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RESEARCH REPORT

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Linked Min.B Configuration Inside
High Shear Magnetic Surface

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Abstract

Arrangement of the l = m baseball coils to form the toroidally linked Min.B configuration with large rotational transform is studied analytically and numerically. By an optimization the closed magnetic isobars are obtained inside the last closed magnetic surface having practical volume and high shear if l = 3 baseball coils are arranged.

1. INTRODUCTION

An invention of the twisted coils [1] have given a light to overcome the technological difficulty of large experiments of the stellrators. The foundamental difficulty, however, still remains to be overcome in the stellarators since the idea of the stellarator is based on ideal hydromagnetics. One of the foundamental difficulty of the stellarators is that particles with low collision rates escape rapidly from plasma by ∇B drift as soon as they enter the loss cone. In order to overcome this difficulty of the original stellarators as a fusion reactor, Dawson, Furth and Tenny [2] have suggested that the stellarators can work as the two component reactors if the Min.B mirror machine section is introduced to confine energetic particles. A limit of their idea leads us to the concept of the linked Min.B configuration [3] inside toroidal magnetic surface with large rotational transform. The linked Min.B configuration inside toroidal magnetic surfaces with rotational transform is superior to that without rotatioal transform in that the equilibrium of the passing particles is easily confirmed and that the effects of the error field to the existence of the magnetic surface can be minimized. The concept given here can be said as an extention of the MIRICLE concept originally proposed by Hall and McNamara [4]. The simplest realization of this concept can be completed in the axially symmetric configuration which can be adjusted [5] so that there exists a region where the magnitude of the magnetic field passes through a minimum. In the axially asymmetric configuration, however, the realization of this concept is not yet completed to date.

The present investigation is devoted to realize the linked Min.B configuration inside the axially asymmetric magnetic surfaces with large rotational transform by arranging the strongly twisted coils, i.e. $l = m$ baseball coils, and to show that the closed magnetic isobars are formed inside the last closed magnetic surface of having a practical aspect ratio if $l = 3$ baseball coils are arranged. An important parameter to arrange the baseball coils is the choice of the multi-polarity of the coils under the condition that the major radius of the torus is given. In the choice the following conditions should be kept in mind.

(1) The closed magneitc isobars must be present at least in the last closed magnetic surface.

(2) The volume of the last closed magnetic surface should be as large as possible.

The first condition is easily satisfied if $l = 2$ baseball coils are adopted. In this case, however, it is difficult to satisfy the second condition without going to rather impractically large aspect ratio, R/a, where R and a are the radius of the torus and that of the vacuum chamber respectively. In section 2 the analytic model of the present configuration is discussed in a linear geometry and it is confirmed that the Min.B region can be realized in the closed magnetic surface with rotational transform. In section 3 the numerically studies are made to find out how to arrange the $l = m$ ($m > 2$) baseball coils to realize a practical devices. We will see that the toroidal arrangement of the $l = 3$ baseball coils gives us the desirable machine parameters, i.e.

large rotational transform and the large volume of the last closed magnetic surface.

2. ANALYTIC MODEL

We consider the following magnetic scalar potential.

$$\Phi = B_0 z + \frac{b_0}{k} I_0(kr)\sin kz$$

$$+ \frac{b_\ell}{2}\left[\, \alpha^{-1} I_\ell(\ell\alpha r)\sin\ell(\theta - \alpha z)\right.$$

$$\left. + \beta^{-1} I_\ell(\ell\beta r)\sin\ell(\theta + \beta z)\right] , \qquad (1)$$

where $I_\ell(x)$ is the modified Bessel function of l-th order and $-1 < b_0/B_0 < 0$. It should be noted that the term in the bracket in R.H.S. of Eq.(1) can be rewritten by

$$\alpha^{-1} I_\ell(\ell\alpha r)\sin\ell(\theta - \alpha z) + \beta^{-1} I_\ell(\ell\beta r)\sin\ell(\theta + \beta z)$$

$$= 2\alpha^{-1} I_\ell(\ell\alpha r)\sin\ell\{\theta - (\alpha - \beta)z/2\}\cos\{\ell(\alpha + \beta)z/2\}$$

$$+ [\beta^{-1} I_\ell(\ell\beta r) - \alpha^{-1} I_\ell(\ell\alpha r)]\sin\ell(\theta + \beta z).$$

Therefore, it is apparent that this term represents the periodic-multipole configuration in the limit of $\beta \to \alpha$. In the following considerations in this section the attention is paid to the case of $\beta \simeq \alpha$.

