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MHD Instability of Plasma Column Induced by Injection of Strong Current Ion Beams

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Abstract

It is theoretically predicted that a MHD instability is induced by the strong ion beam injection perpendicularly into the plasma column. Its growth rate is of order kC_A, where C_A is the Alfven velocity and k is the wave number along the magnetic field.

1. Introduction

Plasma heating by beam injection [1] gives us a hope to have a toroidal plasma in which the fusion ignition condition is satisfied. The technology which supports this heating method is wonderfully progressing to a higher state with some important discoveries [2] to produce the intense ion beam. From the point of view of creating the intense neutral particles for neutral beam heating, the cross section of the charge transfer becomes extremely small, when the energy of ions in the charge neutralizer attains of order 50 keV $\sim$ 1 MeV. This suggests that the ratio Q, of the fusion power released to the power required to produce the neutral beam for the injection becomes extremely small. Since the fusion reactors will, by definition, use nuclear fusion events as their source of power, a better reactor should be not only to maximize the rate of fusion reaction by choosing the energy range of the reacting particles but also to minimize the necessary power required to attain the fuel plasma in a self-heated state. As an alternative to the energetic neutral beam injection, the high energy ion beam should be injected directly into a fuel plasma without passing through a neutralizer. In this case the electrons should be supplied to satisfy the charge neutrality in the plasma. Then after a few slowing down time of the ion beam its energy decreases to a certain value where the cross section of the fusion reaction is maximum.

When both the energy and the current of the injected ion beam become high enough, the effect of the poloidal current of the beam on the toroidal field should be taken into account to analyse the MHD stability of the plasma column. This situation is very similar to the problem which occurs in the infinitely long Ion-layer in the plasma without field reversal. This type of the combined ion beam/background plasma system has recently been considered by some authors [3].

The purpose of the present investigation is to see the condition for the hydrodynamic instability of the plasma column with an rotating ion beam. The plasma column with an embedded ion beam which rotates around the axis of the plasma column can be thought of as a model of the equilibrium state of the toroidal plasma characterized by a marked non-Maxwellian ion velocity distribution such as might be produced by the transverse injection of a strong current ion beam into a thermal plasma.

In the present analysis the assumption is made that the background plasma is a cold, perfect MHD fluid in which a cold, charge-neutralized ion beam rotates. In Sec.2 we consider the equilibrium state, and in Sec.3 we present the simultaneous equations for the perturbed quantities. The set of equations is reduced into a single differential equation in Sec.4, and the dispersion relations are exhibited in Sec.5. In order to afford an insight into the MHD instability induced by the ion beam the dispersion relations are approximately solved in Sec.6 and it is shown that the perpendicular injection of the ion beam has a tendency to make the system unstable.

2. Equilibrium

The geometry of the plasma column with an embedded ion beam rotating around the axis of symmetry is shown in Fig.1, where the cylindrical coordinate system is used and the rotating ion beam is assumed to be a infinitely long column with radius a. A cold, dense plasma of uniform mass density ρ fills the metal cylinder to the wall whose radius is d. The macroscopic flow velocity and the current in the backgound plasma are assumed to be vanished in the equilibrium. Then the structure of the magnetic field in equilibrium is determined by

$$\frac{1}{B_z}\frac{dB_z}{dr} = \frac{ne^2\mu_0}{M} r \qquad (1)$$

and $$B_\theta = B_r = 0 \quad ,$$
where $\vec{B} = \vec{B}(B_r, B_\theta, B_z)$ and e, M and n are the charge, the mass and the density of the ion beam. The quantity μ_0 is the magnetic permeability of vacuum.

In the case of the ion beam density to be uniform, the magnetic field in the equilibrium becomes

$$\begin{aligned} B_z &= B_0 \exp\left(\frac{ne^2\mu_0}{2M} r^2\right) , \qquad 0 < r \leq a, \\ &= B_0 \exp\left(\frac{ne^2\mu_0}{2M} a^2\right) , \qquad a < r \leq d, \end{aligned} \qquad (2)$$

where B_0 is the strength of the magnetic field on the z axis.

