INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Formation of Closed Magnetic Surfaces by Force-Compensated Helical Current

K. Ikuta and Y. Hamada

IPPJ-227 August 1975

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

ABSTRACT

An existence of the closed magnetic surface in the torsatron type fusion machine is considered under the condition that the mechanical forces in the helical coils are compensated. The closed magnetic surface is not destroyed by the condition if $\ell \geqq 3$, where ℓ is the multipolarity of the helical field.

In construction of the high field systems with the helical conductors one of the important technical problem is the compensation of the mechanical forces due to the interaction between the coils. Some authors have considered this problem [1-2]. However, a question arises whether the closed magnetic surface is destroyed or not in such force-compensated constructions.

In the present paper a systematic consideration is given to the existence of the closed magnetic surfaces in the force-compensated helical symmetric systems. Apparently, the compensation of the forces in the helical conductors is difficult to attain in the so called Stellarator systems, but it can be easily completed in the torsatron type apparatus. Thus, we consider only the torsatron type configuration.

After Aleksin's work [3] the magnetic field of the single spiral current with pitch distance L on a cylinder of radius a becomes, with accuracy up to ε,

$$H_r = -\frac{2I}{ca}\frac{\sin\theta}{1 + r^2 - 2r\cos\theta},$$

$$H_\psi = \frac{2I}{ca}\frac{r - \cos\theta}{1 + r^2 - 2r\cos\theta}, \qquad (1)$$

$$H_z = \frac{2I}{ca}\varepsilon\frac{1 - r\cos\theta}{1 + r^2 - 2r\cos\theta},$$

where $\varepsilon = 2\pi a/L << 1$, $\theta = \phi - \varepsilon z$ and r and z are dimensionless coordinates of the "point of observation" expressed in units of the radius a. The notations I and c are the current along the apiral and the velocity of light respectively.

In the torsatron systems ℓ - helical windings are wound relative to each other by angles $2\pi/\ell$ on the surface of the cylinder. These helical currents attract each other except $\ell = 1$. In this situation a simple way to compensate the attractive force is to introduce an uniform magnetic field along the z axis, whose strength is H_0. In the case of $\ell = 2$, the components of the force $\vec{F}$ which acts on the helical conductor becomes

$$F_r = \varepsilon I H_0 - \frac{I^2}{ca} \tag{2}$$

and

$$F_\phi = F_z = 0 . \tag{3}$$

Then, the requirement of the force compensation $F_r = 0$ leads to

$$H_0 = \frac{I}{ca} \frac{1}{\varepsilon} . \tag{4}$$

And the magnetic surface of the $\ell = 2$ torsatron system is of the form

$$\Psi = \frac{I}{ca} \ell n(1 - 2r^2 \cos 2\theta + r^2) + \frac{\varepsilon}{2} H_0 r^2 = \text{const.} \tag{5}$$

An important conclusion from Eqs. (4) and (5) is that there is no closed magnetic surface in the force-compensated $\ell = 2$ torsatron. By inserting the Eq.(4) into the expression (5) we have

$$\Psi \simeq \frac{I}{ca} r^2 \left(\frac{1}{2} + 2 \cos \theta\right) \tag{6}$$

around $r = 0$.
The expression (6) clearly shows that the magnetic axis is hyperbolic in this case.

In order to overcome the lack of closed magnetic surfaces, the multipolarity ℓ must satisfy the condition $\ell \geq 3$, provided that $\varepsilon << 1$.
To illustrate this, for simplicity, we choose $\ell = 3$.
Then, the components of the force F become

$$F_{\phi} = F_z = 0$$

and

$$F_r = I(\varepsilon H_0 - \frac{2I}{ca}). \qquad (7)$$

In this case the magnetic surface under the condition of $F_r = 0$ is

$$\psi = \frac{I}{ca}[r^2 - \ell n(1 - 2r^3 \cos\theta + r^2)]. \qquad (8)$$

Therefore, we have the closed magnetic surfaces around $r = 0$.

From the practical point of view it is important to see the cross-section of the magnetic surface composed of the force-compensated helical current. The magnetic surfaces which are useful for the confinement of plasma is separated in space by the separatrix on which there are singular points. These singular points can be determined from the conditions:

$$\frac{\partial \Psi}{\partial r} = 0$$

and

$$\frac{\partial\Psi}{\partial\theta} = 0 \ . \tag{9}$$

Then we have three singular points at $r = 1/3$ and $3\theta = \cos^{-1}(-1)$. Since the cross-section of the separatrix is very similar to the shape of the equilateral triangle, the cross section ratio Q of the separatrix to that of the cylinder on which the spiral coils are wound is approximately

$$Q \simeq 1/2\pi\sqrt{3} \simeq 0.1 \ . \tag{10}$$

It is easy to see the strength of the uniform magnetic field in order to attain the force compensation for the general torsatron field of multipolarity ℓ. From the tedious calculation the necessary strength H_0 becomes

$$H_0 = \frac{I}{ca}\,\frac{\ell-1}{\varepsilon} \ . \tag{11}$$

Then, the magnetic flux function Ψ is

$$\Psi = \frac{I}{ca}\Big[\frac{\ell-1}{2}\, r^2 - \ell_n(1 - 2r^n\cos \ell\theta + r^{2\ell})\Big]. \tag{12}$$

Thus, the higher multipolarity has a merit to obtain larger Q value.

We conclude, therefore, that the formation of the closed magnetic surfaces by using the force-compensated helical current is possible if the multipolarity ℓ larger than 2 is adopted and that the higher multipolarity has a merit to obtain larger cross section of the separatrix.

References

[1] Hubert P., C.R. Acad. Sc. Paris, 272 (1971) 661

[2] Georgievsky, A.V. Ziser, V.E. Litvinenko, Yu.A., *Proceedings of the* FIFTH SYMPOSIUM ON ENGINEERING PROBLEMS OF FUSION RESEARCH (Princeton University, 1973) p.511

[3] Aleksin, V.F., Soviet Physics-Technical Physics 6 (1962) 934