The equation of the averaged magnetic surfaces after Morozov and Solovev becomes

$$\Psi = \frac{b_\ell^2}{8B_0}\left[\frac{I_\ell(\ell\alpha r)}{\alpha^3 r}\frac{\partial I_\ell(\ell\alpha r)}{\partial r} - \frac{I_\ell(\ell\beta r)}{\beta^3 r}\frac{\partial I_\ell(\ell\beta r)}{\partial r}\right]$$

$$- \frac{b_0 b_\ell}{4B_0 kr}\left[\sigma_i \frac{I_\ell(\ell\alpha r)}{\alpha^2} + \sigma_j \frac{I_\ell(\ell\beta r)}{\beta^2}\right]\frac{\partial I_0(kr)}{\partial r} \cos\ell\theta, \qquad (2)$$

where

$$\sigma_i = \begin{cases} 1, & \ell\alpha/k = 1 \\ 0, & \ell\alpha/k \neq 1 \end{cases}$$

and

$$\sigma_j = \begin{cases} 1, & \ell\beta/k = 1 \\ 0, & \ell\beta/k \neq 1. \end{cases}$$

In the case of $\ell = 2$ with $\sigma_j = 0$ Eq.(2) reduces to

$$\Psi = \frac{b_2^2}{8B_0}(\alpha - \beta)r^2\left[1 + \frac{2b_0}{b_2}\frac{\alpha}{\alpha - \beta}\sigma_i \cos 2\theta\right], \qquad (3)$$

where the Bessel function is expanded around the Z axis. The condition of closed magnetic surface becomes

$$\left|\frac{2\, b_0}{b_2(\alpha - \beta)}\right| < 1 \qquad (4)$$

The magnetic isobars near the point, $r = 0$, $z = 2n\pi/k$, becomes

$$B^2=(B_0+b_0)^2+ \frac{b_2{}^2}{4}(\alpha+\beta)^2[1+ \frac{2(B_0+b_0)b_0k^2}{b_2{}^2(\ +\ )^2}]r^2-k^2(B_0+b_0)b_0z^2 \quad (5)$$

And the condition that the isobars form a family of ellipsoids becomes

$$1 + \frac{2(B_0 + b_0)b_0k^2}{b_2{}^2(\alpha+\beta)^2} > 0 . \quad (6)$$

We note that the condition (4) is compatible with the condition (6). A configuration which satisfies both (4) and (6) is the one we desire. It is worth while noting that the situation is not so simple in the case of $\ell > 2$. In the case of $\ell > 2$ the multipole field is always too weak to compensate for the radial decrease in the bumpy component. In this case closed magnetic surface are still formed but instead of a single minimum at $r = 0$, $z = 2n\pi/k$. There are ℓ minima situated off the axis in the linear geometry. In the toroidal geometry of practical aspect ratio, however, we have always a single minimum by the effect of the toroidal perturbation and the weakness of the multipole field compared with the bumpy component offers an advantage to have a large volume of the last closed magnetic surface.

3. NUMERIAL STUDIES

In this section we discuss about the realization of the toroidally linked Min.B configuration with large rotational transform by arrangement the $\ell = 3$ baseball coils.

3.1 SINGLE UNIT COIL

The single unit coil is wound on the surface of the cylinder of radius a and of the length 2b as shown in Fig.1. An important fact in designing the Min.B field is to decide the curve of the conductor along which the current flows. The curve on the surface is given by

$$x = a \cos\theta \ ,$$

$$y = b\gamma|\sin\ell\theta|^{\nu} \ , \qquad (7)$$

$$z = a \sin\theta \ ,$$

where $0 < \nu \leqq 1$, ℓ is the multipolicity which is an integer and $\gamma = \text{sign}(\sin\ell\theta)$. The bird's eye view of the curve on the surface of the cylinder is shown in Fig.2, where $\ell = 3$, $\nu = 1/4$ and $a = b$. The magnetic isobars in the plane of z = const. and y = const. are shown in Fig.3 and Fig.4. From the numerical study we see that the parameter ν must be less than 1/2, otherwise isobars never form a family of closed surfaces.

3.2 TOROIDAL ARRANGEMENT

The baseball coils wound on the cylinder is arranged toroidally as shown in Fig.5. N pairs of coils are set along the torus with small gaps adjacent to the next coils. Let a small gap be 2δ, then we have

$$s + t = \frac{\pi}{N} \quad , \tag{8}$$

where

$$s = \sin^{-1} \frac{b}{R - a} \quad , \qquad t = \sin^{-1} \frac{\delta}{R - a}$$

and R is the major radius of the torus. It should be noted that the phase difference Δ between the neighboring coils is given by

$$\Delta = \frac{\pi}{\ell} - \frac{2p\pi}{N} \tag{9}$$

where p is an integer. The presence of the magnetic surfaces is guaranteed by this arrangement (i.e. helical periodic multipole arrangement originally suggested by Lenard [6]), even though the Min.B regions are present locally in the torus. The bird's eye view of the toroidal arrangement of coils in an optimized case is given in Fig. 6.

