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Anomalous Diffusion of Alpha Particles in the Nonlinear Trapped-Ion Regime

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The neoclassical theory shows us a diffusion coefficient across magnetic lines of force in a quiescent plasma.
It is well known that the plasma in a toroidal device may be seriously impaired by the development of drift instabilities.
In the next generation of tokamaks as well as in a fusion reactor, the ions enter the banana regime and the dissipative trapped-ion mode may appear[1).

In this letter we study the anomalous diffusion coefficient of α particles in a fusion reactor plasma due to the fluctuating electric field of the trapped-ion mode. The toroidal geometry effect is included in terms of the particle trapping and the magnetic drift due to the field curvature is assumed to be small[2).

The particle flux across the magnetic lines of force under perturbed electric field E ($\equiv -ik_y\varphi$) is

$$\Gamma_\sigma = [\, n E_y \,]_{av} / B_z$$

$$= \sum_k ik_y [\, n_{\sigma,k,\omega}\, \varphi_{-k,\omega} \,]_{av} / B_z \quad . \qquad (1)$$

For a single mode of electrostatic perturbation

$$\Gamma_\sigma = -n_\sigma k_y \frac{q_\sigma}{B_z T_\sigma} |\varphi|^2 \chi_\sigma'' \qquad (1')$$

where the magnetic field and the density gradient are taken in the z and x directions respectively. The susceptibility of the particles χ_σ (σ denotes species.) is defined by

$$\frac{\tilde{n}_\sigma}{n_\sigma} = \frac{q_\sigma \varphi}{T_\sigma} (\chi_\sigma' + i \chi_\sigma'') . \qquad (2)$$

In the trapped-ion regime where the effective ion collision frequency ($\nu_{+} = (\nu_i)_{eff} \equiv \nu_i/\epsilon$, where $\epsilon = r/R$ is the inverse aspect ratio) is less than the ion bounce frequency (ω_{bi}), particles are also trapped in magnetic wells. To know the susceptibility of α particles, we divide them into two classes, one is the trapped particles, the number density of which is $n_{\alpha t}$, and one is the untrapped particles (circulating particles), $n_{\alpha\, un}$. We assume the density of α particles $n_{\alpha} = n_{\alpha t} + n_{\alpha\, un}$ is small compared to the electron and ion densities, $n_{e,i}$, so that the ion-trapped mode is entirely determined by the host ions and electrons.

The basic equation to obtain the phase relation between the density perturbation $\tilde{n}_\alpha$ and the electrostaic perturbation $\tilde{\varphi}$ is[3])

$$\frac{\partial \tilde{f}_\alpha}{\partial t} + v_{/\!/}\frac{\partial \tilde{f}_\alpha}{\partial z} - \frac{q_\alpha}{m}\frac{\partial \varphi}{\partial z} - \frac{1}{B}\frac{\partial \tilde{\varphi}}{\partial y}\frac{\partial f_{\alpha 0}}{\partial x} = -\nu \tilde{f}_\alpha + \nu \frac{\tilde{n}_\alpha}{n_\alpha} f_{\alpha 0} , \tag{3}$$

where $/\!/$ and $\perp$ denote the components parallel and perpendicular to the magnetic line, ν is the effective collision frequency of α particles. Let $\tilde{f} = f_1 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$ and $\tilde{\varphi} = \varphi e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$ we get

$$f_{\alpha 1} = \frac{q_\alpha \varphi}{T_\alpha} f_{\alpha 0}\left\{ -1 - \frac{J_0^2\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right)}{k_{/\!/} v_{/\!/} - \bar{\omega}}(\bar{\omega} + \omega_{*\alpha})\right\} + \frac{\nu}{i}\frac{\tilde{n}_\alpha}{n_\alpha} f_{\alpha 0}\frac{J_0^2\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right)}{k_{/\!/} v_{/\!/} - \bar{\omega}} , \tag{4}$$

where $\omega_{c\alpha}$ is the cyclotron frequency of the α particle, $\bar{\omega} = \omega + i\nu$, $k_\perp^2 = k_x^2 + k_y^2$, $\omega_{*\alpha}$ is the drift frequency ($\equiv \frac{T_\alpha}{ZeB} \kappa k_y$) and J_0 is the zeroth order Bessel's function.

Each α particle has the kinetic energy of 3.52 MeV, then the unperturbed distribution function is written in the form,

$$f_{\alpha 0} = \frac{1}{4\pi V_\alpha^2}\,\delta(v - V_\alpha), \tag{5}$$

where v_α is the reaction velocity ($M_\alpha V_\alpha^2/2 = 3.52$ MeV). For circulating particles we integrate Eq. (4) using Eq. (5) over the range of $v_\parallel^2/v_\perp^2 \geq \epsilon$, assuming the effective collision frequency of the untrapped particles is small, and we get

$$\tilde{n}_{un} = -\frac{q_\alpha \varphi}{T_\alpha} n_{un}(1 + (\omega + \omega_{*\alpha})\, I_{un}), \tag{6}$$

$$I_{un} \equiv \frac{1}{V_\alpha}\int_{V_\alpha/\sqrt{1+\frac{1}{\epsilon}}}^{V_\alpha} J_0^2\left(\frac{k_\perp}{\omega_{c\alpha}}\sqrt{V_\alpha^2 - V_\parallel^2}\right)\frac{dV_\parallel}{k_\parallel V_\parallel - \omega}. \tag{6'}$$

