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Analytical Model for Super-Compression of Multi-Structured Pellet

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Abstract

We present a one-dimensional analytical model which can be applied to the super-compression of the multi-structured pellet. The main result shows that the time dependence of the input power E for the optimal compression is given by $E \propto (1 - t/t_s)^{-(3G+1)/2G}$ where $G=(\rho_1/\rho_2)^{1/4}$, ρ_1 and ρ_2 are the densities of the D-T fuel and the high Z material respectively, and t_s if the characteristic time interval.

§1. Introduction

To compress the density of the D-T fuel pellet to more than 1000 times of the solid density is necessary for the laser-fusion scheme. One of the possible method to realize this compression is launching successive shock waves which arrive simultaneously at the pellet center. Recently the multi-structured pellet, which is the D-T microsphere covered with materials having high atomic numbers Z, is proposed to use. These high Z materials which form the outer shell prevent the pellet from pre-heating due to high-energy electrons and hence regist the reduction of the compression due to pre-heating at the pellet center. So far S.Mikoshiba and B.Ahlborn[1] have presented a one-dimensional analytical model for the super-compression of the D-T solid fuel. But their analysis cannot be adapted to the multi-structured pellet. In this paper we present a one-dimensional analytical model which can be applied to the super-compression of the multi-structured pellet. The main result shows that the time dependence of the input power E for the optimal compression is given by $E \propto (1 - t/t_s)^{-(3G+1)/2G}$ where $G=(\rho_1 / \rho_2)^{1/4}$, ρ_1 and ρ_2 are the densities of the D-T fuel and the high Z material respectively, and t_s is the characteristic time interval.

§2. Model for Super-Compression

To simplify the analysis, we limit ourselves to a one-dimensional plane configuration. As shown in Fig.1, the pellet consists of a D-T fuel (region 1) and a high Z material (region 2). A large number of successive shock waves are launched from the point A (the point of the critical density for the laser light which is irradiated from the right hand side in Fig.1), impinge upon the contact discontinuity B and are transmitted through B to the region 1. This phenomena are schematically shown in Fig.2. As each shock is weak, we describe the Mach number M of the shock wave as $M=1+\Delta M$, where $0<\Delta M<<1$ and the Mach number is determined in the reference frame moving with the velocity before the shock wave. If we use the suffices i and t to the incident and transmitted shock wave respectively, we have

$$\frac{\Delta M_i}{\Delta M_t} = \left(\frac{\rho_{20}}{\rho_{10}}\right)^{1/4} = \frac{1}{G}, \qquad (1)$$

for $\gamma=5/3$ (see Appendix A), where the the suffices 1 and 2 denote the region 1 and 2 respectively, and the suffix 0 means the value before the shock wave reaches. Equation (1) implies that the transmitted shock should be weaken. In the region 2 on the other hand, a rarefaction wave is reflected shown as B - R in Fig.2. We denote the velocity of the ablation surface (A-F), the shock surface (A-B or B-O' or A'-B'or------) and the contact discontinuity (B-B'-E) by V, U and W respectively. The n-th shock leaves the surface A-F with the velocity U_n at $t=t_n$, reaches the surface B-E at $t=t'_n$ and propagates toward the center O with the velocity U'_n in the region 1. Let us consider that successive shock waves are launched from the surface A-F into the region 1 as a result of the anomalous absorption of the laser energy at the surface A-F of the critical density. In order to converge the shock waves in the region 1 to the point O at the same time, the input power E of the laser light irradiated to the pellet should increase in time in the definite way. If we assume that the n-th and (n+1)-th shock waves arrive at the point O at the same time t_s, then the following two equations must be satisfied,

$$U'_n\,(t_s-t'_n) = U'_{n+1}(t_s-t'_{n+1})+W_n(t'_{n+1}-t'_n)\ , \tag{2}$$

$$U_n(t'_n-t_n)+W_n(t'_{n+1}-t'_n)-V_n(t_{n+1}-t_n)$$
$$=U_{n+1}(t'_{n+1}-t_{n+1}) + O(\Delta M) \tag{3}$$