3. Equations for Linearized Quantities

The linearized equations describing the cold background plasma, the field and the ion cloud are

$$\rho\frac{\partial\vec{u}}{\partial t} = \vec{J}_p \times \vec{B}_0 \quad , \tag{3}$$

$$\vec{E} + \vec{u} \times \vec{B}_0 = 0 \quad , \tag{4}$$

$$\nabla \times \vec{B}_1 = \mu_0(\vec{J}_p + \vec{J}_b), \tag{5}$$

$$\frac{\partial\vec{B}_1}{\partial t} + \nabla \times \vec{E} = 0, \tag{6}$$

$$M(\frac{\partial\vec{v}_1}{\partial t} + \vec{v}_0\cdot\nabla v_1 + v_1\cdot\nabla v_0) = e(\vec{E} + \vec{v}_0 \times \vec{B}_1 + \vec{v}_1 \times \vec{B}_0), \tag{7}$$

$$\frac{\partial n_1}{\partial t} + \nabla\cdot(n_0\vec{v}_1 + n_1\vec{v}_0) = 0 \quad , \tag{8}$$

where $\vec{u}$ and $\vec{v}$ are the velocities of the plasma and the ion beam respectively, and $\vec{E}$ and n are the electric field and the density of ion cloud, and $\vec{J}_p$ and $\vec{J}_b$ are the current densities of plasma and beam, respectively. The suffixes 0 and 1 mean the unperturbed and the linealized quantities respectively.

From Eq.(3) we immediately have

$$\vec{B}_0\cdot \frac{\partial\vec{u}}{\partial t} = 0 \quad . \tag{9}$$

Therefore, $\vec{B}_0\cdot\vec{u} = 0$, provided it vanishes initially. Then, from Eq.(4)

$$\vec{u} = \vec{E} \times \vec{B}_0/B_0{}^2. \tag{10}$$

It is convenient to introduce the vector potential $\vec{A}$ with gauge chosen such that

$$\vec{B}_1 = \nabla \times \vec{A}$$

and

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \quad .$$

Then from Eqs. (3), (5) and (10) we have

$$\vec{B}_0 \times \vec{I} = 0 \quad , \tag{11}$$

where

$$\vec{I} = \left(\frac{\mu_0 \rho}{B_0{}^2}\right) \frac{\partial^2 \vec{A}}{\partial t^2} + \nabla \times \nabla \times \vec{A} - \mu_0 \vec{J}_b \quad . \tag{12}$$

In order to solve the differential equation with respect to $\vec{A}$ the perturbed current density $\vec{J}_b$ of ion beam must be obtained. The quantity $\vec{J}_b$ can be calculated in principle by Eqs. (7) and (8) for the general configuration of the ion beam. In the present analysis we consider the case of having a cylindrical symmetry, and equilibrium quantitites are functions of the distance r from the axis of the cylinder. Taking advantage of the symmetry of the undisturbed system to Fourier-analyse the perturbed quantities we write

$$f = f(r,\, m,\, k) \exp i\, (m\theta + kz - \omega t) \quad , \tag{13}$$

where f represents the perturbed quantity, and m and k are the mode number and the wave number respectively. By using Eq. (4) we have

$$\vec{E} \cdot \vec{B}_0 = 0 \quad . \tag{14}$$

This means that the z component of the vector potential is vanished in the cylindrical ion beam considered here, i.e.

$$\vec{A} = \vec{A}(A_r, A_\theta, 0) \quad . \tag{15}$$

From (11) and (12) we see that the problem to be solved at present is reduced to a dual simultaneous partial differential equation. In order to solve the equation we must, at first, calculate the linearized current density of ion beam. The componenets of the perturbed velocity are

$$v_r = \frac{e}{M}\frac{1}{\omega^2-\Omega^2}[\omega(\omega-m\Omega)A_r-2i\omega\Omega A_\theta-i\omega\Omega r\frac{\partial A_\theta}{\partial r}] \ , \tag{16}$$

$$v_\theta = \frac{e}{M}\frac{1}{\omega^2-\Omega^2}[i\Omega(\omega-m\Omega)A_r+(\omega^2+\Omega^2)A_\theta+\Omega^2 r\frac{\partial A_\theta}{\partial r}] \ , \tag{17}$$

$$v_z = \frac{e}{M}\frac{\Omega}{\omega}krA_\theta \ , \tag{18}$$

where $\Omega \equiv eB_{z0}/M$.