The similar calculation for trapped particles with $\nu = V_\alpha/\epsilon$ and $k_\parallel = 0$ gives the total density perturbation as

$$\tilde{n}_\alpha = -\frac{q_\alpha \varphi}{T_\alpha} n_\alpha \left[1 + (\omega + \omega_{*\alpha})(I_{un} + I_t)\right], \tag{7}$$

$$I_t \equiv \frac{1}{V_\alpha}\int_0^{V_\alpha/\sqrt{1+\frac{1}{\epsilon}}} J_0^2\left(\frac{k_\perp}{\omega_{c\alpha}}\sqrt{V_\alpha^2 - V_\parallel^2}\right)\left(-\frac{1}{\omega + iV_\alpha/\epsilon}\right) dV_\parallel. \tag{7'}$$

The susceptibility χ_α'' is obtained, noting Eq. (2),

$$\chi_\alpha'' = -\,\mathrm{Im}\,((\omega + \omega_{*\alpha})(I_{un} + I_t)). \tag{8}$$

For parameters, typical of the fusion reactor, $T_\alpha = 3.52$ MeV, $\mathrm{Re}\,\omega = \frac{\sqrt{\epsilon}}{2}\omega_* \equiv \omega_0$ [1] ($T_e = T_i$) and $k_\parallel qR = 1$, the resonance condition for circulating particles ($k_\parallel v_\parallel \sim \omega_0$) is not satisfied. The major contribution to χ_α'' comes from the trapped particle part since $\mathrm{Im}\,\omega = 0$ on the nonlinearly saturated stage.

From the definition of the diffusion coefficient, i.e., $\Gamma_\alpha = - D_\alpha(\nabla n_\alpha)$ and from Eqs.(1') and (8), one obtains

$$D_\alpha = \left(\frac{k_\perp}{\kappa}\right) \frac{z T_e}{e B} \frac{T_e}{T_\alpha} \left(\frac{e\varphi}{T_e}\right)^2 (\omega_0 + \omega_{*\alpha}) \ \mathrm{Im}\, I_t, \qquad (9)$$

where $\mathrm{Im}\, I_{un} \ll \mathrm{Im}\, I_t$. The saturation level of the ion-trapped mode was analyzed by LaQuey et. al.[4] and they give $e\varphi/T_e \sim \sqrt{\epsilon}\, \omega_0/\nu_-$ ($\nu_- \equiv \nu_e/\epsilon$; effective electron collision frequency) under the condition of $\omega_0/\nu_- < 1$. They conclude the saturation mechanism of this mode to be the effective transfer of energy from long-wavelength to short-wavelength mode which are then Landau damped by ion bounce resonance.

Since $\epsilon \ll 1$ we evaluate $\mathrm{Im}\, I_t$ as

$$\mathrm{Im}\, I_t = \sqrt{\frac{\epsilon}{1+\epsilon}} \frac{\nu_\alpha/\epsilon}{\omega_0^2 + (\nu_\alpha/\epsilon)^2} J_0^2\left(\frac{k_\perp V_\alpha}{\omega_{c\alpha}}\right) \lesssim \sqrt{\epsilon} \frac{\nu_\alpha/\epsilon}{\omega_0^2 + (\nu_\alpha/\epsilon)^2}. \qquad (10)$$

Substituting Eq.(10) and the saturation level, $e\varphi/T_e \sim \epsilon^{3/2} \frac{\omega_0}{\nu_e}$, into Eq.(9) we obtain

$$D_\alpha = \frac{\omega_*^2 \nu_\alpha}{\kappa^2 \nu_e^2} \epsilon^{5/2}$$

$$= D_{KP} \left(\frac{k_\perp^2}{\kappa^2} \frac{4 \nu_\alpha}{\nu_e} \right), \qquad (11)$$

where ω_0 is small compared with $\omega_{*\alpha}$ ($T_\alpha \ll T_{e,i}$) and D_{KP} is the diffusion coefficient suggested by Kadomtsev and Pogutse for the host plasma[5], namely $D_{KP} \sim \epsilon^{5/2} (T_e/eB)^2 \kappa^2 / 4\nu_e$.

Eq.(11) shows that α particles with the high kinetic energy diffuse satisfying $D_{n.c.} \ll D_\alpha \ll D_{KP}$, where $D_{n.c.}$ donotes a ordinary neoclassical diffusion coefficient of α particles. The diffusion coefficient becomes larger for the shorter-wavelength mode which are Landau damped with a proper condition[2), so that we expect the diffusion coefficient of the smaller $k_{\perp}$.

In this letter we have studied the anomalous diffusion of α particles due to the fluctuating potential of the trapped-ion mode. The diffusion coefficient of α particles is expected to be much larger than that derived from the neoclassical theory, but much less than the diffusion coefficient of the host ions and electrons due to this mode. As α particles lose their energy, they diffuse faster than the α particles with the higher kinetic energy.

When the number density of α particles is large enough to have an influence on the ion-trapped mode our analysis is not valid, since we assume $n_{\alpha} \ll n_{e,i}$. With the same principle of this analysis the calculation of the anomalous diffusion coefficiet of impurity ions has been done and is reported elsewhere.[6)]

References

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