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Modified Nonlinear Schrödinger Equation for Alfvén Waves Propagating along the Magnetic Field in Cold Plasmas

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RESEARCH REPORT

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

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Abstract

The basic nonlinear equation which describes the Alfvén waves, with small but finite amplitude propagating along the magnetic field in cold plasmas is derived modifying the reductive perturbation method proposed by Taniuti and Wei. Then as a result, the nonlinear dispersion relation is obtained through a procedure which clarifies the physical meaning. Furthermore, the modified nonlinear schrödinger equation is proposed which describes the modulated Alfvén wave more correctly than the previous works. Our analysis agrees well with the numerical calculation of the initial value problem using the basic equations.

§1. Introduction

In recent years, nonlinear behaviors of waves in plasmas have been extensively analysed by many authors.[1,2] Taniuti and Wei[3] proposed the reductive perturbation method as a generalized scheme for deriving nonlinear differential equations of the waves. The method is shown by many authors to be a powerful procedure to extract the equation describing a particular wave with finite amplitude from a set of fundamental equations in which various waves can be included. Their method has been successfully applied to various waves such as ion acoustic[4] and magnetosonic waves.[5] However, the method by Taniuti and Wei cannot be applied to Alfvén waves propagating along static magnetic fields. This is because the eigenvalues of the zero-order matrix obtained from the basic equations degenerate in the left and right-hand circularly polarized Alfvén waves (left Alfvén wave and right Alfvén wave). This fact contradicts their assumption[3] on which the reductive perturbation method is substantiated.

In the present paper, a different way of reductive perturbation is proposed which is effective even in the case that the degeneracy does exist. In the present method, the right and left Alfvén waves can be separately extracted from the basic equations describing the magnetohydrodynamic waves which conserve the nonlinearity of the waves. The nonlinear equation for each wave thus obtained is analysed in detail. Using the equation, we derive the nonlinear dispersion relation of Alfvén waves through a procedure that the physical meaning of the relation is clarified. Next, the propagation of modulated Alfvén waves is analysed, which results in an equation named, in this paper, the modified nonlinear Schrödinger equation (M. N. S. equation). Hasegawa[6] obtained the nonlinear Schrödinger equation (N. S. equation) describing the behavior of the modulated magnetohydrodynamic waves propagating along the magnetic field. He used the reductive perturbation method for the envelope of the modulated waves which was proposed by Taniuti and Yajima.[7] The present M.N.S. equation includes an additional higher-order term which plays an important role when the wavelengths of the envelope and the carrier wave become the same order. Thus the features of the steepening of the envelope and the modulational instabilities are affected significantly by the additional term except an early stage of the time evolution.

In order to check our theoretical analysis, we numerically calculated our equations. The results of the nonlinear dispersion relation and the steepening of the envelope in the modulated waves agree with the theoretical analysis.

In §2, the M.N.S. equation for modulated Alfvén waves is derived. The numerical results and discussions are presented in §3. The analytical results of the modulational instabilities and the envelope solitons will be described in the subsequent paper.

§2. Theoretical Analysis

2.1. Formulation of the basic equations

The motions of electrons and ions in plasmas are described by the following equations: the equations of continuity, the equations of motion which are coupled with Maxwell's equations. The magnetohydrodynamic wave with long wavelength, i.e., Alfvén waves in a uniform magnetized plasma is here considered. In a long wavelength limit, the charge separation between electrons and ions can be ignored. This allows us to assume that the plasma is quasi-neutral. Furthermore the displacement current in Maxwell's equation, is neglected assuming that the phase velocity of the Alfvén waves is much lower than that of light in vacuum.

Eliminating the electric field and the electron velocity, from the equations, one obtains the equations for a plane Alfvén wave propagating along x-axis in a uniform plasma. Then, the equations[5)] are

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} n u = 0 , \tag{1}$$

$$\frac{du}{dt} + n^{-1} \frac{\partial}{\partial x} \frac{B_y^2 + B_z^2}{2} = 0 , \tag{2}$$

$$\frac{dv}{dt} - n^{-1} B_x \frac{\partial B_y}{\partial x} = - \omega_{ce}^{-1} \frac{d}{dt}\left(n^{-1} \frac{\partial B_z}{\partial x}\right) , \tag{3}$$

$$\frac{dw}{dt} - n^{-1} B_x \frac{\partial B_z}{\partial x} = \omega_{ce}^{-1} \frac{d}{dt}\left(n^{-1} \frac{\partial B_y}{\partial x}\right) , \tag{4}$$

$$\frac{dB_y}{dt} - B_x \frac{\partial v}{\partial x} + B_y \frac{\partial u}{\partial x} = \omega_{ci}^{-1} \frac{\partial}{\partial x}\left(\frac{dw}{dt}\right) , \tag{5}$$

$$\frac{dB_z}{dt} - B_x \frac{\partial w}{\partial x} + B_z \frac{\partial u}{\partial x} = - \omega_{ci}^{-1} \frac{\partial}{\partial x}\left(\frac{dv}{dt}\right) , \tag{6}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} \quad ,$$

