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RESEARCH REPORT

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Turbulent Diffusion Coefficient of Plasma

which Depends Inversely on Density.

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Abstract

A possible interpretation of the plasma confinement scaling in the Alcator experiment is presented. In the regime where $\nu_c \leq \omega_{*emax}$, the diffusion coefficient is modified frome the pseudo-classical diffusion $D_{p.c.}$ to $D_{p.c.}(\omega_{*emax}/\nu_c)^2$ due to the electron inertia effect. This Electron-Inertia diffusion shows $\tau \propto n a^2 I^2 T_e^{-7/2}$ and $\tau = 25$ms for the experimental parameters if the numerical constant is taken as 1.2.

A new measurement about confinement time of tokamak plasma was recently reported by Alcator Group[1], that is with plasma parameters, B_t = 75kG, q = 5, a = 0.1m, n = $5 \times 10^{13} \sim 5 \times 10^{14}$ /cc, the confinement time τ is directly proportional to the plasma density n. The temperature and the magnetic field cannot be independently varied: thus the dependence is not clearly known. The group also has reported that $T_e \propto I^{3/5} \cdot f(q_0) \sim I \cdot f(q_0)$ is the commonly observed scaling from which one for τ can be derived. The scaling law of this energy confinement cannot be derived from the mere pseudo-classical scaling[2].

On the other hand, it is well known that in the low frequency regime ($\omega < \omega_{*e\,max} \lesssim \nu_c$) where $\omega_{*e\,max}$ is the maximum drift frequency defined by $\omega_{*e\,max} = v_d/4\rho_i$ [3], $v_d = k_B T_e \nabla n/eBn$ and ρ_i is the ion Larmour radius and ν_c denotes the collision frequency, the scaling for plasma confinement is well explained by the pseudo-classical formula[2,4]. The pseudo-classical diffusion shows that if there is an instability and/or a fluctuation in the confined plasma there is a net flux across the magnetic lines of force. The derivation of this diffusion coefficient is based on the assumption that one should be able to derive it without the detailed knowledge of the nature of fluctuations.

In this letter we shall show that the pseudo-classical diffusion coefficient continues to new one (we call this Electron Inertia Diffusion) in the regime where $\nu_c \leq \omega_{*e\,max}$. In this regime the pseudo-classical diffusion coefficient is modified by the factor $[\,1 + (\omega_{*e\,max}/\nu_c)^2]$ due to the electron inertia effect, that is $D = D_{p.c.}[\,1 + (\omega_{*e\,max}/\nu_c)^2]$. This electron inertia diffusion gives $\tau \propto I^2 a^2 n T^{-7/2}$ and well explaines the result of Alcator experiments.

With the increase of the plasma density while other parameters are fixed, this Electron-Inertia Diffusion leads to the pseudo-classical diffusion. If we increase the electron temperature, the trapped electron diffusion may also appear[5] (Fig.1 and 2).

Particle flux of a plasma with electrostatic fluctuations across a magnetic field is given by

$$\Gamma_x = \frac{1}{B_z} \langle \tilde{n} \tilde{E}_y \rangle_{av.} = -D(\nabla n_0)_x , \tag{1}$$

where the uniform magnetic field and a density gradient are taken in the direction of the z-axis and the x-axis respectively. For a single mode of fluctuation we choose the density perturbation as

$$\tilde{n} = n_1 \, e^{i(\omega t + k_y y + k_z z)} f(x), \tag{2}$$

where n_1 is taken to be real constant and f denotes the localization of the oscillation in the x direction, which is normalized as the maximum value of f equals to unity. We define α as the phase difference between $\tilde{n}$ and $\tilde{\phi}$ (α is determined later). For low frequency oscillation such as drift waves, for a finite k_z, electrons tend to be in thermal equilibrium along the magnetic lines, and $\tilde{n}$ and $\tilde{\phi}$ can be expressed as

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\phi}}{k_B T_e} e^{-i\alpha} , \qquad |\alpha| \ll 1 . \tag{3}$$

From Eqs.(1),(2) and (3) the diffusion constant for this single mode is obtained as

$$D_1 = ak_y\left(\frac{k_B T_e}{eB}\right)\left|\frac{n_1}{n_0} f(x)\right|^2 \sin\alpha \ , \tag{4}$$

where $a^{-1} = -\nabla n_0/n_0$.