In this case $a/R = 0.2$, $b/R = 0.18$, $\delta/R = 0.01$, $\nu = 1/2$, $\ell = 3$ and $p = 1$.

The family of the magnetic isobars in the z=0 plane is shown in Fig.7 by the Mercator's method of projection. We see that the families of the closed isobars is linked periodically in the toroidal direction. An important innovation of the present design of the torus can be made clear if we see that the families of the closed isobars are present inside the last closed surface. To see this, the distribution of the magnetic isobars in the minor crosssection is plotted in the last magnetic surface as shown in Fig.8. From the numerical studies it is confirmed that the number of the coils must satisfy

$$N \geqq 13 \tag{10}$$

in order to have the closed isobar surfaces inside the last closed magnetic surface, provided that

$$b/a \gtrsim 0.8.$$

The rotational transform ι in the example shown in Fig.8. is $1.1\,\pi$ on the last closed surface. Since the average radius $\bar{a}$ in the Min.B region is $\bar{a} = a/3$ from the numerical result, the shear parameter Θ can be estimated as

$$\Theta \equiv \frac{\iota}{\bar{a}} \simeq 3.3\pi/a$$

4. CONCLUSION

From these investigation we see that the closed isobars can be inside the high shear magnetic surface. This mirror section should be useful for the high energy ion beam injection to realize the concept of the two component reactor. By choosing the design parameters, especially the coil number N and the ratio b/a, this system can have the optimum mirror ratio of the Min.B region inside the high shear magnetic surface.

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Figure Captions

Fig. 1 Coordinate system and a single sylinder on which coil is wound.

Fig. 2 A single unit coil seen from Z axis. Dotted line indicates the lower part of the coil which could not be seen from above.

Fig. 3 Intersection of magnetic isobars of the single unit coil with plane z=constant. Constants indicate the magnitude of the magnetic isobars normalized at the center.

Fig. 4 Intersection of magnetic isobars of the single unit coil with plane y=constant. Constnats indicate the magnitude of the magnetic isobars normalized at the center.

Fig. 5 Toroidal arrangement of the cylinders on which coils are wound.

Fig. 6 Schematic picture of 13 unit coils seen from above. Dotted lines indicate the lower part of the coils which could not be seen from above. Constants indicate the azimuthal angle along torus.

Fig. 7 Intersection of magnetic isobars of the toroidally arranged 13 units coils with equatorial plane of the torus. Note that the major radius side of the picture is drawn on a smaller scale in order to depict the intersections in the Cartesian coordinate system.

Fig. 8 Intersections of magnetic isobars and last closed magnetic surface with planes normal to the magnetic axis. Constants indicate the azimuthal angle along the torus. Alphabet corresponds the strength of the magnetic isobars, i.e. a=1.1, b=1.05, c=1.0, d=0.995 and e=0.99. Dotted lines are the intersections of the last closed magnetic surfaces at 0°, 83° and 166°.