Here, B, x, t and n are normalized, respectively, by the static magnetic field B_o, the wavelength L, the reciprocal of the wave frequency ω_o and the mean plasma density n_o. Also, ω_{ce} and ω_{ci} denote the cyclotron frequencies, respectively, of electrons and ions normalized by ω_o. The frequency ω_o is given by V_A/L, where V_A is the velocity of the Alfvén wave, i.e., $B_o/[4\pi n_o(m_e+m_i)]^{\frac{1}{2}}$. In the above equations, u, v and w denote respectively, x, y and z components of the ion velocity normalized by V_A. The equations constitute the fundamental equations of the present analysis.

We restrict to the propagations of the Alfvén waves along a static magnetic field in uniform plasmas. As is well known, the dispersion relation of the Alfvén waves with an infinitesimal amplitude are given from Eqs.(1) - (6), as

$$\frac{\omega}{k} = 1 \mp \mu k \ , \tag{7}$$

where

$$\mu = \frac{1}{2}\left(\omega_{ci}^{-1} - \omega_{ce}^{-1}\right) \ , \tag{8}$$

ω and k are, respectively, the frequency and the wave number, which are normalized, respectively, by ω_o and L^{-1}. In Eq. (7), the upper and the lower signs denote, respectively, the left and right Alfvén waves. Taking into account the linear dispersion relation, Eq.(7), we introduce the following stretched space and time variables,

$$\xi = \varepsilon(x-t)\ ,\quad \tau = \varepsilon^2 t\ , \tag{9}$$

in order to carry out a perturbation expansion of Eqs.(1) - (6), where ε is a positive number smaller than unity.

The variables have the power series expansions in ε about a homogeneous equilibrium state,

$$n = 1 + \tilde{n} = 1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + \cdots \tag{10}$$

$$u = \tilde{u} = \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \cdots \tag{11}$$

$$v = \tilde{v} = \varepsilon^{\frac{1}{2}}(v^{(1)} + \varepsilon v^{(2)} + \cdots) \tag{12}$$

$$w = \tilde{w} = \varepsilon^{\frac{1}{2}}(w^{(1)} + \varepsilon w^{(2)} + \cdots) \tag{13}$$

$$B_y = \tilde{B}_y = \varepsilon^{\frac{1}{2}}(B_y^{(1)} + \varepsilon B_y^{(2)} + \cdots) \tag{14}$$

$$B_z = \tilde{B}_z = \varepsilon^{\frac{1}{2}}(B_z^{(1)} + \varepsilon B_z^{(2)} + \cdots)\ . \tag{15}$$

The expansion given by Eqs.(10) - (15) is the best choice of the present analysis in the sense that leading order terms, respectively, of the quantities in Eqs.(10) - (15) can be the largest among various ways of expansion. The detailed discussions on the choice of expansion in the reductive perturbation method will be published elsewhere.

In order to simplify the analysis, the plasma is assumed to be in equilibrium state at $\xi \rightarrow \pm\infty$, then following boundary conditions are valid that $n^{(i)}$, $u^{(i)}$, $v^{(i)}$, $w^{(i)}$, $B_y{}^{(i)}$ and $B_z{}^{(i)}$ approach, respectively, zero as ξ goes infinity where $i = 1, 2, \ldots .$ Substituting Eqs.(9) - (15) into Eqs.(1) - (6), and equating terms with the identical powers of ε, the following equations are obtained. First for ε^1:

$$n^{(1)} = u^{(1)} = \frac{1}{2}(B_y^{(1)2} + B_z^{(1)2}) , \tag{16}$$

$$v^{(1)} = -B_y^{(1)} \tag{17}$$

and

$$w^{(1)} = -B_z^{(1)} . \tag{18}$$

Next for ε^2:

$$\frac{\partial n^{(2)}}{\partial \xi} - \frac{\partial u^{(2)}}{\partial \xi} = \frac{\partial n^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi} n^{(1)} u^{(1)} , \tag{19}$$