And the perturbed density becomes

$$\begin{aligned} n_1 = & -\frac{i}{r}\frac{en_0}{M}\frac{1}{(\omega^2-\Omega^2)(\omega+m\Omega)}[\{\omega(\omega-m\Omega)(1+d_n+\frac{2\Omega^2}{\omega^2-\Omega^2}d_B) \\ & - m\Omega(\omega-m\Omega) - m\Omega\omega d_B\}A_r+\omega(\omega-m\Omega)r\frac{dA_r}{dr} - i\omega\Omega r^2\frac{d^2A_\theta}{dr^2} \\ & + i\{m(\omega^2+\Omega^2) - 2\omega\Omega(1+d_n+\frac{\omega^2+\Omega^2}{\omega^2-\Omega^2}d_B)\}A_\theta \\ & + i\{m\Omega^2-\omega\Omega(4+d_n+\frac{\omega^2+\Omega^2}{\omega^2-\Omega^2}d_B)\}r\frac{dA_\theta}{dr}] \\ & + \frac{en_0}{M}\frac{\Omega}{\omega(\omega+m\Omega)}k^2rA_\theta \ , \end{aligned} \tag{19}$$

where d_n and d_B are defined by

$$d_n \equiv \frac{r}{n_0}\frac{dn_0}{dr} \quad \text{and} \quad d_B \equiv \frac{r}{\Omega}\frac{d\Omega}{dr} \quad .$$

4. Reduction to an Ordinary Differential Equations

We now wish to rewrite (11) in dimensionless form except the vector potential. We define dimensionless radius x:

$$r = ax \quad . \tag{20}$$

We introduce dimensionless parameters $\tilde{\omega}$, K and λ as follows.

$$\tilde{\omega} = \omega/\Omega \ , \quad \lambda^2 = \omega^2_{pb} a^2/c^2$$

and (21)

$$K^2 = \left(\frac{\omega^2}{c_A^2} - k^2\right)a^2 \quad ,$$

where $\omega^2_{pb} = ne^2/\varepsilon_0 M \ , \quad c_A^2 = B^2_{z_0}/\mu_0\rho.$

Then the r component of Eq.(11) becomes

$$\left\{\frac{m^2}{x^2} - K^2 - \lambda^2 \frac{\tilde{\omega}(\tilde{\omega}-m)}{\tilde{\omega}^2 - 1}\right\} A_r$$

$$= -i\left\{\left(\frac{m}{x^2} + \lambda^2 \frac{\tilde{\omega}}{\tilde{\omega}^2-1}\right) x \frac{dA_\theta}{dx} + \left(\frac{m}{x^2} + \frac{2\lambda^2\tilde{\omega}}{\tilde{\omega}^2-1}\right) A_\theta\right\} \quad . \tag{22}$$

The θ component becomes more complicated than the r component given above, i.e.

$$- \frac{d}{dx}\left\{\frac{1}{x}\frac{d}{dx}(xA_\theta)\right\} - K^2 A_\theta = -im \frac{d}{dx}\left(\frac{A_r}{x}\right) + \frac{\lambda^2}{\tilde{\omega}^2-1}\left\{i(\tilde{\omega}-m)A_r + (\tilde{\omega}^2+1)A_\theta + x\frac{dA_\theta}{dx}\right\}$$
$$+ \frac{i\lambda^2}{(\tilde{\omega}+m)(\tilde{\omega}^2-1)}\Big[\tilde{\omega}(\tilde{\omega}-m)x\frac{dA_r}{dx} + \Big\{(\tilde{\omega}-m)\tilde{\omega}\Big(1+d_n+\frac{2d_B}{\tilde{\omega}^2-1}\Big) - m(\tilde{\omega}-m) - m\tilde{\omega}d_B\Big\}A_r$$
$$+ i\Big\{m(1+\tilde{\omega}^2) - 2\tilde{\omega}\Big(1+d_n+\frac{\tilde{\omega}^2+1}{\tilde{\omega}^2-1}d_B\Big)\Big\}A_\theta + i\Big\{m-\tilde{\omega}\Big(4+d_n+\frac{\tilde{\omega}^2+1}{\tilde{\omega}^2-1}d_B\Big)\Big\}x\frac{dA_\theta}{dx} - i\tilde{\omega}x^2\frac{d^2A_\theta}{dx^2}\Big]$$
$$+ \frac{\lambda^2 K^2}{\tilde{\omega}(\tilde{\omega}+m)} A_\theta \quad , \tag{23}$$

where $\tilde{k}^2 = k^2 a^2$.

