}{\omega + i\nu - \omega'_{Ei} - k_{\parallel} v_z} \left(\frac{n_{1i}}{n_0} + \frac{m_i v_z}{T_i} u_{1iz} \right) J_0^2(\mu) \\
& + i \frac{m_i v_{\perp}}{T_i} u_{1ix} J_0(\mu) J_1(\mu) \left. \right\} f_{0i} \\
& + (\theta\text{-dependent terms and higher order terms}),
\end{aligned} \tag{9}$$

where the ions are supposed to have the unit charge, and

$$v = v_{ii}, \quad \mu = k_{\perp} v_{\perp} / \Omega_i, \quad \text{and } \omega'_{Ei} = -(k_{\perp} E_0 / B_0) (1 - v_{\perp}^2 / 2 \Omega_i^2 a^2).$$

Finally we obtain

$$\frac{n_{1i}}{n_0} = \frac{e\phi_1}{T_i} \frac{-\omega_{*i} (1 - F(b)) - b(\omega - \omega_{Ei} + \frac{1}{4} i b v_{ii})}{\omega - \omega_{Ei} + i v_{ii} F(b)}, \tag{10}$$

where $b = (1/2) k_{\perp}^2 \rho_i^2 = k_{\perp}^2 T_i / m_i \Omega_i^2$, $\omega_{*i} = e T_i / m_i \Omega_i$, and

$$F(b) \approx b \frac{\omega - \omega_{Ei} + \frac{1}{4} i v_{ii} b}{\omega - \omega_{Ei} + i v_{ii}}$$

$$\omega_{Ei} = \omega_E (1 - \delta), \quad \delta = \rho_i^2 / 2a^2, \quad \omega_E = -k_{\perp} E_0 / B_0.$$

In the course of calculation, the limit of $k_{\parallel}^2 T_i / m_i \ll |\omega - \omega_{Ei}|$ was taken and then the ion parallel velocity u_{1iz} was neglected. Note that the finite ion gyroradius correction is included in the expression of ω_{Ei} .

For the electrons, one may carry out similar calculations. It is not necessary to take account of finite gyroradius correction, however, the electron diffusion along the magnetic field must be considered. Under the condition that $|\omega - \omega_E + i\nu_{ei}|^2 \gg k_{\parallel}^2 T_e/m_e$ and with $D_{\parallel} \equiv T_e/m_e \nu_{ei}$, we have

$$\frac{n_{1e}}{n_0} = \frac{e\phi_1}{T_e} \frac{\omega_{*e} + ik_{\parallel}^2 D_{\parallel}}{\omega - \omega_E + ik_{\parallel}^2 D_{\parallel}} . \quad (11)$$

§3. Dispersion Relation

By equating (10) and (11), we obtain the dispersion relation for drift-dissipative mode as

$$\frac{\omega_{*e}(1-F(b)) - b(T_e/T_i)(\omega - \omega_{Ei} + \frac{1}{4} i b \nu_{ii})}{\omega - \omega_{Ei} + i \nu_{ii} F(b)} = \frac{\omega_{*e} + ik_{\parallel}^2 D_{\parallel}}{\omega - \omega_E + ik_{\parallel}^2 D_{\parallel}} . \quad (12)$$

If we set $F(b) \approx (1/4)b^2$, then the dispersion relation used by Dodo et al is recovered and the validity of their assumption for finite ion gyroradius correction is proved.

The de-stabilizing effect of the non-uniform static electric field can be seen, for example, in a following form of the solution of (12). For $k_{\parallel}^2 D_{\parallel} \gg \omega_{*e} \gg \delta\omega_E$, $b \nu_{ii}$, it turns out to be

$$\omega \approx \frac{\omega_{*e}}{1+\lambda b} - \delta\omega_E + i \left\{ \frac{\omega_{*e}}{k_{\parallel}^2 D_{\parallel}} \left[\frac{1}{1+\lambda b} \frac{\delta\omega_E}{\omega_{*e}} + \frac{\lambda b}{(1+\lambda b)^3} \right] - \frac{b^2}{4} \nu_{ii}^2 \right\} , \quad (13)$$

where $\lambda = T_e/T_i$. As $k_{\perp}^2 D_{\perp}/\omega_{*e}$ decreases, (the other parameters included in (12) are fixed), the growth rate increases and will reach the maximum value at some value of $k_{\perp}^2 D_{\perp}/\omega_{*e}$, while the real part of ω gradually decreases.

Acknowledgements

The author would like to thank Dr. T. Dodo and Dr. T. Watanabe for helpful discussions.

References

- 1) T. Dodo, M. Fujiwara, K. Miyamoto and A. Ogata:
Proceedings of the Sixth European Conference on Controlled
Fusion and Plasma Physics, Vol. II (1973) p338.
- 2) A. Mohri and M. Fujiwara: Nuclear Fusion 14 (1974) 67.
- 3) K. Miyamoto, M. Fujiwara, K. Kawahata, Y. Terashima,
T. Dodo and K. Yatsu: the paper presented at the Fifth
International Conference on Plasma Physics and Controlled
Nuclear Fusion Research (1974), CN-33/B5-1.
- 4) T. E. Stringer and G. Schmidt: Plasma Physics 9 (1967) 53.