$$\frac{\partial u^{(2)}}{\partial \xi} - \frac{\partial}{\partial \xi}(B_y^{(1)} B_y^{(2)} + B_z^{(1)} B_z^{(2)}) = \frac{\partial u^{(1)}}{\partial \tau} , \tag{20}$$

$$\frac{\partial v^{(2)}}{\partial \xi} + \frac{\partial B_y^{(2)}}{\partial \xi} = \frac{\partial v^{(1)}}{\partial \tau} - \omega_{ce}^{-1} \frac{\partial^2 B_z^{(1)}}{\partial \xi^2} , \tag{21}$$

$$\frac{\partial w^{(2)}}{\partial \xi} + \frac{\partial B_z^{(2)}}{\partial \xi} = \frac{\partial w^{(1)}}{\partial \tau} + \omega_{ce}^{-1} \frac{\partial^2 B_y^{(1)}}{\partial \xi^2} , \tag{22}$$

$$\frac{\partial B_y^{(2)}}{\partial \xi} + \frac{\partial v^{(2)}}{\partial \xi} = \frac{\partial B_y^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi} u^{(1)} B_y^{(1)} + \omega_{ci}^{-1} \frac{\partial^2 w^{(1)}}{\partial \xi^2} , \tag{23}$$

$$\frac{\partial B_z^{(2)}}{\partial \xi} + \frac{\partial w^{(2)}}{\partial \xi} = \frac{\partial B_z^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi} u^{(1)} B_z^{(1)} - \omega_{ci}^{-1} \frac{\partial^2 v^{(1)}}{\partial \xi^2} . \quad (24)$$

Substituting Eqs.(16) - (18) into Eqs.(21) - (24), we obtain equatons regarding the stretched variables, ξ and τ in time and space, respectively. Turning the equations back to the forms in the real space and time coordinates, x and t, the following equations are derived,

$$\frac{\partial B_y}{\partial t} + \frac{1}{4}\frac{\partial}{\partial x}\left[B_y\left(4 + B_y^2 + B_z^2\right)\right] - \mu \frac{\partial^2 B_z}{\partial x^2} = 0, \quad (25)$$

$$\frac{\partial B_z}{\partial t} + \frac{1}{4}\frac{\partial}{\partial x}\left[B_z\left(4 + B_y^2 + B_z^2\right)\right] + \mu \frac{\partial^2 B_y}{\partial x^2} = 0, \quad (26)$$

where, for simplicity, the superscript (1) of B_y and B_z are omitted. Equations (25) and (26) together with Eqs.(16) - (18) are the basic equations for nonlinear Alfvén waves obtained in the present analysis.

First, the nonlinear dispersion relation of the Alfvén waves with small but finite amplitudes is investigated in some detail. In order the Eqs.(25) and (26) to be applicable to describe the nonlinear behavior of the wave, B_y and B_z should be, respectively, smaller than unity, since $\tilde{B}_{y,z} \simeq \varepsilon^{\frac{1}{2}} B_{y,z}{}^{(1)} = O(\varepsilon^{\frac{1}{2}}) < 1$, as is shown in Eqs.(14) and (15).

In the linear theory, B_y and B_z can be written as

$$\begin{aligned} B_y &= a \cos(kx - \omega t) \\ B_z &= \pm a \sin(kx - \omega t), \end{aligned} \quad (27)$$

where a is the amplitude normalized by the static magnetic field B_o. If a small amplitude a of Eq.(27) is assumed as initial values of B_y and B_z in Eqs.(25) and (26), $B_y^2 + B_z^2 = a^2$ which is constant in time and space. Then, Eqs.(25) and (26) become essentially the linear differential equations regarding B_y and B_z. The linear equations thus obtained are Fourier transformed in time and space. Then the nonlinear dispersion relation $\omega(k, a^2)$ can be written as

$$\omega(k, a^2) = \omega(k) + \left(\frac{\partial\omega}{\partial a^2}\right)_0 a^2, \qquad \left(\frac{\partial\omega}{\partial a^2}\right)_0 = \frac{k}{4}, \tag{28}$$

where ω and k denote, respectively, the frequency and the wave number, $\omega(k)$ is the linear dispersion relation given by Eq.(7). Here the notations by Karpman dn Krushkal[8] have been adopted. Hasegawa[6] derived the N.S. equation somewhat different from Eq.(35) for a magnetohydrodynamic wave propagating along the magnetic field. He obtained the nonlinear dispersion relation, Eq.(36) in Ref.6, which tends to Eq.(28) in a long wavelength limit, comparing the corresponding terms in his N.S. equation and that by Karpman and Krushkal.[8] The nonlinear dispersion relation, Eq.(28), is derived in a procedure which clarifys the physical meaning rather than that by Hasegawa. Also, it should be emphasized that the assumption a << 1 is not necessarily required in Eq.(28). Then, no harmonic waves are generated even when the initial unmodulated amplitude is relatively large, whereas, the phase velocity of the wave increases with amplitudes.