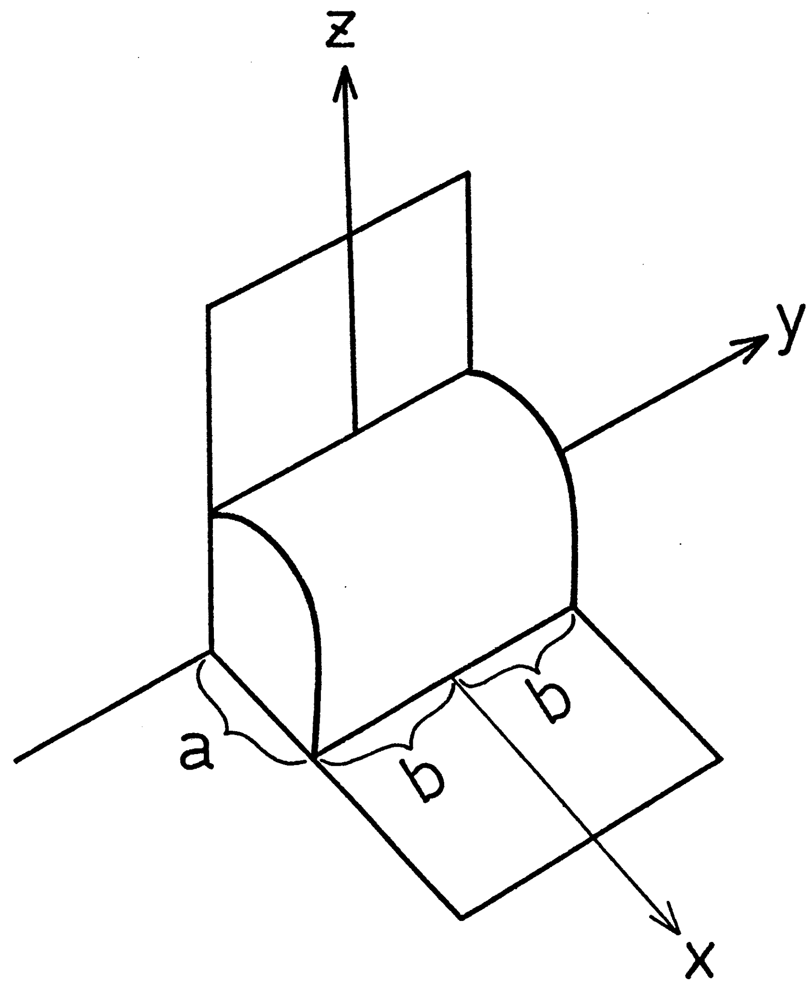

Fig. 1

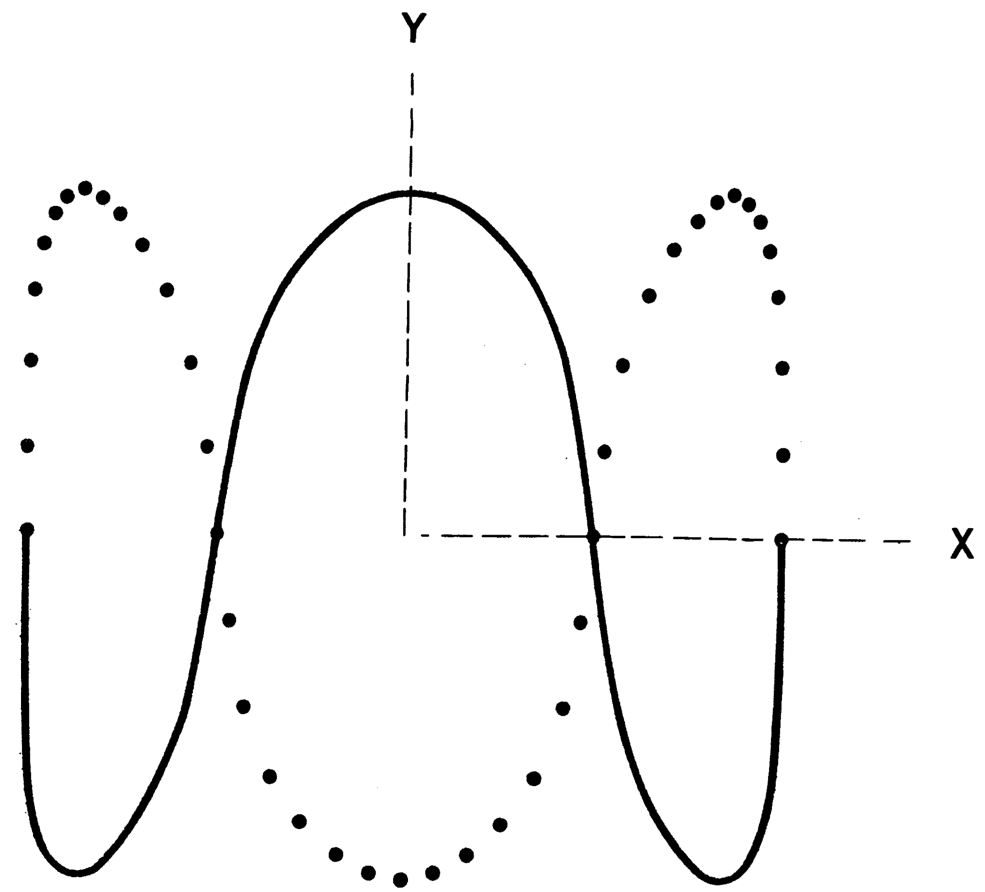

Fig. 2

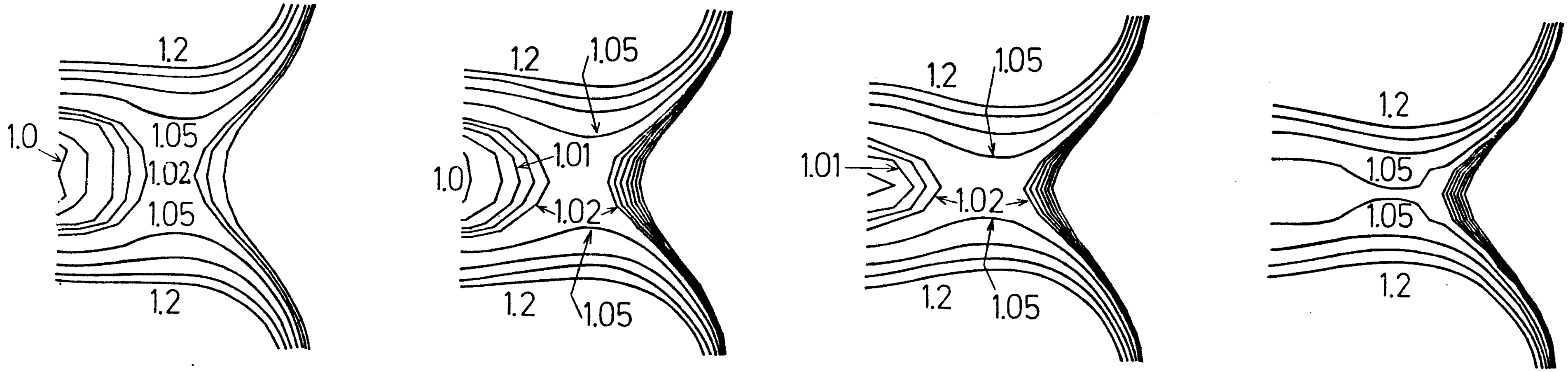

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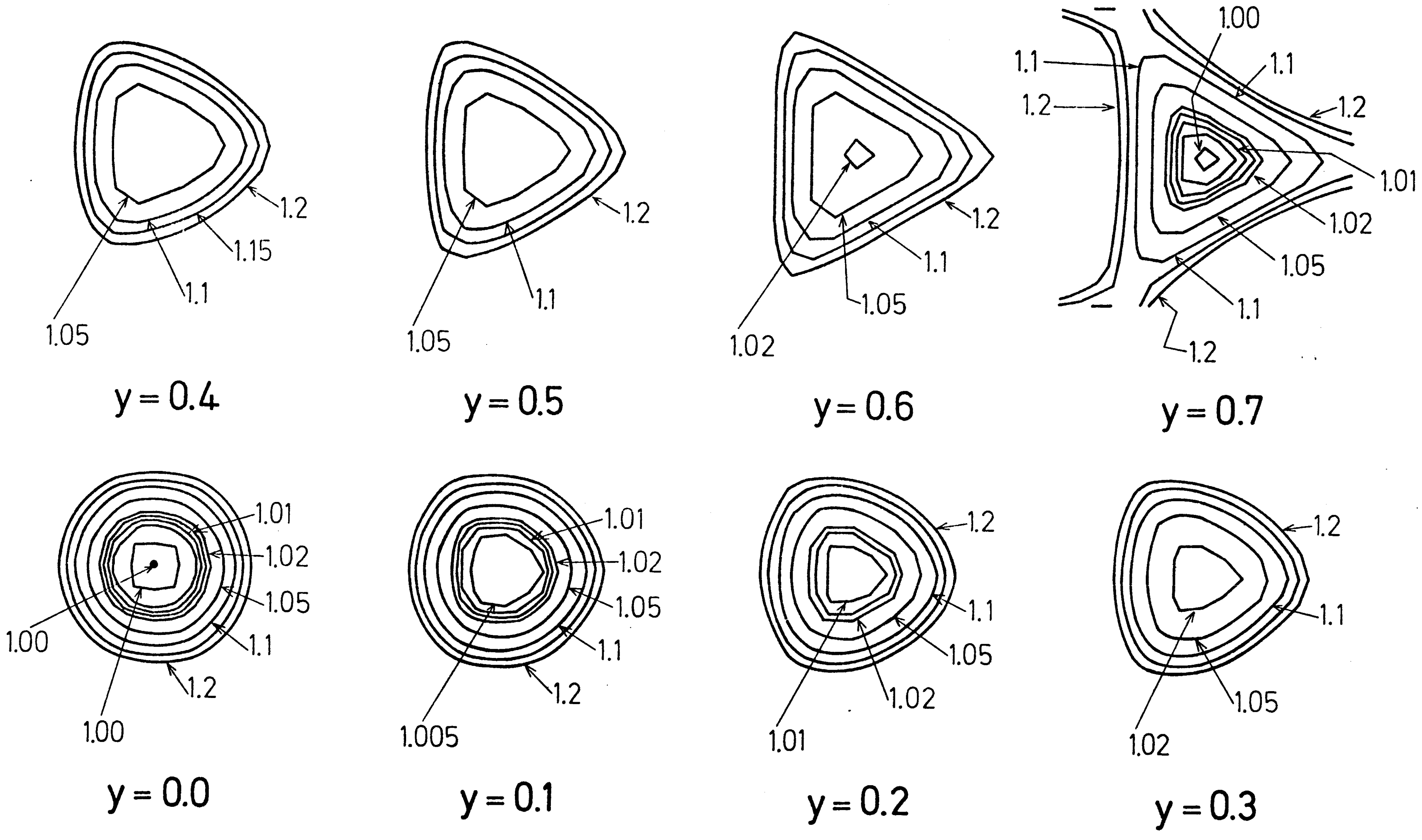
Fig. 3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1.2
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1.00
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1.1
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1.01
1.02
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1.1
1.2
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1.00
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y = 0.1
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y = 0.2
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y = 0.3