There is another way of deriving the diffusion constant proposed by Kruskal and Kulsrud[6). The plasma expansion by the diffusion v_D is related to the dissipation as $|v_D \nabla p| = \eta J^2$. This method was successfully used to derive the Pfirsch-Schlüter and the pseudo-classical diffusion constant[7,8), hence we also assume that this result is also applicable in this regime. The diffusion coefficient for a single mode can be expressed as

$$D_1/D_{cl} = \nu_{eff} J^2/\nu_c J_{dia}^2$$

$$= \nu_{eff} J^2/\nu_c [(1/B)^2 (k_B T_e + k_B T_i)^2 (n_0/a)^2] \ , \tag{4'}$$

where D_{cl} is the classical diffusion coefficient $(\nu_c/\omega_e)(k_B T_e/eB)$, ω_e is the electron cyclotron frequency, ν_{eff} is the collision frequency of the turbulent plasma. From Eq.(4')

$$D_1 = \nu_{eff} J^2 \frac{B\, k_B T_e}{\omega_e e(k_B T_e + k_B T_i)^2} (a/n_0)^2 \ . \tag{5}$$

The turbulent current must satisfy $\nabla \cdot \vec{J} = 0$ (for the fluctuation with $k \ll k_D$, k_D : Debye wave number). In the case of the low frequency instabilities where $|k_\perp/k_\parallel| \gg 1$, $\nabla \cdot \vec{J} = 0$ leads to

$$J^2 \doteqdot J_{\parallel}^2 \quad . \tag{6}$$

In the high density plasma, that is with a high collision frequency, we consider that the equation of motion still holds as[8)]

$$m_e \frac{\partial \Gamma_{\parallel}}{\partial t} = -\nabla_{\parallel}(n k_B T_e) + e n \nabla_{\parallel} \tilde{\phi} + \frac{m}{e} \nu_{eff} J_{\parallel} \quad , \tag{7}$$

where the electron inertia term cannot be neglected when ν_{eff} is reduced to ω_{*max}. By expressing $\tilde{\Phi}$ in terms of the density perturbation $\tilde{n}$ using Eq.(3), Eq.(7) can be solved for $J_{\parallel}$ ($= -e\Gamma_{\parallel}$), we therefore get noting Eq.(6),

$$J^2 = \frac{4e^2}{m^2} \frac{k_{\parallel}^2 n_1{}^2}{\omega^2 + \nu_{eff}^2} |f(x)|^2 (k_B T_e)^2 \sin^2 \frac{1}{2}\alpha \quad . \tag{8}$$

In deriving Eq.(8), the condition

$$| \tilde{n} k_B T_e (1 - i \sin \alpha) | \gg | e \tilde{n} \tilde{\phi} | \tag{9}$$

is assumed. This condition is discussed at the end of this letter. From Eqs.(5),(6) and (8), we obtain

$$D_1 = \frac{4k_B T_e}{eB} \frac{\nu_{eff}\, \omega_e}{\omega^2 + \nu_{eff}^2} \left[\frac{a k_{\parallel}}{1+T_i/T_e}\right]^2 \left|\frac{n_1}{n_0} f(x)\right|^2 \sin^2 \frac{1}{2}\alpha \quad . \tag{10}$$