If a wave whose amplitude is initially modulated, the envelope steepens as the wave propagates. This is because the portion of large amplitude propagates faster than that of small amplitude. Then the break-down time t_B of the modulated wave can be defined as the period that the part of the maximum amplitude of the wave, which is initially modulated sinusoidally, over-takes that of the minimum amplitude. In this case, the initial condition of the modulated wave is expressed as

$$\begin{aligned} B_y(x,0) &= (a_0 + a_1 \sin Kx)\cos kx, \\ B_z(x,0) &= \pm(a_0 + a_1 \sin Kx)\sin kx, \end{aligned} \tag{29}$$

where K and k are, respectively, the wavenumbers of the envelope and the carrier wave. Using Eq.(28), the maximum and minimum phase velocities, V_{pmax} and V_{pmin}, respectively, of the wave can be written as,

$$V_{pmax} = V_p^0 + (a_0 + a_1)^2/4 \ ,$$
$$V_{pmin} = V_p^0 + (a_0 - a_1)^2/4 \ ,$$

where V_p^o is the phase velocity for the linear Alfvén wave. The break-down time t_B is then written as

$$t_B = \frac{\pi/K}{V_{pmax} - V_{pmin}} = \frac{\pi}{a_0 a_1 K} \ . \tag{30}$$

Next, the time evolution of the modulated Alfvén wave, Eq.(29), is analysed. It is restricted during a time less than t_B. For this purpose the rotating vector components are introduced as

$$B_{L,R} = B_y \pm i B_z \, , \qquad\qquad (31)$$
$$v_{L,R} = v \pm i w \, ,$$

where suffixes L and R denote the left and right-hand circularly polarized waves, respectively. Substituting Eq.(31) into Eqs.(16) - (18), one directly obtains

$$n = u = \frac{1}{2}|B_L|^2 = \frac{1}{2}|B_R|^2 = \frac{1}{2}|B_\perp|^2 , \qquad\qquad (32)$$
$$v_{L,R} = - B_{L,R} \, .$$

Also, substituting Eq.(31) into Eqs.(25) and (26), the basic equation is obtained which describes the nonlinear evolution of a finite amplitude Alfvén wave propagating along the static magnetic field. For simplicity, the following notations are introduced

$$\varphi = B_{L,R} \, , \quad u_0' = \frac{\partial^2 \omega(k)}{\partial k^2} = \mp 2\mu \, . \qquad\qquad (33)$$

Then, Eqs.(25) and (26) are written

$$\frac{\partial \varphi}{\partial t} + \frac{1}{4}\frac{\partial}{\partial x}\left[\varphi(4 + |\varphi|^2)\right] - \frac{i u_0'}{2}\frac{\partial^2 \varphi}{\partial x^2} = 0^{*} . \qquad (34)$$

2.2. Modified nonlinear Schrödinger equation

Taniuti and Yajima[6)] proposed a reductive perturbation method to analyse generally the propagation of the modulated waves. They used the transformation in time and space, such as

$$\xi = \varepsilon(x-t)\ , \quad \tau = \varepsilon^2 t$$

and the perturbation expansion for the wave amplitude, as

$$\varphi = \sum_{l,m=1}^{\infty} \varepsilon^m \varphi^{(m)} e^{il(k_0 x - \omega_0 t)} ,$$

where k_o and ω_o are given by a linear dispersion relation. If their method is applied to the Alfvén waves propagating along the magnetic field, the following equation for the modulated wave, is obtained after some straight-forward calculations.

$$i\frac{\partial \varphi^{(1)}}{\partial \tau} + \frac{u_0'}{2}\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} - \frac{k}{4}|\varphi^{(1)}|^2 \varphi^{(1)} = 0. \tag{35}$$

This is well-known as nonlinear Schrödinger equation.[9] Equation (35), however, is valid only in a case that the wavelength of the envelope is much longer than that of the carrier wave. In contrast to Eq.(35), the Eq.(34) derived here which is available even in the case that the both wavelengths are comparable each other. Thus the Eq.(34) describes the nonlinear behavior of the modulated Alfvén waves more exactly than Eq.(35). The function ψ in Eq.(35)