In eq.(3), the term $O(\Delta M)$ comes from the interaction between the shocks and the rarefaction waves in region 2, whose interaction change the shock velocity so we denote this change as

$$\int_{t_n}^{t'_n} U_n\, dt = U_n(t'_n - t_n) + O(\Delta M) \text{ etc.}$$

By use of

$$\tau'_n = t_s - t'_n\ , \qquad \tau_n = t_s - t_n\ , \tag{4}$$

eqs. (2) - (3) reduce to

$$\frac{\tau'_{n+1} - \tau'_n}{\tau'_n} = \frac{U'_n - U'_{n+1}}{U'_{n+1} - W_n}\ , \tag{5}$$

$$(U_{n+1}-W_n)\,\tau'_{n+1} + (W_n-U_n)\,\tau'_n$$
$$= (U_{n+1}-V_n)\,\tau_{n+1} + (V_n-U_n)\,\tau_n + O(\Delta M)\ , \tag{6}$$

To solve eqs.(5) and (6) we use the following Rankine-Hugoniot relations for weak shocks,

$$u_n = u_{n-1} + \frac{4}{\gamma+1} c_{n-1} \Delta M, \tag{7}$$

$$U_n = u_{n-1} + c_{n-1}(1+\Delta M), \tag{8}$$

$$c_n = c_{n-1}\left(1+\frac{2(\gamma-1)}{\gamma+1}\Delta M\right), \tag{9}$$

where u_n is the fluid velocity and c_n is the sound speed behind the shock n. The increment of the Mach number ΔM in the region 2 should be replaced by ΔM^* which includes the interaction between the shocks and the rarefaction waves, but you can see that the leading term of eq.(3) is of the order of the sound velocity, that is O(1), accordingly the terms of $O(\Delta M)$ and therefore $O(\Delta M^*)$ is negligibly small.

The Rankine-Hugoniot relations in the region 1 are obtained if u_n, c_n, U_n and ΔM in eqs.(7)-(9) are replaced by W_n, c'_n, U'_n and $\Delta M'$ respectively. By the use of eqs.(7)-(9), eqs.(5) and (6) reduce to

$$\frac{\tau'_{n+1} - \tau'_n}{\tau'_n} = -2\Delta M' \quad , \tag{10}$$

$$\tau'_{n+1} - \tau'_n = (1-N_n)(\tau_{n+1} - \tau_n) \quad , \tag{11}$$

where N_n is the Mach number of the ablation surface. In the above equations, only the leading terms are retained. The velocity of the ablation surface is given by

$$V_n = u_n + c_n N_n .$$

Introducing a new variable $\xi(=n\Delta M)$ and taking the limit of $n \to \infty$, $\Delta M \to 0$ and $n\Delta M \to \xi$, we obtain the following differential equations for τ' and τ (the suffix n being dropped) instead of eqs.(10)and (11),

$$\frac{d\tau'}{d\xi} = -2G\tau' \quad , \tag{12}$$

$$\frac{d\tau'}{d\xi} = (1-N)\frac{d\tau}{d\xi} \quad , \tag{13}$$

where eq.(1) is used with the identities $\Delta M_i=\Delta M$, and $\Delta M_t=\Delta M' =G\Delta M$.

The solution of eqs.(12) and (13) is given by

$$\tau \sim \exp(-2G\xi) \tag{14}$$

for the range of moderate value of ξ , where N can be neglected because N diminishes as $\exp(-\xi)$ with respect to ξ.(see eq.(16)) In the derivation of eq.(14), the fact that G is constant is used which is derived in Appendix A.

§3. Power Input

The absorbed power E in the ablation surface can be found as

$$E = (1-N^2)^2\rho^*c^{*3} /2(\gamma^2-1)N, \tag{15}$$

from the Chapman-Jouguet deflagration condition[1-3]. Here suffix * refers to the values including the effects of the reflected rarefaction waves discussed in Appendix B. We can express N as a function of ξ as follows[1) ,

$$N = \frac{1}{4}\exp(-\xi) \quad . \tag{16}$$

On the estimation of the term ρ^*c^{*3}, we must consider both the effects of launching shock waves and of rarefaction waves coming back from the contact discontinuity. As shown in Appendix B, finally we derive

$$\rho^* c^{*3} \sim \exp\left[\frac{2(3\gamma-1)}{\gamma+1} G\xi\right] . \qquad (17)$$