In order to pick out the decisive terms in (22) and (23) on the stability we introduce an ordering in a small quantity. In the present investigation we define the small quantity.

$$\varepsilon \equiv \omega/(eB_0/M) << 1 \quad . \tag{24}$$

And it is found that the consistent scheme which is suitable for the problem of interest is

$$\begin{aligned} \tilde{\omega} &\sim O(\varepsilon) , \\ \lambda, K, \tilde{k}, \omega a/c_A &\sim O(\varepsilon^{1/2}) \quad . \end{aligned} \tag{25}$$

Then, since the differential equation reduced from Eqs.(22) and (23) are different between the case of m=0 and the case of m≠0, these are treated in the separate subsections.

A. Case of m≠0

In this case the ratio A_r/A_θ is of order unity. By dropping terms of order ε^2 we have a single differential equation with respect to A_θ. Inside of the beam

$$\begin{aligned} &\frac{d^2 A_\theta}{dx^2} + \left\{\frac{3}{x} + 2x\left(\frac{K^2}{m^2} - \frac{\omega^2 a^2 \lambda^2}{K^2 c_A{}^2}\right)\right\}\frac{dA_\theta}{dx} \\ &+ \left[\frac{\tilde{k}^2\lambda^2}{m\tilde{\omega}} + \frac{K^2}{m^2}(2 + m^2) - \frac{2\omega^2 a^2 \lambda^2}{K^2 c_A{}^2}\right. \\ &+ \left.\left\{1 - m^2 + \frac{\tilde{k}^2\lambda^2}{K^2}\left(1 - \frac{m}{\tilde{\omega}}\right)\right\}\frac{1}{x^2}\right] A_\theta = 0 \quad . \end{aligned} \tag{26-a}$$

Outside of the beam there is a perfect MHD plasma whose governing equation can be obtained by letting $\lambda \to 0$ in Eq.(26-a). We introduce the quantities Ψ and ρ in place of A_θ and x, i.e.

$$A_\theta = \Psi \exp\{-\frac{x^2}{2}(\frac{K^2}{m^2} - \frac{\omega^2 a^2 \lambda^2}{K^2 c_A{}^2})\} \quad ,$$

and

$$\rho = x^2(\frac{K^2}{m^2} - \frac{\omega^2 a^2 \lambda^2}{K^2 c_A{}^2}) \quad ,$$

whereupon we obtain the equation for ψ

$$\frac{d^2\Psi}{d\rho^2} + \frac{2}{\rho}\frac{d\Psi}{d\rho} + [\frac{\gamma}{\rho} - \frac{1}{4} + \frac{M^2}{\rho^2}]\Psi = 0 \quad , \tag{26-b}$$

where

$$\gamma = \frac{\frac{\lambda^2 \tilde{K}^2}{m\tilde{\omega}^2} + \frac{2\omega^2 a^2 \lambda^2}{K^2 c_A{}^2} + \frac{K^2}{m^2}(m^2-2)}{4(\frac{K^2}{m^2} - \frac{\omega^2 a^2 \lambda^2}{K^2 c_A{}^2})}$$

and

$$M^2 = \frac{1}{4}\{\frac{\tilde{K}^2 \lambda^2}{K^2}(1 - \frac{m}{\tilde{\omega}}) - (m^2-1)\} \quad .$$

Eq.(26-b) has exactly the same form of the schrödinger equation with respect to the motion of charged particle in a Coulomb field.

B. Case of m=0.

In this case the ratio A_r/A_θ becomes of order ε. And the reduced equation is

$$\frac{d^2 A_\theta}{dx^2} + (\frac{1}{x} - 4\lambda^2 x)\frac{dA_\theta}{dx} + \{(\frac{\lambda^2 \tilde{K}^2}{\tilde{\omega}^2} + K^2 - 4\lambda^2) - \frac{1}{x^2} + \frac{\lambda^4 \tilde{K}^2}{\tilde{\omega}^2}x^2\} A_\theta = 0 \, . \tag{27-a}$$

Eq.(27-a) is also reduced to a well-known form which appears in the quantum mechanics. Here, the quantities ψ and ρ are introduced by

$$A_\theta = \Psi x \exp(\lambda^2 x^2)$$

and

$$\rho = x^2 \left(- \frac{\lambda^4 \tilde{k}^2}{\tilde{\omega}^2} \right)^{1/2}$$

whereupon we have

$$\frac{d^2\psi}{d\rho^2} + \frac{2}{\rho}\frac{d\psi}{d\rho} + \left(\frac{\Gamma}{\rho} - \frac{1}{4}\right)\psi = 0 \quad , \tag{27-b}$$

where $$\Gamma = \frac{\dfrac{\lambda^2 \tilde{k}^2}{\tilde{\omega}^2} + K^2}{4\left(-\dfrac{\lambda^4 \tilde{k}^2}{\tilde{\omega}^2}\right)^{1/2}} \quad .$$

The equation which governs outside of the beam is reduced to the well-known Bessel's form by letting $\lambda \to 0$ in Eq.(27-a).

