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Condition for Absolute Confinement of Alpha Particles in Axisymmetric Tori

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

Abstract

More than 50% of fusion generated α particles can be absolutely trapped in the axisymmetric tori if the toroidal current I is larger than $I > 11$ MA.

The orbit confinement of the fusion generated alpha particles is one of the key for the confinement devices to work as a steady fusion reactor. In the nuclear reactions of interest to fusion reactors the generated alpha particles has energy of order 1 MeV. Then, a question arise whether the assumption of having the same confinement time of alpha particle as the fuel ions [1] is appropriate or not to the reactor studies since the transit time of alphas across the toroidal field by ∇B drift is much shorter than the slowing down time of them in the standard tokamak plasma. More over, it excite no little apprehension whether the single alpha particle of having MeV energy can be trapped or not in the collision free plasma in the magnetic devices of the practical interests.

The purpose of the present paper is to see the necessary condition in order for an axisymmetric torus to act as an absolute trap of fusion generated alpha particles. In order to have a definite solution the assumption is made that the fusion reaction occurs only on the elliptic magnetic axis where the plasma pressure is maximum.

The model of the axisymmetric toroidal plasma, for simplicity, is given by the following magnetic flux function ψ.

$$\Psi = \frac{3}{4} B_0 r^2 \left(1 - \frac{r^2+z^2}{R^2}\right) , \qquad r^2 + z^2 \leqq R^2$$

$$= -\frac{B_0 r^2}{2} \left[1 - R^3 (r^2+z^2)^{-3/2}\right] , \qquad r^2 + z^2 > R^2 \tag{1}$$

where the plasma is confined in the spherical region (i.e.

$r^2 + z^2 \leq R^2$) and the plasma vacuum interface is assumed to be a sphere of radius R [2].

In the present model we have one elliptic magnetic axis at

$$r = R/\sqrt{2} \qquad \text{and} \qquad z = 0. \tag{2}$$

Thus, the magnetic axis described by (2) is the source of alpha particles with energy E, where the energy E is typical for the reactions.

In the axisymmetric devices with a field which is constant in time, both the energy and the angular momentum are conserved. Then, the energy integral can be written in the form [3]

$$\begin{aligned} E &\equiv \frac{m\, v_0{}^2}{2} \\ &= \frac{m}{2}\,(v_r{}^2+v_z{}^2) + \frac{1}{2mr^2}\,(P_\theta - e\Psi)^2 , \end{aligned} \tag{3}$$

where v_0, P_θ, m and e are the velocity, the angular momentum, the mass and the charge of alpha particle respectively. Here, the angular momentum P_θ is defined by

$$P_\theta \equiv mrv_\theta + e\Psi , \tag{4}$$

where v_θ is the component of the velocity in the toroidal direction.

We now wish to rewrite (3) in dimensionless form, with B_0 and R appearing explicitly as a decisive parameter. We define dimensionless coordinates (x, y):

$$r = R\,x\ , \qquad\qquad z = R\,y \tag{5}$$

Then, the plasma is inside the sphere

$$x^2 + y^2 \leqq 1\ .$$

We introduce a dimensionless function g and the parameters β and T as follows.

$$\Psi = \frac{B_0 R^2}{2}\ g\ , \tag{6}$$

$$\beta = v_{\theta 0}/R\Omega\ , \tag{7}$$

$$T = (v_0/R\Omega)^2\ , \tag{8}$$

where $\Omega \equiv eB_0/m$ and $v_{\theta 0}$ is the initial velocity component in the toroidal direction. Then, the energy integral (3) can be rewritten by

$$T = (v_r^{\ 2} + v_z^{\ 2})/R^2\Omega^2 + \frac{1}{4x^2}(\sqrt{2}\beta + g_0 - g)^2\ , \tag{9}$$

where g_0 is the value of the magnetic surface on which the reaction takes place. Under the present assumption $g_0 = 3/8$. We can then estimate the region of possible particle positions:

$$Q \equiv T - \frac{1}{4x^2}(\sqrt{2}\beta + g_0 - g)^2 \geqq 0, \quad (10)$$

where

$$- T^{1/2} \leqq \beta \leqq T^{1/2} . \quad (11)$$

The condition for absolute confinement is that the equality

$$T - \frac{1}{4x^2} (\sqrt{2}\beta + g_0 - g)^2 = 0 \quad (12)$$

describes a family of closed curves in (x, y) space for any β restricted by the inequality (11) and that the quantity Q is positive in the closed domains. A maximum value of the dimensionless radius x on the closed curve is a function with respect to T and β:

$$x_{max} = f(T, \beta). \quad (13)$$

In order to heat the plasma ions effectively by the confined alpha particles they should be inside the last closed magnetic surface.
Thus, we have

$$x_{max} < 1. \quad (14)$$

The quantity x_{max} takes its maximum at $\beta = - T^{1/2}$. Therefore, the whole alpha particles produced can be confined inside the closed magnetic surfaces if and only if

$$f(T, -T^{1/2}) < 1 \quad . \tag{15}$$

The important parameter of any confinement systems to be a fusion reactor is that the number ratio C of alpha particles confined in the last closed magnetic surface to the whole alpha particles produced must be larger than a critical value. The precise estimate of the critical C is difficult to obtain at present. And we give a simple measure that more than a half of the alphas produced is inside the last closed surface.

Then, we have

$$f(T, 0) \leqq 1 \ , \tag{16}$$

where the assumption is made that whole alpha particles produced have the same probability with respect to the initial velocity vector.

From (16) we have for the present model

$$T \leqq (3/16)^2 \simeq 0.0352 \quad . \tag{17}$$

Since the most likely fuel cycle for first generation fusion reactors is D-T reaction, the alpha carries the energy of 3.5 MeV. Then the velocity v_0 is

$$v_0 = 1.3 \times 10^7 \text{ m/sec.}$$

And the condition (17) reduces to

$$R B_0 \geqq 2.74 \quad \text{Wb/m} . \tag{18}$$

Since the expression of the total toroidal current for the present model (i.e. Eq.(1)) is $I = \frac{5B_0R}{\mu_0}$, the inequality (18) can be reduced to the necessary toroidal current as follows.

$$I \geqq 10.74 \quad \text{MA}. \tag{19}$$

REFERENCES

[1] Kammash, T., in Fusion Reactor Physics, Ann Arbor Science Publishers, Inc. (1975), 21.

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[3] Morozov, A.I., Solov'ev, L.S., in Review of Plasma Physics (Leontovich, M.A., Ed.) Consultant Bureau (1966), 2, 260.