Equations (4) and (10) are two expressions of the same diffusion coefficient. Equating Eq.(4) with Eq.(10), $\sin\alpha$ is determined as

$$\sin\alpha = \frac{\omega^2 + \nu_c^2}{\omega_e \nu_c} (1 + T_i/T_e)^2 \frac{ak_\perp}{(ak_{/\!/})^2} , \tag{11}$$

where we assume $\nu_{eff} = \nu_c$, and obtain

$$D_1 = \frac{k_B T_e}{eB} \left|\frac{n_1}{n_0} f(x)\right|^2 \left|\frac{k_\perp}{k_{/\!/}}\right|^2 (1 + T_i/T_e)^2 \frac{\nu_c}{\omega_e} (1 + \omega_{*emax}^2 / \nu_c^2)$$

$$= D_{p.c.} [1 + (\omega_{*emax}/\nu_c)^2], \tag{12}$$

where k_y is evaluated by $k_\perp/2 \sim 1/2\rho_i$ [9], and $\omega \sim \omega_{*emax}$.

When the temperature is high enough for the maximum drift frequency ω_{*emax} to exceed ν_c, the diffusion coefficient becomes $D_{p.c.} (\omega_{*emax} / \nu_c)^2$. In this regime the confinement time obeys the scaling law, with the assumption $|n_1/n_0| = 1/ak_\perp$ [8] which is used for evaluating the pseudo-classical diffusion, as

$$\tau \propto a^4 B_p^2 n T_e^{-7/2} \qquad (\nu_c \lesssim \omega_{*emax}) . \tag{13}$$

With the increase of n, τ increases directly proportional to n until this diffusion becomes the pseudo-classical diffusion (Fig. 2). From these results, the thermal conductivity χ is expected to have a form as

$$\chi = \chi_{p.c.} (\omega_{*emax} / \nu_c)^2 , \quad \chi_{p.c.} = C_0 \nu_c \rho_{pe}^2 \tag{14}$$

where C_0 is a constant to be determined from the parameters of devices and saturation levels of the fluctuations, and ρ_{pe} is the electron Larmour radius in the poloidal magnetic field. For the values of parameters of the Alcator experiments, $C_0 = 1.2$ gives the confinement time $\tau = 25$ms, which is equal to the observed value.

From the energy balance equation $nk_B T/\tau = \eta J^2$ and Eq.(13), we get

$$T_e^{9/2} \propto \eta I^4. \tag{15}$$

Assuming that $\eta \propto T_e^{-3/2}$, we find that the electron temperature T_e depends on the plasma current I as

$$T_e \propto I^{2/3}, \tag{16}$$

which well compares to the experimental results $T_e \propto I^{\nu} \cdot f(q_0)$ (here $3/5 < \nu < 1$). The experimental q-dependence of T_e may result from the MHD instabilities (q : safety factor).

Another possible interpretation of the Alcator experiments is that this diffusion is due to the trapped electron instability[5]. However for the values of parameters of Alcator experiments, the electron bounce frequency $\omega_b = \sqrt{\epsilon}\, v_e/qR$ (v_e: electron thermal velocity, $\epsilon = a/R$) is comparable to or smaller than the effective collision frequency of the trapped electrons ν_c/ϵ, so that the regime of these parameters may not belong to the trapped particle regime. In addition to it, the diffusion due to the trapped electron instability is given by[5]

$$D_t = \frac{\gamma}{k_\perp^2} = \sqrt{\epsilon}\,\frac{\omega_*\omega_{*T}}{k_\perp^2 \nu_{eff}}\,\frac{\nu_{eff}^2}{\omega_{*e}^2+\nu_{eff}^2} = D_0 \cdot \frac{\nu_{eff}^2}{\omega_{*e}^2+\nu_{eff}^2}\,, \tag{17}$$

where D_0 is independent of $k_\perp$, so that the smaller $k_\perp$ gives the larger D_t because ω_{*e} is directly proportional to $k_\perp$. The $k_\perp$-dependence of the diffusion due to the trapped electron instability is different from that of the Electron-Inertia diffusion. The Electron-Inertia diffusion becomes larger by the fluctuations with larger $k_\perp$. Experimental results of the FM-1[9] and ATC[10] have been recently reported that $k_\perp \rho_i$ is the order of unity. Assuming that this fact holds in the Alcator experiments, we find that the diffusion in the Alcator is explained by this Electron-Inertia dffusion.