Fig. 4

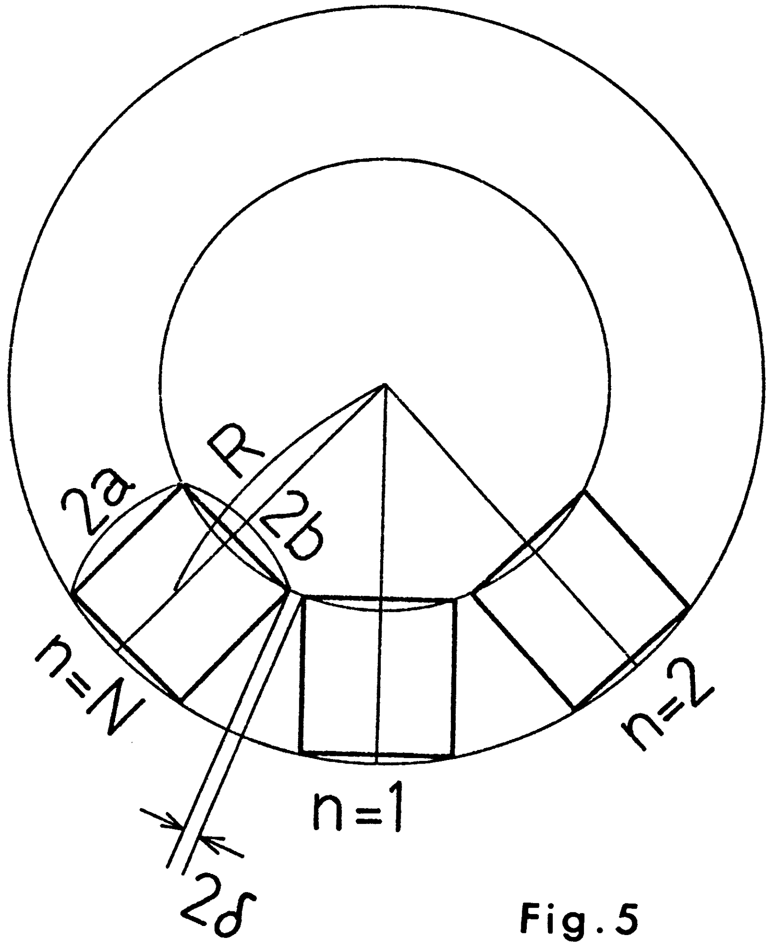

Fig. 5

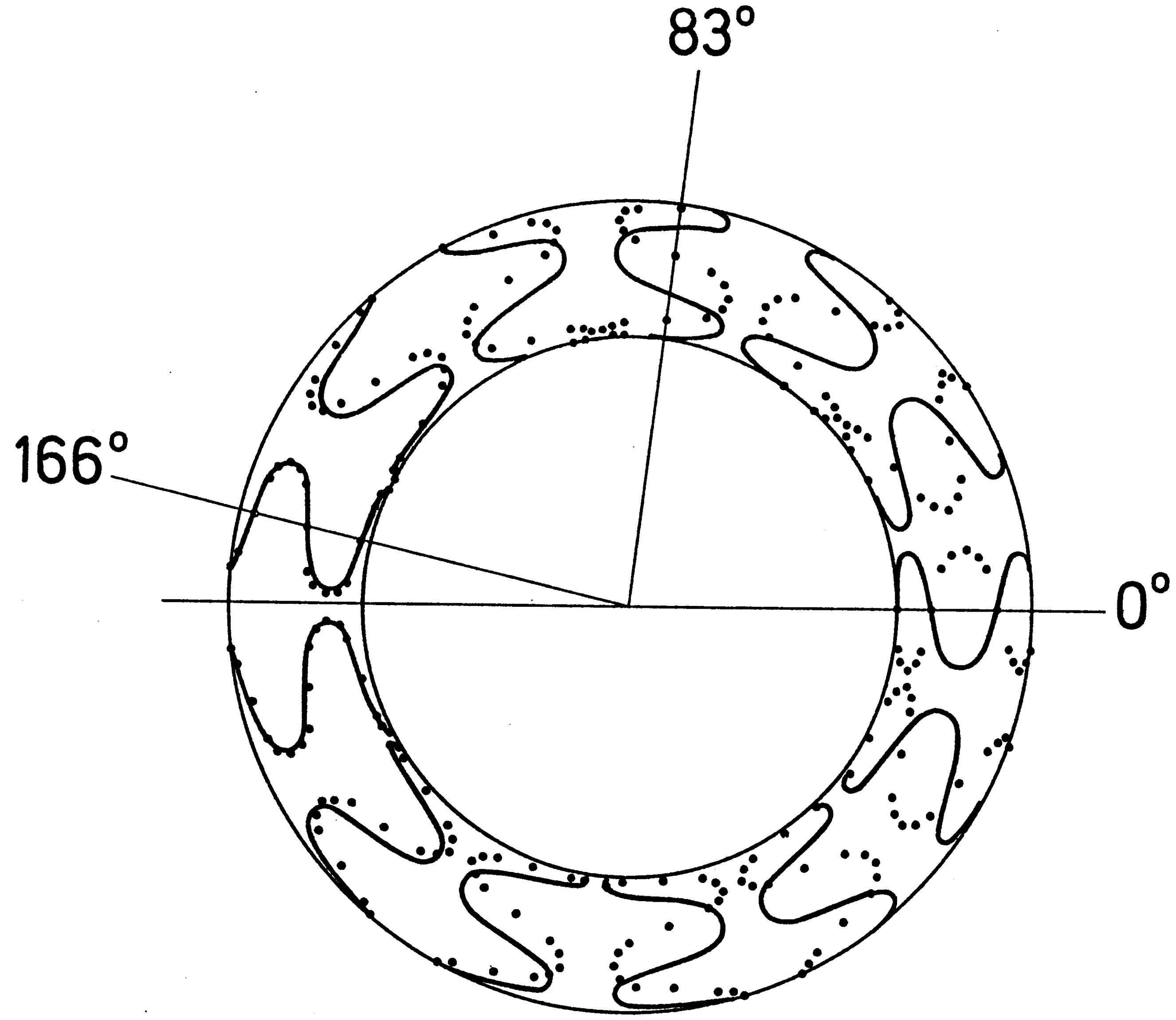