5. Dispersion Relations

By solving Eqs.(26) and (27) inside and outside of the beam and joining with the appropriate boundary conditions we have the dispersion relations. At x=1 the following boundary conditions should be satisfied.

$$\text{i)}\ A_{\theta p} = A_{\theta b} \quad ,$$

$$\text{ii)}\ \frac{dA_{\theta p}}{dx} = \frac{dA_{\theta b}}{dx} \quad , \tag{28}$$

where the suffixes p and b correspond to the outer and the inner solutions respectively.

And also at $x = d/a \equiv x_w$,

$$A_{\theta p} = 0 \quad . \tag{29}$$

A. Case of $m \neq 0$

Inside of the beam (i.e. $|x| \leq 1$) the solution of Eq.(26-a) is found to be

$$A_{\theta b} = C_1 \ \rho^q \exp(-\rho) F(1 + q - \gamma \ ; \ 2q + 2; \rho \ )$$

where

$$q = \frac{1}{2} \ [\ -1 + \ \sqrt{1 - 4M^2} \ ]$$

and $F(s,t,z)$ is the confluent hypergeometric function defined by

$$F(s,t,z) = 1 + \frac{s}{t}\frac{z}{1!} + \frac{s(s+1)}{t(t+1)}\frac{z^2}{2!} + \cdots\cdots \ . \qquad (31)$$

Outside of the beam the parameter q is displaced by q', where $q' = \frac{1}{2}(-1+|m|)$. Thus, the solution of (26-a) in the plasma region becomes

$$A_{\theta p} = \rho^{q'} \exp(-\rho) [C_2 F(1 + q'-\gamma; \ 1 + |m| \ ; \ \rho)$$
$$+ C_3\{F(1+q'-\gamma; \ 1+|m|;\rho)\ell n\rho + F^*(1+q'-\gamma; \ 1+|m|;\rho)\}],$$

(32)

where

$$F^*(s;t;z) = (-1)^t (t-1)! \sum_{u=0}^{t-2} \frac{(-1)^n (t-n-2)! \ z^{1-t+n}}{(s-1)(s-2)\cdots\cdot(s-t+n+1)}$$

$$+ \ (t-1)! \sum_{n=0}^{\infty} \frac{s(s+1)\cdots\cdot(s+n-1)}{(t+n-1)!n!}[\sum_{r=0}^{n-1} (\frac{1}{s+1} - \frac{1}{t+r} - \frac{1}{r+1})]z^n \ ,$$

(33)

and C_2 and C_3 are costants.

In order to obtain a simplified dispersion relation an assumtion is made that the ratio d/a is sufficiently large. From both this assumption and the condition (29) we have

$$C_3 = 0 \quad . \qquad (34)$$

By using the equality (34) and the boundary conditions (28) the dispersion relation becomes

$$(q-A) + \frac{1+q-\gamma}{2(1+q)} A \frac{F(2+q-\gamma;\ 2q+3;\ A)}{F(1+q-\gamma;\ 2q+2;\ A)}$$

$$= (q' - \frac{K^2}{m^2}) + \frac{1+q'-\gamma}{1+|m|} \frac{K^2}{m^2} \frac{F(2+q'-\gamma;\ 2+|m|;\ \frac{K^2}{m^2})}{F(1+q'-\gamma;\ 1+|m|;\ \frac{K^2}{m^2})} , \tag{35}$$

where $$A = \frac{K^2}{m^2} - \frac{\omega^2 a^2 \lambda^2}{K^2 C_A{}^2} .$$

B. Case of m=0.

Inside of the beam the solution of Eq.(27-a) becomes

$$A_{\theta b} = C_1\ x \exp(\lambda^2 x^2 - \frac{\rho}{2}) F(1 - \Gamma;\ 2;\rho), \tag{37}$$

where F is also the confluent hypergeometric function defined by (31). Outside of the beam Eq.(27-a) reduces to the well-known Bessel's differential equation. And by using the requirement (29) the solution becomes

$$A_{\theta p} = C_2 \{J_1(Kx) - \frac{J_1(Kx_w)}{N_1(Kx_w)} N_1(Kx\ )\} . \tag{38}$$

