INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Production of Strong Current Ion Beams in a Non-Uniform Magnetic Field

Kazunari Ikuta, Akihiro Mohri
and
Masaru Masuzaki

IPPJ-224 June 1975

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

Abstract

The possibility is discussed of producing strong current ion beams in a magnetic field without neglecting the diamagnetic effect of the drifting electrons across the magnetic field. In certain cases the diamagnetic effect can improve the ion current density. A design of the possible ion beam source is also illustrated.

The increasing necessity of the high current ($\sim 10^5$ A) ion beam source in the 1 MV range in the field of CTR and the nuclear science have motivated to explore the new mechanism to produce the high current ion beam. Recently Sudan and Lovelace[1)] has suggested the importance of the suppression of the electron current along the electric field in the plasma in order for the plasma to serve as the ion source. However, they have not taken into account of the effect of the electron current across the electric field. In case the strong electric field is induced across the magnetic field, the applied magnetic field may be distorted with the diamagnetic effect of the drifting electrons. The consideration of this effect is one of the key whether the present approach is feasible or not.

The present paper discusses the effect of the electron current across the electric field and shows a possible answer to produce the intense ion beams. An important assumption is that the thickness of the plasma slab is less than the ion Larmor radius but it is sufficiently thicker than the electron Larmor radius, and that the applied electric field is so strong that the pressure of the ions and the electrons can be neglected. In that situation the ions are almost free from the magnetic field in crossing the plasma slab, which are approximately described by

$$M \vec{v}_i \cdot \nabla \vec{v}_i = \vec{Z}e\ E \qquad (1)$$

where M, Ze and $\vec{v}_i$ are the mass, the charge and the velocity of ions respectively, and $\vec{E}$ is the electric field which is

determined by the electric potential Φ , i.e. $\vec{E} = -\nabla\Phi$.
While the electrons are described as the mass-less magnetic fluid as follows.

$$0 = -e(\vec{E} + \vec{v}_e \times \vec{B}) \tag{2}$$

where $\vec{v}_e$ is the electron velocity and B the magnetic field. The magnetic field is determined by the electron current, since the electron drift velocity is sufficiently larger than the ion velocity in the situation considered here. Thus the magnetic field is subjected to

$$\nabla \times \vec{B} = -\mu_0 n_e e \vec{v}_e \tag{3}$$

and

$$\nabla \cdot \vec{B} = 0 \quad , \tag{4}$$

where n_e is the density of electron.

An important relation is obtained from Eqs.(2) and (3) with respect to the magnetic field and the electric field.
Let $\vec{B}$ and $\vec{v}_e$ be $\vec{B} = (0, 0, B)$

$$\vec{v}_e = (0, v_y, 0) \ ,$$

where the Cartesian coordinate (x,y,z) is used. And the components of Eq.(2) become

$$n_e e \frac{\partial\Phi}{\partial x} - \frac{\partial}{\partial x}\left(\frac{B^2}{2\mu_0}\right) = 0$$

and (5)

$$n_e e \frac{\partial\Phi}{\partial y} - \frac{\partial}{\partial y}\left(\frac{B^2}{2\mu_0}\right) = 0 \quad .$$

This states that the Jacobian

$$J\left(\frac{\Phi,\ B^2/2\mu_0}{x,\ y}\right) = 0 \qquad (6)$$

implies that $\frac{B^2}{2\mu_0}$ is, in general, a function of Φ alone;

$$\frac{B^2}{2\mu_0} = f(\Phi) \ . \qquad (7)$$

This relation is important to obtain the analytic solution in the present consideration.

To facilitate analysis, the plasma slab is assumed to be infinitely long in the y direction so that the quantities are considered as functions of the distnace x from the anode surface.