Fig. 6

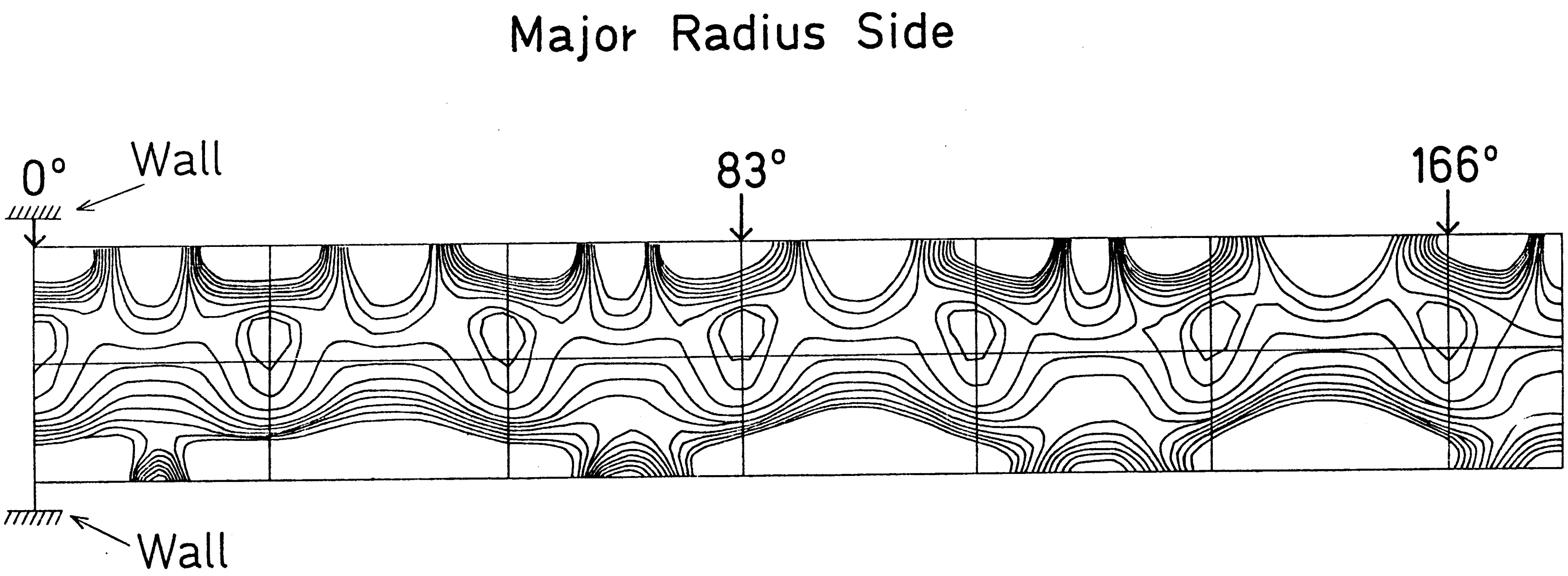

Fig.7