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* After the present work had been completed, we found that the similar equation to Eq.(34) was independently derived also by Einer Mjφℓhus, in the internal report No.48, Department of Applied Mathematics, University of Bergen, Norway (1974).

is expressed as

$$\varphi(x,t) = \psi(x,t)\, e^{i(kx-\omega t)}, \tag{36}$$

where k and ω are given in the nonlinear dispersion relation, Eq.(28). Substituting Eq.(36) into Eq.(34), one obtains

$$\psi_t + u_0\psi_x - \frac{iu_0'}{2}\psi_{xx} + i\left(\frac{\partial\omega}{\partial a^2}\right)_0 (|\psi|^2 - |\psi_0|^2)\psi + \frac{1}{4}(|\psi|^2\psi)_x = 0, \tag{37}$$

where

$$\left(\frac{\partial\omega}{\partial a^2}\right)_0 = \frac{k}{4}, \quad u_0 = 1 \mp 2\mu k,$$
$$u_0' = \frac{\partial^2\omega(k)}{\partial k^2} = \mp 2\mu,$$

where $|\psi_0|$ is the wave amplitude without modulation, the subscripts t and x denote the derivatives in time and space, respectively. We call Eq.(37) the modified nonlinear Schrödinger equation. It includes the last term, as an additional higher-order term, to Hasegawa's N.S. equation.

Next, the physical meaning of the term of Eq.(37) is considered. The envelope $\psi(x,t)$ in Eq.(36) is written in a form

$$\psi(x,t) = a(x,t)\, e^{i\phi(x,t)}. \tag{38}$$

Substituting Eq.(38) into Eq.(37), the following two equations are obtained from the real and imaginary parts.

$$a_t + u_0 a_x + \frac{u_0'}{2}(2a_x\phi_x + a\phi_{xx}) + \frac{3}{4}a^2 a_x = 0, \tag{39}$$

$$\phi_t + u_0\phi_x + \frac{u_0'}{2}\phi_x^2 + \left(\frac{\partial\omega}{\partial a^2}\right)_0 (a^2 - a_0^2) - \frac{u_0}{2a}a_{xx} + \frac{1}{4}a^2\phi_x = 0 \tag{40}$$

The last terms in Eqs.(39) and (40) introduced from the last term of Eq.(37) is equivalent to the nonlinear term in modified Korteweg-deVries equation[10) (M. K-dV equation).

It is well known that the nonlinear term in M. K-dV equation plays a role to steepen the waveform so as to balance a dispersion term, resulting in solitary waves. Thus, using the analogy, the last term in Eq.(37) will enhance the steepening of the envelope of the modulated Alfvén wave. The effect of the last term in Eq.(37) can be neglected if the wavelength of the envelope is much longer than that of the carrier wave. The last term becomes essentially important when both wavelengths are in same order. Thus, it is concluded that Eq.(37) has a wider applicability than the usual N.S. equation to describe the evolution of the modulated Alfvén waves.

§3. Numerical Computation

Next, the results of a numerical computation of the nonlinear equations are shown, in order to check the validity of our analytical results. First the frequency shift of finite amplitude Alfvén waves, which is predicted by Eq.(28) is examined. The Eqs.(25) and (26) are numerically computed in time and space. Here, 2-step Lax-Wedroff method is used to solve a set of difference equations which are derived from Eqs.(25) and (26). An initial condition is given by

$$B_y(x, 0) = a\cos kx ,$$
$$B_z(x, 0) = -a\sin kx ,$$

for right Alfvén waves. It is expected from Eq.(28) that, for a given k, the freuqency and therefore the phase velocity would increase with wave amplitude. A wavenumber $k = 0.01$ is assumed which corresponds to a long wave length. An example of phase velocity ω/k vs a is shown in Fig.1. The open circles are numerically computed ones which exactly fit to the theoretical line at $a = 0.01$ and 0.5, respectively. The solid line is the nonlinear dispersion relation given by Eq.(28). It is confirmed that ω/k numerically calculated for left Alfvén waves also agrees that by Eq.(28).

The spectrum of wavenumber k at each step of the calculations is observed. The generation of higher harmonic components in time and space has not been observed even in the case of $a = 0.5$. This fact suggests that the condition $a \ll 1$ is not necessary in the present analysis. In the case of ion acoustic and magnetosonic waves, large amplitude collapses into solitons as is well known in K-dV equation. Whereas, large amplitude Alfvén waves propagate without any distortion. The phase velocity only increases as the wave amplitude increases.