Combining eqs.(14)-(17), we obtain

$$E \sim \exp\left[\frac{2(3\gamma-1)G+(\gamma+1)}{\gamma+1}\ \xi\right] ,$$

$$\sim \tau^{-\frac{2(3\gamma-1)G+(\gamma+1)}{2(\gamma+1)G}}$$

$$\sim \tau^{-\frac{3G+1}{2G}} \quad (\text{for } \gamma=5/3). \qquad (18)$$

Equation (18) yields

$E \sim \tau^{-2}$ for solid D-T,

$E \sim \tau^{-2.43}$ for solid D-T covered with solid glass ,

$E \sim \tau^{-3.05}$ for solid D-T covered with gold,

$E \sim \tau^{-3.95}$ for gas D-T covered with solid glass .

We analyzed the super-compression of the multi-structured pellet and obtained the relation (18). This relation should be useful as a 'structure law' when we engage in laser-fusion experiments.

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Appendix A

A contact discontinuity has no discontinuity with respect to the pressure p or the fluid velocity u across it. Hence if the regions in Fig.2 are indicated by the suffices, it follows that

$$p_1=p_2 \ , \ p_4=p_5, \ u_1=u_2 \ , \ u_4=u_5. \tag{A-1}$$

As the shocks are weak, we can put $1+\Delta M_t$, $1+\Delta M_i$ and $1-\Delta M_R$ for the Mach number of the incident shock, transmitted shock and reflected rarefaction waves respectively, where

$$0 < \Delta M_t, \ \Delta M_i, \ \Delta M_R << \ 1.$$

Then from eq.(A-1), we obtain

$$\Delta M_i=\Delta M_t + \Delta M_R, \tag{A-2}$$

and

$$\Delta M_t\sqrt{\frac{\gamma+1}{2\gamma+4\Delta M_t}} = \Delta M_i\sqrt{\frac{\rho_1}{\rho_2}\frac{\gamma+1}{2\gamma+4\Delta M_i}} - \Delta M_R\sqrt{\frac{\rho_1 p_3}{\rho_3 p_1}\frac{\gamma+1}{2\gamma+4\Delta M_R}} \ . \tag{A-3}$$

Equations (A-2) and (A-3) reduce to

$$\frac{\Delta M_i}{\Delta M_t} = \left(\frac{\rho_2}{\rho_1}\right)^{1/4} \ , \tag{A-4}$$

with the aid of the Rankine-Hugoniot relations

$$p_3=p_2\left(1+\frac{4\gamma}{\gamma+1}\Delta M_i\right),$$
$$\rho_3=\rho_2\left(1+\frac{4}{\gamma+1}\Delta M_i\right). \tag{A-5}$$

The increment of the Mach number ΔM_i may reduce to ΔM_i^* due to the interaction between the shocks and the rarefaction waves but this effect is of the order of the density ratio, that is,

$$\Delta M_i = \Delta M_i^* \ (1 + O(\Delta M_i)),$$

so we can assume ΔM_i remains constant in the interaction processes. This fact does not mean that the quantities ρ, p, u and U remain constant but means that the quantities vary in the way of ΔM_i remaining constant. As the quantities in eqs.(A-1)-(A-5) are not ones throughout the region 2 but ones in the vicinity of the contact discontinuity, we have no difficulty to use ΔM_i in eq.(A-5) instead of ΔM_i^*.

The ratio ρ_5/ρ_4 is obtained from

$$\rho_4 = \rho_1 \frac{1}{G^4}\left[1 + \frac{4}{\gamma(\gamma+1)}\Delta M_i\right]\left[1 - \frac{4}{\gamma(\gamma+1)}\Delta M_R\right],$$

and

$$\rho_5 = \rho_1 \left[1 + \frac{4}{\gamma(\gamma+1)}\Delta M_t\right],$$

as

$$\rho_5/\rho_4 = G^4 + O(\Delta M^2). \qquad \text{(A-6)}$$

Equation (A-6) means that the density ratio across the contact discontinuity remains constant in all successive times.