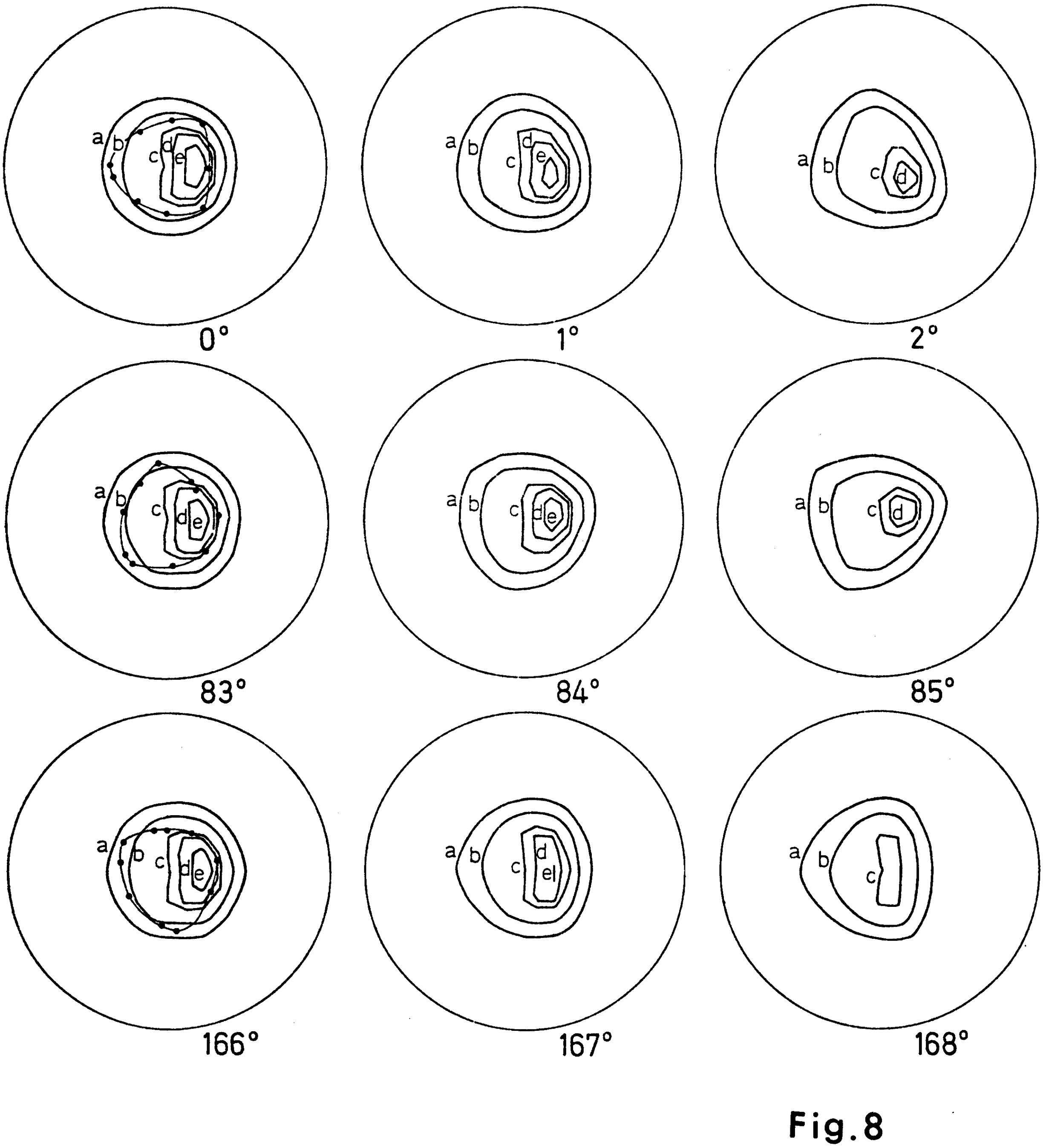
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Fig.8