The propagations of the modulated Alfvén waves are also computed numerically. We use the initial condition given by Eq.(29) for the left Alfvén wave. In the computation, a_o =

0.4, $a_1 = 0.1$, $k = 0.01$, $K = k/8$, $\mu = 0.5$ and therefore $t_B = 2 \times 10^4$ are used. The width Δx and Δt, respectively, in space and time are chosen in the difference equations such as $\Delta x = \Lambda_o/512$ and $\Delta t = \Delta x/2$, where $\Lambda_o = 2\pi/K$. The computation are made in a reference frame moving with the Alfvén velocity, in order to conserve the numerical accuracy. An example of the wave patterns is shown in Fig.2, for times $t = 0.015\ t_B$, $0.21\ t_B$ and $0.42\ t_B$. The solid and dashed lines are, respectively, $|B_\perp|$ and B_z. It is shown that the initial sinusoidal envelope steepens and approaches to a shock-like pattern. To the right of each wave pattern, the spectrum of the wavenumber k is shown. At $t = 0.015\ t_B$, the spectrum of B_z is concentrated around the carrier wavenumber $k = 0.01$. At $t = 0.21\ t_B$ and $t = 0.42\ t_B$, the spectrum becomes broad, yielding higher wavenumbers.

The time evolution of the maximum value, $|B_\perp|_{max}$, of the envelope is plotted in Fig.3(a). At the time around $t = 0.21\ t_B$, $|B_\perp|_{max}$ increases all of a sudden and approaches to a saturated level. Afterwards, $|B_\perp|_{max}$ randomly oscillates around the saturated level. In Fig.3(b), the wave energy W in the plasma is followed in time where

$$W = \int_{-\pi/K}^{\pi/K} |B_\perp|^2\, dx$$

and W_o is the initial value of the energy. The energy W should be an invariant because no physical mechanism for dissipation exists in the equations. As is shown in the figure, W begins to decrease in numerical computation at

$t = 0.24\ t_B$. At $t = 0.48\ t_B$, W is 1.2 % smaller than the initial value. It will come from the fact that a sharp spike in front of the shock pattern is formed, which is smoothed out in the numerical calculation due to the finite interval of the spatial difference. Since the decrement are quite small, the appearance of the saturated level in $|B_\perp|_{max}$ is reliable. The numerical results of the almost steady-state envelope suggest the existence of the envelope soliton which will be discussed in detail in the subsequent paper.

References

1) A.C. Scott, F.Y.F. Chu and D.W. McLaughlin, Proc. IEEE 61 (1973) 1443.

2) Many authors, Supplement of the Progress of Theoretical Physics, No.55 (1974).

3) T. Taniuti and C.C. Wei, J. Phys. Soc. Japan 10 (1968) 941.

4) H. Washimi and T. Taniuti, Phys. Rev. Letters 17 (1966) 996.

5) T. Kakutani, H. Ono. T. Taniuti and C.C. Wei, J. Phys. Soc. Japan 24 (1968) 1159.

6) A. Hasegawa, Phys. Fluids 15 (1972) 870.

7) T. Taniuti and N. Yajima, J. Math. Phys. 10 (1969) 1369.

8) V.I. Karpman and E.M. Krushkal, Soviet Phys. JETP 28 (1969) 277.

9) V.E. Zakharov and A.B. Shabat, Soviet Phys. JETP 34 (1972) 62.

10) M Wadati, J. Phys. Soc. Japan 32 (1972) 1681; 34 (1973) 1289.

Figure Captions

Fig.1 Wave amplitude a vs phase velocity ω/k. The solid line is the theoretical values calculated from the nonlinear dispersion Eq.(28). The open circles are the values of numerical computation of Eq.(7) and (28).

Fig.2 Wave patterns and the spectra of wavenumbers of a modulated left Alfvén wave at (a) $t = 0.015\ t_B$, (b) $t = 0.21\ t_B$ and (c) $t = 0.42\ t_B$. The initial conditions are described in the text. To the right of each wave pattern, the spatial Fourier spectrum is shown. The solid and dashed lines are, respectively $|B_\perp|$ and B_z.

Fig.3 (a) The change in time of the maximum value, $|B_\perp|_{max}$, of the envelope in the calculation given in Fig.2; (b) the decrease of the wave energy W in time whose invariance assures the accuracy of the computation.