Appendix B

The successive shocks launching from the ablation surface compress and heat the high Z material. Such compressions are described as

$$c_n = c_0 \left[1 + \frac{2(\gamma-1)}{\gamma+1} \Delta M \right]^n , \qquad \text{(B-1)}$$

$$\rho_n = \rho_0 \left[1 + \frac{4}{\gamma+1} \Delta M \right]^n , \qquad \text{(B-2)}$$

where the suffix 0 denote the initial values. In the limit of $n \to \infty$, $\Delta M \to 0$ and $n\Delta M \to \xi$, eqs. (B-1)and (B-2) reduce to

$$c_n = c_0 \exp\left[\frac{2(\gamma-1)}{\gamma+1} \xi \right] , \qquad \text{(B-3)}$$

$$\rho_n = \rho_0 \exp\left[\frac{4}{\gamma+1} \xi \right] . \qquad \text{(B-4)}$$

On the other hand, the rarefaction waves coming back from the contact discontinuity reduce c_n and ρ_n to c_n^* and ρ_n^* respectively according to

$$c_n^* = c_n \exp\left[\frac{2(\gamma-1)}{\gamma+1} (G-1)\xi \right] , \qquad \text{(B-5)}$$

$$\rho_n^* = \rho_n \exp\left[\frac{4}{\gamma+1} (G-1)\xi \right] . \qquad \text{(B-6)}$$

These results are based on the fact that the Mach number of the rarefaction waves is given by

$$\Delta M_R = \Delta M_i - \Delta M_t$$

$$= \Delta M\ (1-G), \qquad (\Delta M = \Delta M_i) \qquad \text{(B-7)}$$

where eqs.(A-2) and (A-4) are used. Combining eqs.(B-3), (B-4), (B-5) and (B-6), we obtain

$$\rho_n^* c_n^{*3} \sim \exp\left[\frac{2(3\gamma-1)}{\gamma+1} G\xi\right] \quad . \tag{B-8}$$

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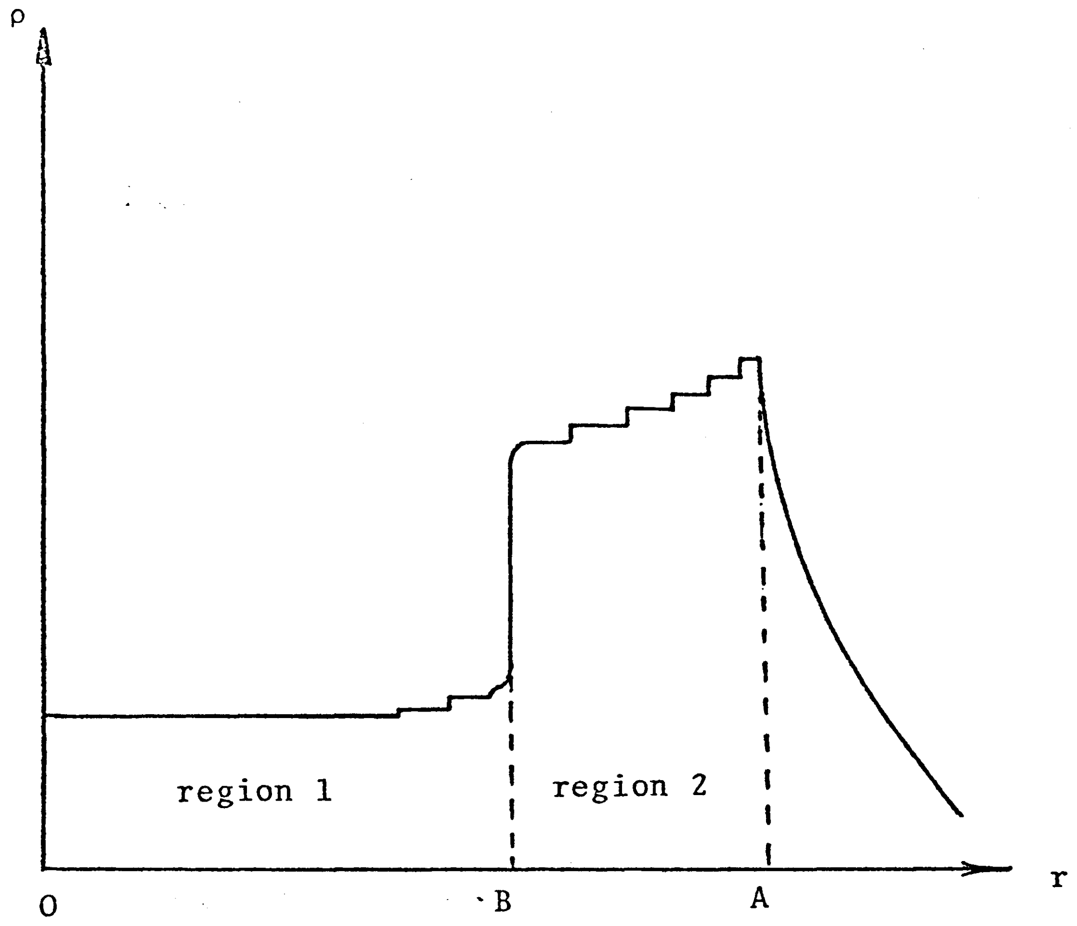