When the fluctuation level decreases and the condition (9) is not satisfied, it is predicted that the neo-Bohm diffusion appears[8]. Using the saturation amplitude $|n_1/n_0| \sim 1/ak_\perp$[8], Eq.(9) with Eq.(12) can be rewritten as

$$\left(1 + \frac{T_i}{T_e}\right)^2 \frac{T_e}{T_i}\,\frac{q^2R^2}{a\rho_i}\,\frac{m_e}{M_i} \gg 1, \quad \text{for } k_\perp\rho_i,\ k_\parallel qR \sim 1 \tag{9'}$$

This condition is satisfied for the parameter regime of the Alcator. But in the FM-1 experiments, $(1+T_i/T_e)^2 T_e q^2 R^2 m_e/T_i a M_i \rho_i$ reduces close to unity, and in fact, the neo-Bohm diffusion appears[11]. This suggests that in low $(1+T_i/T_e)^2 R^2 q^2 m_e T_e/T_i a M_i \rho_i$ (small aspect ratio R/a and high temperature) machines we can expect that the plasmas obey the neo-Bohm diffusion scaling.

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Figure Captions

Fig. 1 Various diffusion regimes are shown in terms of the plasma density and the electron temperature T_e (the case of high q operation). The shaded portion indicates the Electron-Inertia regime, and the rest is the peudo-classical regime. The trapped particle regime is denoted by the dotted portion.

Fig.2 The confinement time τ v.s. the palsma density n (both in logarithmic scales). Two curves of the different electron temperature are shown. All other parameters are fixed. The dashed line shows the maximum value of τ (at $V_L = \omega_{*e\,max}$).

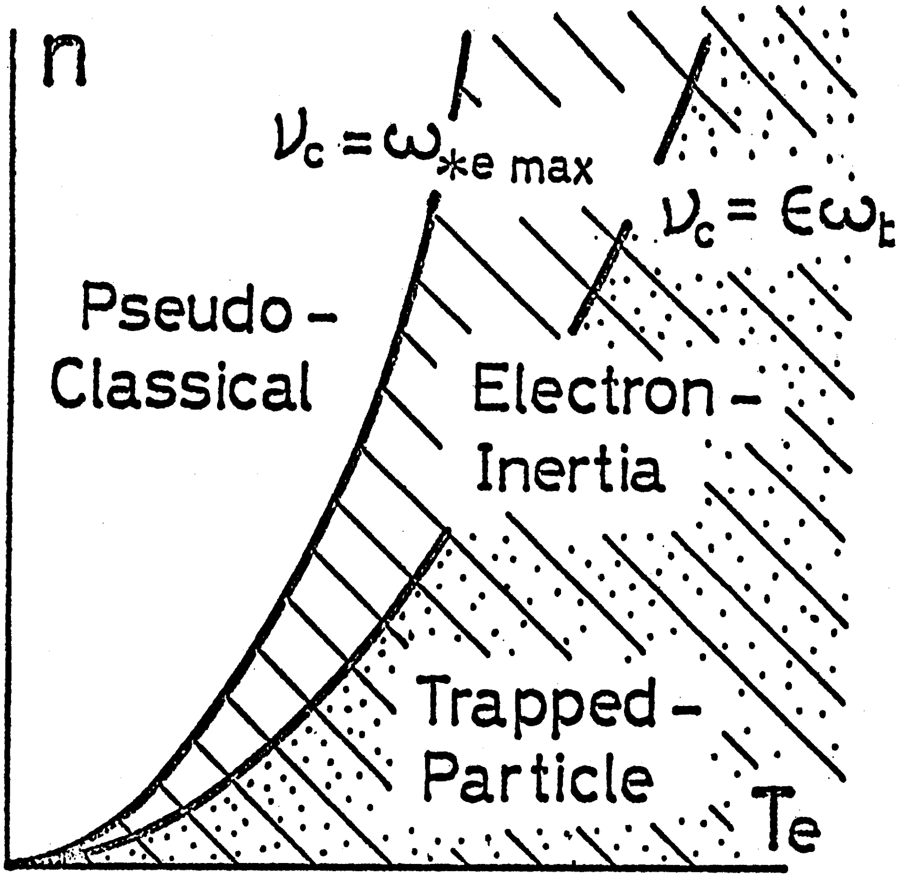
n
$\nu_c = \omega_{*e\,max}$
$\nu_c = \epsilon\omega_t$
Pseudo-
Classical
Electron-
Inertia
Trapped-
Particle
Te