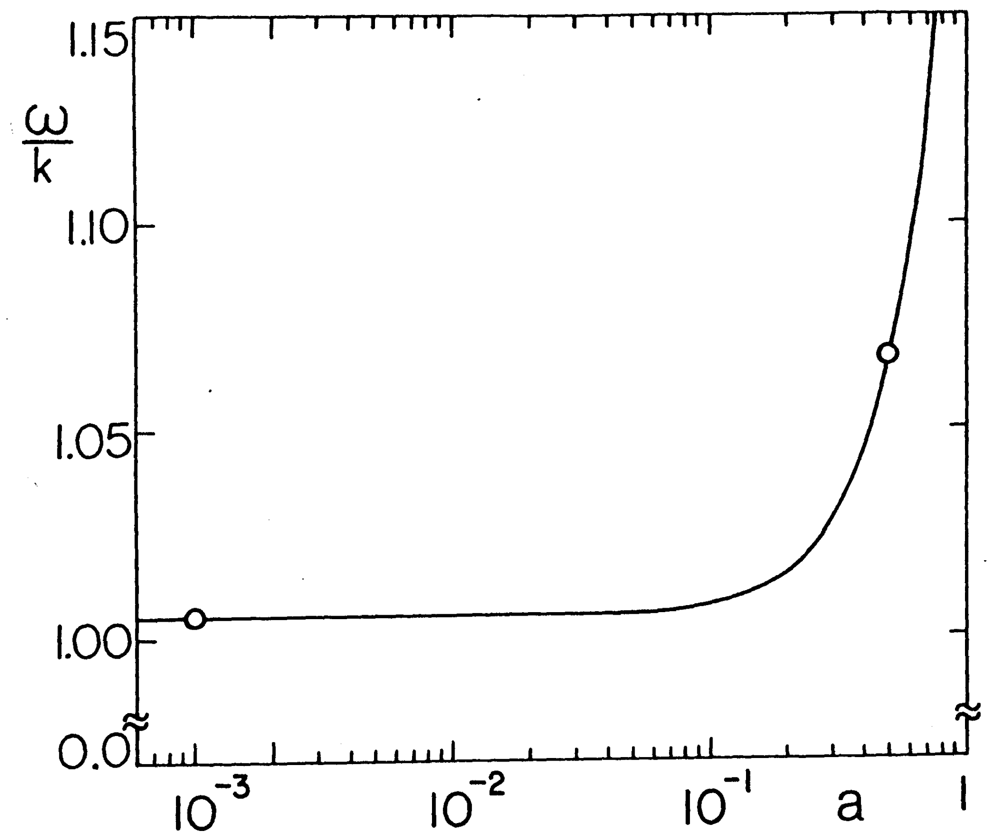

Fig. 1

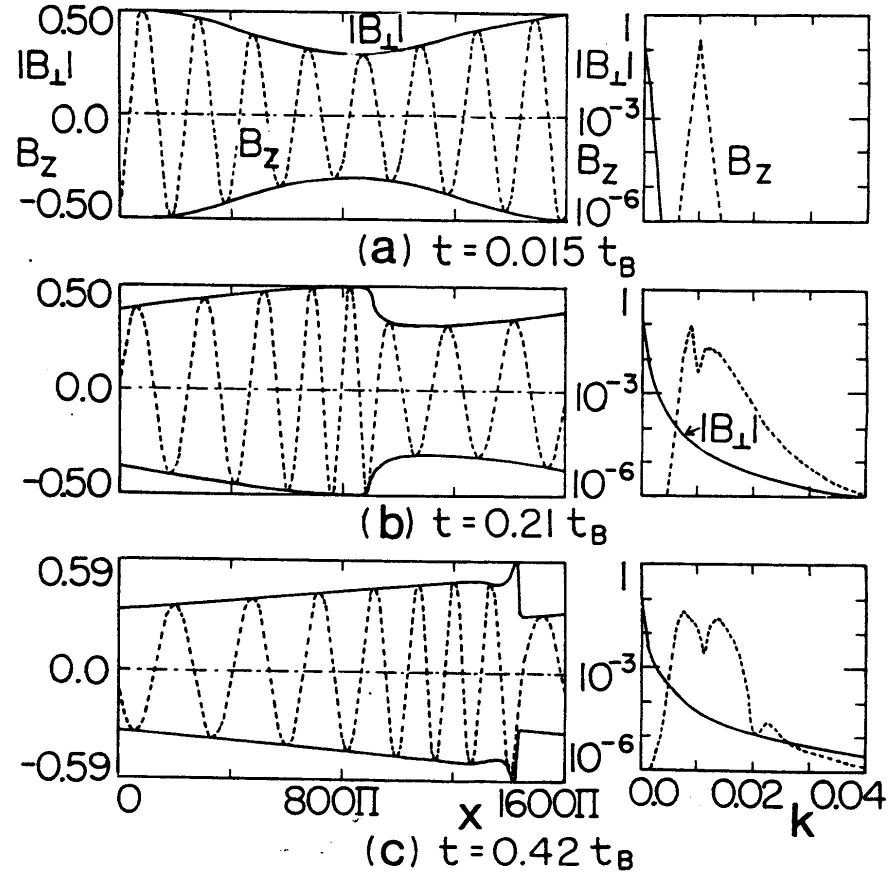
0.50
$|B_\perp|$
0.0
B_z
-0.50
$|B_\perp|$
B_z
1
$|B_\perp|$
10^{-3}
B_z
10^{-6}
B_z
(a) $t = 0.015\, t_B$
0.50
0.0
-0.50
1
10^{-3}
$|B_\perp|$
10^{-6}
(b) $t = 0.21\, t_B$
0.59
0.0
-0.59
0
800Π
x
1600Π
1
10^{-3}
10^{-6}
0.0
0.02
k
0.04
(c) $t = 0.42\, t_B$