Fig.1 Density distribution in the pellet.

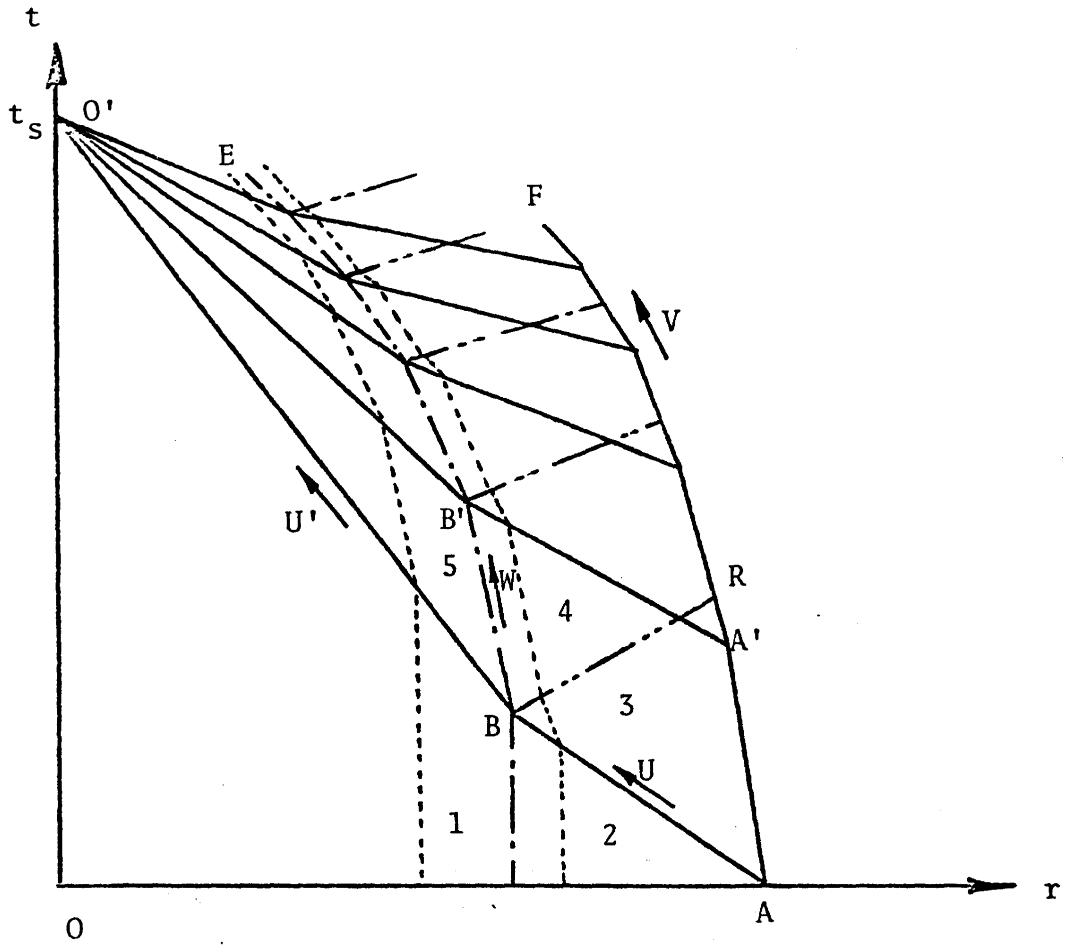

Fig.2 r-t diagram of the ablation surface(solid line, A-F), the successive shocks(solid line, A-B, B-O', A'-B' etc.), the rarefaction waves(double chain line, B-R etc.) and the contact discontinuity(chain line, B-B'-E). Dotted lines denote the pathes of particles..