If

$$\vec{v}_e = (0,\ v_y,\ 0) \quad ,$$

$$\vec{v}_i = (v_x,\ 0,\ 0) \quad ,$$

$$\vec{B} = (0,\ 0,\ B) \quad ,$$

$$\vec{E} = \left(-\frac{d\Phi}{dx}\ ,\ 0,\ 0\right) \ ,$$

Eqs.(1) and (2) becomes

$$\frac{Mv_r^{\ 2}}{2} + Ze\Phi = \frac{Mv_0^{\ 2}}{2} + Ze\Phi_0 \qquad (8)$$

and

$$n_e = \frac{d}{dx}\left(\frac{B^2}{2\mu_0}\right)/e\,\frac{d\Phi}{dx} \ , \qquad (9)$$

where v_0 is the ion velocity at the anode surface and Φ_0 is the applied voltage between the electrodes. In case the applied voltage is sufficiently high, the initial energy of ion at anode surface can be neglected, i.e. $\frac{Mv_0^2}{2} << Z_e \Phi_0$. By combining the equation of continuity with Eq.(5) the ion density is

$$n_i = \left(\frac{M}{2Ze}\right)^{1/2} \Gamma(\Phi_0 - \Phi)^{-1/2} , \tag{10}$$

where Γ is the flux density of ions at the surface of the cathode. Then the potential Φ is determined by Poisson's equation

$$\frac{d^2\Phi}{dx^2} = \frac{e}{\varepsilon_0} \left[\frac{1}{e}\frac{df}{d\Phi} - Z\Gamma \left(\frac{M}{2Ze}\right)^{1/2} (\Phi_0 - \Phi)^{-1/2}\right]. \tag{11}$$

An integration of Eq.(11) could not be performed without an assumption of f with respect to Φ. With the helps of Eq.(7) the reasonably simple solution is obtained if f is chosen so that the dependence of the density profile of the electron on Φ is the same as the ions. In this case the magnetic field is not uniform, i.e.

$$\frac{B^2}{2\mu_0} = \frac{B_0^2}{2\mu_0} - \sqrt{2}\, \gamma J\left(\frac{M}{Ze}\right)^{1/2} (\Phi_0 - \Phi)^{1/2}, \tag{12}$$

where J is the ion current density defined by $J = Ze\Gamma$, and γ is a constant. Then, from Eq.(12) the Poisson's Eq.(11) is reduced to

$$\frac{d^2\Phi}{dx^2} = - \frac{(1-\gamma)}{\sqrt{2}} \frac{J}{\varepsilon_0} \left(\frac{M}{Ze}\right)^{1/2} (\Phi_0 - \Phi)^{-1/2} \tag{13}$$

with a solution

$$\Phi = \Phi_0 - (\frac{3}{2})^{4/3} [\frac{1-\gamma}{\sqrt{2}} \frac{J}{\varepsilon_0} (\frac{M}{Ze})^{1/2}]^{2/3} x^{4/3} . \quad (14)$$

Since the cathode potential $\Phi(a) = 0$, the ion current density becomes

$$J = \frac{4\sqrt{2}}{9} \varepsilon_0 (\frac{Ze}{M})^{1/2} \frac{\Phi_0^{3/2}}{|1-\gamma| a^2} . \quad (15)$$

This formulae obtained is the Child-Langmuir density[2)3)] with the correction of the diamagnetic effect of the electrons, which shows that the non-uniformity of the magnetic field has a possibility to improve the ion current density if $|1-\gamma| < 1$.

Finally it should be noted that in designing a practical diode the important point is not only to provide a closed path of drifting electron but also to surround the diode by the closed magnetic surfaces. The lack of a closed path of electron current can induce the unfavorable space charge gradient and the lack of closed magnetic surface can give rise the parallel electric field along the magnetic lines of force by a small technical error of the set up. And they can allow transport of electrons across the electrodes. One of the simplest idea to satisfy both requirements may be to utilize a toroidal multipole field proposed by Ohkawa ans Kerst,[4)] which is illustrated in Fig.1. In this configuration the plasma may be expected to be reasonably stable for the application as the ion beam source, and the expected current density of ion is

$$J_i \gtrsim 10^3 \ \text{A/cm}^2$$

for the parameters

$$\Phi_0 \simeq 10^7 \ \text{volt}, \quad a \simeq 1 \ \text{cm and}$$

$$B_0 \simeq 10^4 \ \text{gauss.}$$

One of the authors (K.I.) would like to express his sincere thanks to Prof. S. Yoshikawa for the helpful conversations.

References

1) R.N. Sudan and R. V. Lovelace; Phys. Rev. Letters 31 (1973) 1174.