Fig. 2

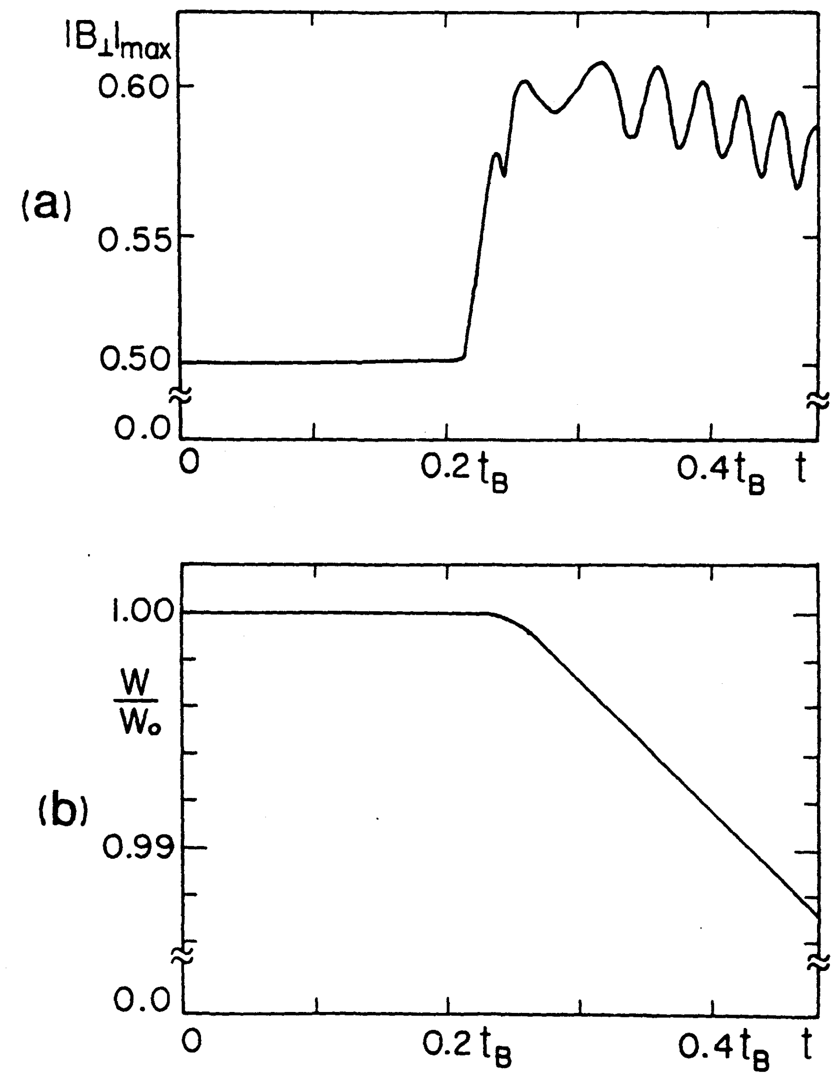
$|B_\perp|_{max}$
0.60
(a)
0.55
0.50
0.0
0
$0.2\,t_B$
$0.4\,t_B$
t
1.00
$\frac{W}{W_o}$
(b)
0.99
0.0
0
$0.2\,t_B$
$0.4\,t_B$
t

Fig. 3