Fig. 1

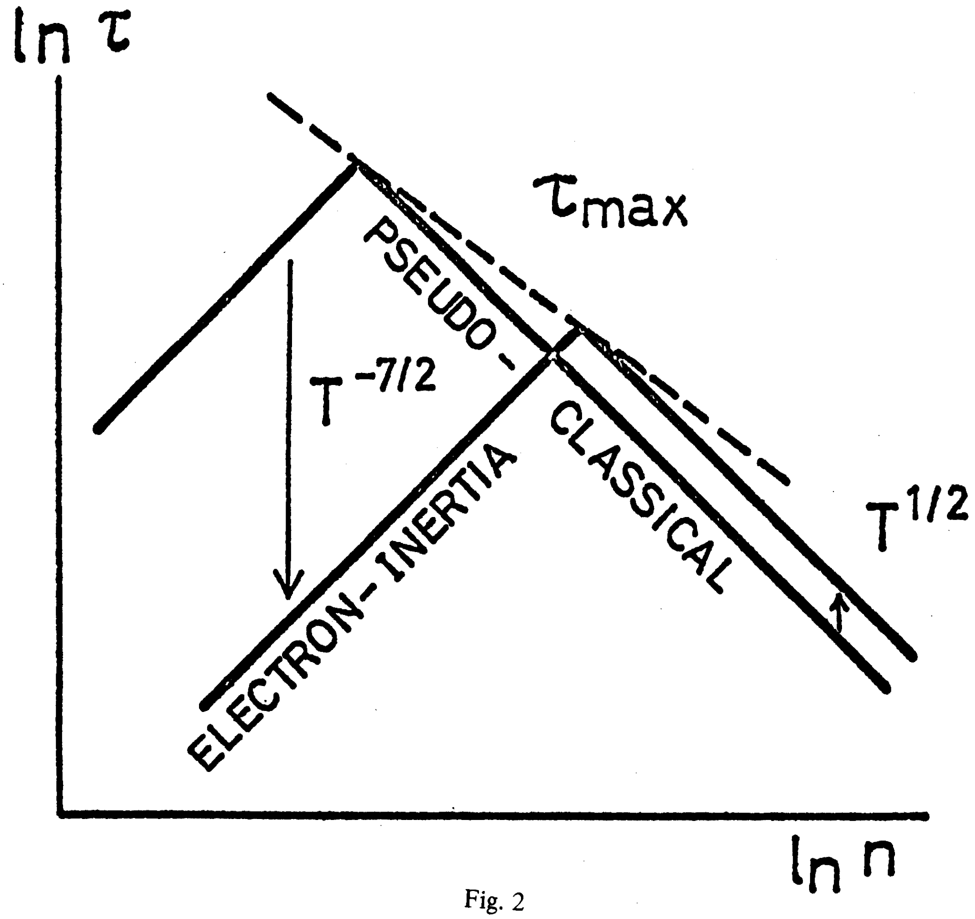

Fig. 2