2) C. D. Child; Phys. Rev. 32 (1911) 492.

3) I. Langmuir; Phys. Rev. 2 (1913) 450.

4) T. Ohkawa and D. W. Kerst; Phys. Rev. Letters 7 (1961) 41.

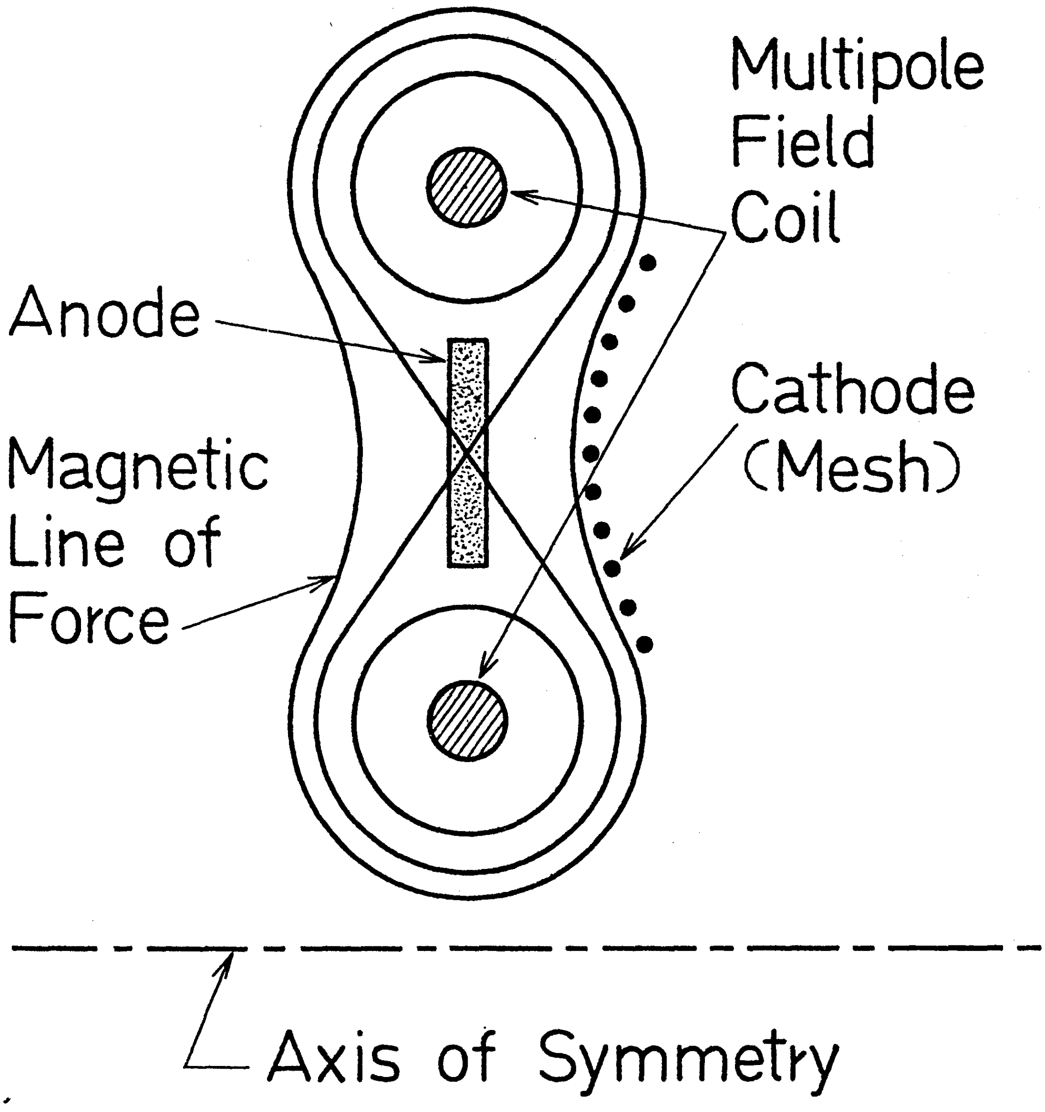

Figure 1. A design of the possible ion beam source.