For $x_w \to \infty$ the dispersion relation becomes

$$\frac{K\ J_0(K)}{J_1(K)} = 2(1+\lambda^2) + (-\frac{\lambda^4 \tilde{K}^2}{\tilde{\omega}^2})^{1/2} [1 - (1+\Gamma) \frac{F(1-\Gamma;3;A^*)}{F(1-\Gamma;2;A^*)}] \tag{39}$$

where $$A^* = (-\frac{\lambda^4 \tilde{K}^2}{\tilde{\omega}^2})^{1/2} .$$

6. Analysis of the Dispersion relations

To afford an insight into the MHD instability induced by the strong current ion beam, the dominant terms are carefully picked from the dispersion relations (35) and (38).

A Case of $m \neq 0$

The dominant terms of the dispersion relation (35) is

$$\left(\frac{\omega}{kC_A}\right)^{5/2}\xi^{1/2} + \left[\left(\frac{\omega}{kC_A}\right)^2 - 1\right]^{1/2} = 0 \quad , \tag{40}$$

where

$$\xi = \frac{k\ C_A\ \lambda^2}{m\Omega} \quad .$$

It should be noted that the parameter ξ is a small quantity rewritten by

$$\xi = \left(\frac{n}{N}\right)^{1/2}\left(\frac{\omega_{pb}\ a}{c}\right)\frac{ka}{m} \quad , \tag{40}$$

where N is the density of back ground plasma ion. And it is clear that there is no real root in Eq.(39) so long as the parameter ξ is small but finite. To the accuracy of order $\xi^{1/3}$ the solution of Eq.(39) becomes

$$\frac{\omega}{kC_A} \simeq \xi^{-1/3}\quad e^{\pm\frac{2}{3}\pi i} - \frac{\xi^{1/3}}{3}\quad e^{\mp\frac{2}{3}\pi i} \quad . \tag{40}$$

Therefore, the intense ion beam contrinutes to the helical overstability.

B Case of m=0

The dominant terms of the dispersion relation (39) is

$$\tilde{\omega}^2 + \frac{\lambda^2\tilde{k}^2}{24} = 0 \quad . \tag{41}$$

Then, the solution of Eq.(41) can be written by

$$\frac{\omega}{kC_A} = \pm i \frac{\lambda^2}{2\sqrt{6}} \left(\frac{N}{n}\right)^{1/2} \tag{42}$$

which implies instability.

7. Discussions

The above analysis could be utilized in predicting the possible M.H.D. instability which might be induced by the perpendicular injection of the intense ion beam in the plasma column. In the case of the beam injection into a toroidal system the beam should have a little toroidal velocity and there is a weak poloidal magnetic field, B_θ, in the equilibrium configuration. This case can be analysed without difficulty by making a small revision of the basic Eq.(12). The quantity $\vec{I}$ should be displaced by $\vec{I}^*$, where

$$\vec{I}^* = \frac{\mu_0 \rho}{B_0^2} \frac{\partial^2 \vec{A}}{\partial t^2} + \nabla \times \nabla \times \vec{A} - \alpha \nabla \times \vec{A} - \mu_0 \vec{J}_b \qquad \cdots\cdot (12)'$$

and

$$\alpha \equiv \frac{\mu_0 \vec{J}_{p0} \cdot \vec{B}_0}{\vec{B}_0 \cdot \vec{B}_0}$$

However, when $\left(\frac{B_\theta}{B_z}\right) / \left(\frac{r}{B_z} \frac{dB_z}{dr}\right)$ is less than the order of $\varepsilon^{1/2}$, the result obtained in the present analysis is valid, even if the poloidal field B_θ is present in the equilibrium configuration. In the case of the torus having an appreciable poloidal field the plasma column (e.g. tokamak plasma) must be analyzed by taking the z component of the vector potential (15) into account.

So long as the perpendicular injection of the ion beam is concerned the effect of the field gradient $\frac{r}{B_z} \frac{dB_z}{dr}$ could not be neglected. This situation is very different from the case of the stability of the field reversed ion ring/layer where $\frac{dB_z}{dr} = 0$ [3].

