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Nonlinear Helical Equilibrium of a Current-Carrying Plasma

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Abstract

Finite amplitude solution of helical equilibria of current-carrying plasmas bounded by a perfectly conducting cylinder is obtained. The nonlinear saturation levels of the fixed boundary MHD modes are also obtained near the marginal stable points. It is shown that by shaping the current density profile the saturation levels can be suppressed.

A finite amplitude helical equilibrium in a toroidal device still requires further investigations since experimentally observed kink instabilities are nonlinearly saturated.

Several authors have studied the development of this problem by computer simulations[1,2] and for a particular case the non-linear theories have been presented[3,4].

In this letter we analyze the amplitude of helical equilibria of the plasma with a fixed boundary, i.e., the plasma is restricted by a perfectly conducting cylinder. The knowledges about the amplitude are necessary for the analysis of the instability itself as well as the other investigations, concerning the enhancement of the diffusion[5] and the fatal instability called disruptive instability[6].

We study the finite amplitude helical equilibrium of the plasma using helical coordinates r, $\varphi = m\theta + k_z z$, $\chi = (mz - k_z r^2\theta)/(m^2 + k_z^2 r^2)$ (r, θ, z are the ordinary cylindrical coordinates with r=0 corresponding to the axis of the cylinder). The considered system is helically symmetric, i.e., χ independent. We define B_φ and A_φ ($\vec{A}$; vector potential) as

$$B_\varphi = k_z r B_\theta - m B_z \quad , \tag{1}$$

$$A_\varphi = k_z r A_\theta - m A_z \equiv \psi \quad . \tag{2}$$

The plasma equilibrium equation, $\vec{J} \times \vec{B} = \nabla p$, can be rewritten in the form as[7]

$$\left(k_z^2 + \frac{m^2}{r^2}\right)\frac{\partial^2\psi}{\partial\varphi^2} + r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{2}{r}\frac{m^2}{[(k_z r)^2 + m^2]}\frac{\partial\psi}{\partial r} +$$

$$\frac{2mk_zB_\varphi}{(k_z r)^2 + m^2} + \frac{\partial}{\partial\psi}\left[\frac{1}{2}B_\varphi^2 + \mu_0(k_z^2 r^2 + m^2)p\right] = 0, \qquad (3)$$

$$B_\varphi = B_\varphi(\psi), \qquad p = p(\psi). \qquad (4)$$

The z component of the current density is given as $J_z(r) = \mu_0^{-1} B_z \partial B_\varphi/\partial\psi + m\partial p/\partial\psi$.

The plasma fills the interior of the conducting cylinder of radius a. The boundary condition is that B_r vanishes at the conducting wall or

$$\frac{\partial\psi}{\partial\varphi} = 0 \quad \text{at } r = a. \qquad (5)$$

In the equilibrium state ψ can be expressed as

$$\psi(r,\varphi) = \psi_0(r) + \alpha\psi_1(r)\cos\varphi + \alpha^2\psi_2(r) + \cdots\cdots \qquad (6)$$

where α denotes the amplitude of the helical deformation which is used for an expansion parameter.

Let us consider the rounded current profile case where

$$B_\varphi = B_0 + \frac{k^2\psi^2}{2B_0} \qquad (7)$$

holds with $k\psi \ll B_0$. We expand Eq.(3) using Eqs.(6) and(7) with

$(ak_z)^2 = (a/R)^2 \ll 1$ (R is the major radius of the torus), in the limit of the strong toroidal field B_z ($\beta = 0$) and retain the terms up to the 2nd order of α , that is,

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi_0}{\partial r} + \frac{2k_z}{m} B_0 + k^2 \psi_0 = 0 , \tag{8}$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi_1}{\partial r} - \frac{m^2}{r^2} \psi_1 + k^2 \psi_1 = 0 , \tag{9}$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi_2}{\partial r} + k^2 \psi_2 + \frac{k_z k^2}{2mB_0} \psi_1^2 = 0 . \tag{10}$$

The solution of Eqs.(8) and (9) are given as

$$\psi_0(r) = - \frac{2k_z B_0}{k^2 m} [1 + AJ_0(kr)] , \tag{11}$$

$$\psi_1(r) = \frac{2k_z B_0}{k^2 m} J_m(kr) , \tag{12}$$

where J_i is the i-th Bessel's function. The constants A and k determine the longitudinal current profile and the rotational transform. The boundary condition (5) gives

$$J_m(ka) = 0. \tag{13}$$

Using the safety factor q defined by $|B_z a k_z / B_\theta|$, and noting that $k_z r B_z + m B_\theta = \partial\psi/\partial r$, Eq.(13) determines the particular q-value q_e[7], $q_e(a) = m/[1+2AJ_1(ka)/ka]$, $q_e(0) = m/(1+A)$, that permits the neighbouring helical equilibrium and gives the marginal points of the helical MHD instability[8]. The solution of Eq.(10) is given by