Thus, the result obtained here is not trivial. Unfortunately, however, the perpendicular injection of the strong current ion beam has a tendency to make the system unstable. A toroidal effect and the appreciable poloidal field may have a potentiality to stabilize the instability predicted in this paper.

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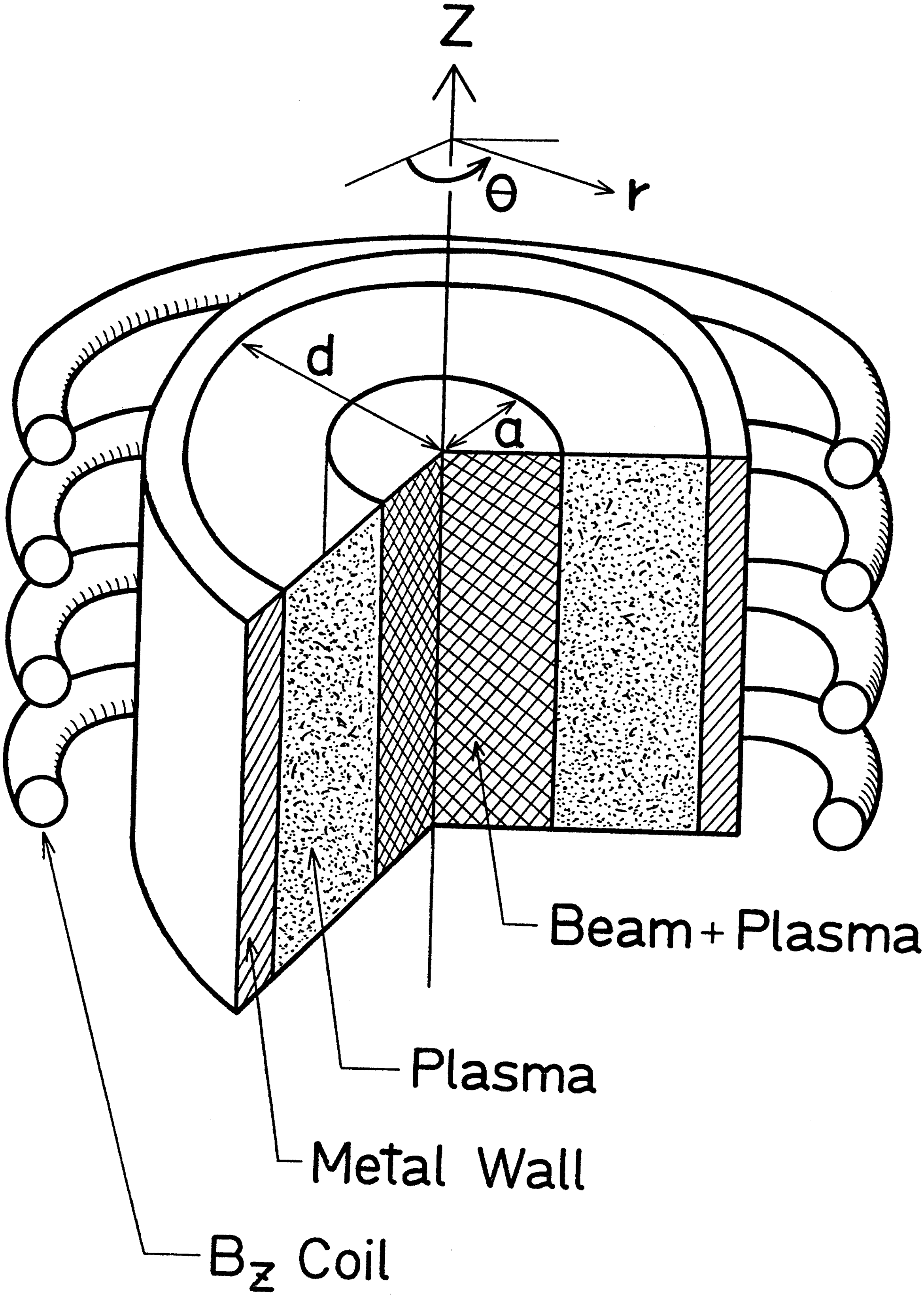
Figure Caption

Geometry


Z
θ
r
d
a
Beam + Plasma
Plasma
Metal Wall
B_z Coil