$$\psi_z(r) = \delta J_0(kr) - \frac{\pi k_z^3 B_0}{m^3 k^4}\left[N_0(kr)\int_0^{kr} sJ_0(s)J_m^2(s)ds + J_0(kr)\int_{kr}^{\infty} sN_0(s)J_m^2(s)ds\right], \quad (14)$$

where N_0 is the Bessel's function of the 2nd kind. An arbitrary constant δ is determined by the conservation of the longitudinal flux, $\int_0^{2\pi}\int_0^{a} r dr d\theta B_z$ = const., as

$$\delta = -\frac{k_z B_0}{k^2 m}\,\frac{J_{m-1}(\lambda)J_{m+1}(\lambda)}{2\{J_0(\lambda) + A(J_0^2(\lambda) + J_1^2(\lambda))\}} \quad (15)$$

$$\equiv -\frac{k_z B_0}{m k^2} C_m , \quad (16)$$

where λ is the minimum zero point of J_m, that is $J_m(\lambda)=0$.

From the nonlinear solution ψ obtained above, the safety factor q is expressed as a function of α as

$$q(a) = q_e(a)\left[1 + \frac{f(\lambda)}{1+2Af(\lambda)} C_m\alpha^2 \right], \quad f(x)=J_1(x)/x. \quad (17)$$

$$q(0) = q_e(0)\left[1 + \frac{1}{2+2A} C_1\alpha^2 \right] \quad (\text{For } m=1). \quad (17')$$

From Eqs.(17) and (17'), we obtain the amplitude α as

$$\alpha_m = \sqrt{K_m[q(a)-q_e(a)]}, \quad K_m=[1+2Af(\lambda)]/C_m f(\lambda) q_e(a), \quad (18)$$

$$\alpha_1 = \sqrt{K_1[q(0)-q_e(0)]}, \quad K_1=2(1+A)/C_1 q_e(0). \quad (18')$$

The saturation amplitude α strongly depends on the current profile. We define the concentration ratio $\kappa = q(a)/q(0)$, which indicates the concentration of the longitudinal current.[8)] From the definition of q, A is interpreted in terms of κ as $A=(\kappa-1)/[1+2f(\lambda)\kappa]$. Figure 1 shows κ vs. α_1 and α_2 where other parameters are fixed. It should be noted that the amplitude α becomes very small around a particular value of κ at which $\alpha=0$. It suggests that by shaping the current profile we can suppress the saturation amplitude of the instability and consequently the plasma transport across the magnetic field can be reduced.

This analysis is performed for the fixed boundary condition case. The similar analyses can be done for the free boundary condition case which is more realistic model of the tokamak plasma.

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Figure Capton

Fig.1 The dependence of the amplitude on the current profile. α_1 and α_2 are shown as a function of κ. Other parameters, $|q(a)-q_e(a)|$ and $|q(0)-q_e(0)|$, are fixed. α_1 (dashed line) and α_2 (solid line).

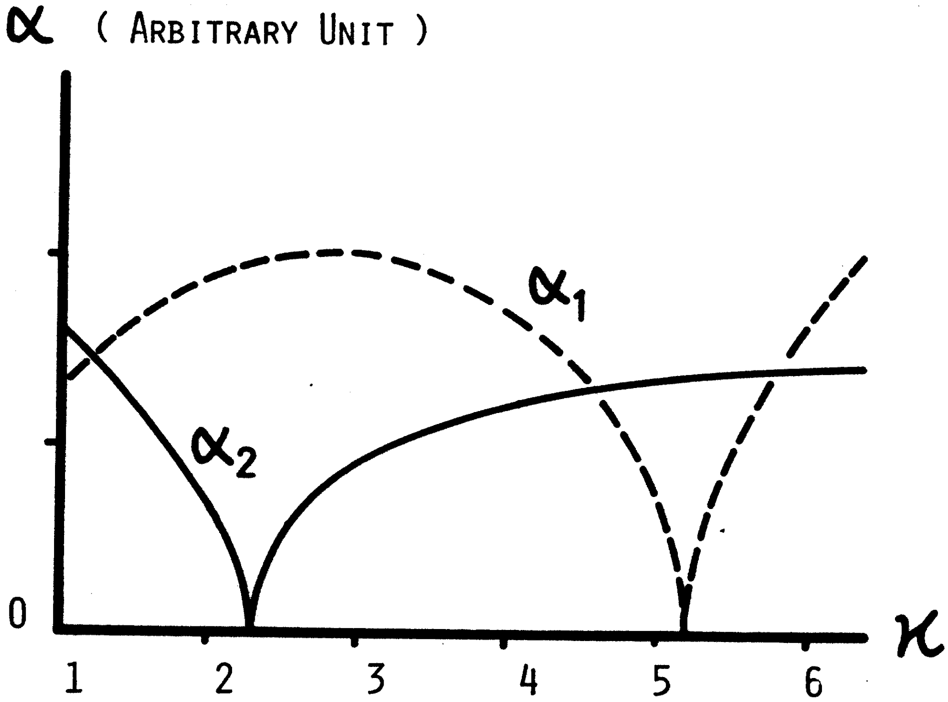

Fig. 1