INSTITUTE OF PLASMA PHYSICS

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RESEARCH REPORT

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Possibility of Medium Energy Neutral Beam Injection into Stellarator Reactor

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I. INTRODUCTION

Neutral beam injection is one of the most promising heating methods for fusion grade plasmas. However the mean free path of neutral deuterium with the medium energy (say 100 keV) is the order of 0.8 meter in the plasma with the ion density of 5×10^{13} cm^{-3}. On the other hand the plasma radii of toroidal reactors are more than 3 m[1] at least, so that the injection of medium energy neutral beam can heat only the boundary region of plasma. The necessary energy of neutral particles to penetrate into the plasma center may be the order of 0.6 $\sim$ 1 MeV. There are a few demerits for the injection of extremely high energy neutral beam, that is, i) difficulty (or high cost) to accelerate the ion beam with the large current, ii) small conversion efficiency of the ion beam to the neutral one, iii) small ratio of ion heating rate to electron heating rate of high energy ions.[2]

In this paper, the orbit of trapped particles in helical ripples of stellarator field are analysed. It is examined how the trapped particles can drift into the plasma center and how possible the neutral beam injection with the medium energy to heat the center core of the plasma is.

II. ORBIT OF TRAPPED PARTICLES IN HELICAL RIPPLES OF STELLARATOR

Denoting the coordinates in the plasma cross section by r and θ and the angle around the major axis of the torus be φ, the field modulation of the toroidal stellarator field is described by

$$B = |B_0| [1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(\ell\theta - n\varphi)] . \qquad (1)$$

where $\varepsilon_t = r/R$ and $\varepsilon_h = \varepsilon_h^{\,o}(r/a)$. The notations R and a are the major and minor radii of the plasma respectively (see Fig. 1).

Fig. 1

The orbit of trapped particles by helical mirror is described with the use of longitudinal invariant J[3),4)]

$$J(r,\theta,W,\mu) = m\oint v_{/\!/}\,dl$$
$$= 16Rn^{-1}(m\mu|B_o|\varepsilon_h)^{1/2}[E(\kappa)-(1-\kappa^2)K(\kappa)] \qquad (2)$$

where $E(\kappa)$ and $K(\kappa)$ are complete elliptic integrals and

$$\kappa^2(r,\theta,W,\mu) = \frac{W-\mu|B_o|[1-\varepsilon_t(r)\cos\theta-\varepsilon_h(r)]-e\phi(r)}{2\mu|B_o|\varepsilon_h(r)} . \qquad (3)$$

The invariants W and μ are the total energy and the magnetic moment respectively and $\phi(r)$ is the electrostatic potential. κ^2 must satisfy $0 < \kappa^2 < 1$. ^{2} is approximately equal to $(v_{/\!/}^{\,2}/v_\perp^{\,2})(2\varepsilon_h)$. The equations of the drift of the trapped particles are given by $d\theta/dt = (eBr)^{-1}(\partial J/\partial W)$ and $dr/dt = -(eBr)^{-1}(\partial J/\partial\theta)/(\partial J/\partial W)$. These equations are reduced to[4)]

$$r\frac{d\theta}{dt} = V_\perp\cos\theta + r\omega_h + r\omega_E$$
$$\frac{dr}{dt} = V_\perp\sin\theta \qquad (4)$$

where

$$V_\perp = \frac{\mu|B_o|}{eB_oR}, \qquad r\omega_h = \frac{\mu|B_o|}{eB_or}2l\varepsilon_h\left(\frac{E(\kappa)}{K(\kappa)}-0.5\right) \qquad r\omega_E = \frac{E_r}{B_o} \qquad (5)$$

The function $F(\kappa) = (E/K - 1/2)$ is shown[5)] in Fig. 2 and is zero at $\kappa^2 = 0.826$.

Fig. 2

The ratio of three terms are the order of $\varepsilon_t : \varepsilon_h : T_e/W$ with the assumption of $e\phi(r) \sim T_e$. In the ordinary stellarators these parameters are in the relation of $\varepsilon_t > \varepsilon_h > T_e/W$. Supper banana orbit can appear when $\varepsilon_h >> \varepsilon_t$ and $E/K = 1/2$.[3] With the use of $s = V_\perp t$, these equations are simplified to

$$r\frac{d\theta}{ds} = \cos\theta + f(r,\theta) \qquad \frac{dr}{ds} = \sin\theta \tag{6}$$

$$f(r,\theta) = -\frac{eR}{\mu|B_0|}\frac{\partial\phi(r)}{\partial r} + \frac{2l\,\varepsilon_h}{\varepsilon_t}\left(\frac{E(\kappa)}{K(\kappa)} - 0.5\right).$$

When the initial values of r, θ, φ, k are r_o, θ_o, φ_o, κ_o respectively, the expersion of κ^2 is

$$\kappa^2(r,\theta) = 0.5 + \left(\frac{r_0}{r}\right)^l\left[\kappa_0^2 - 0.5 - \frac{r\cos\theta - r_0\cos\theta_0}{2\,\varepsilon_h(r_0)\,a} - \frac{e\phi(r) - e\phi(r_0)}{\mu|B_0|\,\varepsilon_h(r_0)}\right] \tag{7}$$

$$\kappa_0^2 = \frac{\alpha_0^2\{1 - (r_0/R)\cos\theta_0 - \varepsilon_h(r_0)\} + \varepsilon_h(r_0)[1 - \cos(l\theta_0 - n\varphi_0)]}{2\,\varepsilon_h(r_0)\,(1 - \alpha_0^2)} \tag{8}$$

where $\alpha_o = (v_{/\!/}/v)_o$ is the initial ratio of the parallel component of the velocity to the absolute value of the velocity. $\kappa^2(r,\theta)$ may become $\kappa^2 > 1$. Then the trapped particles can escape from the helical mirror and the escaping particles becomes banana trapped by the toroidal mirror field (blocked particles[6]) or transit particles. The width Δ of the banana is $\Delta \gtrsim \varepsilon_t^{-1/2} q\rho_i$, q and ρ_i are safety factor and Larmor radius of the high energy ions respectively. When $\kappa^2(r, \theta)$ becomes zero, $\kappa^2(r, \theta) = 0$ itself is the solution of Eq.(6) as $(E/K - 1/2) = 1/2$ at $\kappa^2 = 0$, that is, the orbit is given by

$$W = \mu |B_0| [1 - \varepsilon_t(r)\cos\theta - \varepsilon_h(r)] + e\phi(r) . \qquad (9)$$

Fig. 3

§3. Discussion

Typical examples of the orbits are shown in Fig. 3 when $\varepsilon_t{}^o = a/R = 0.2$, $\varepsilon_h{}^o = 0.1$, $\ell = 2$, $\phi = \phi_o(1 - (r/a)^2)$, $e\phi_o/K_i = -0.071$, K_i being the kinetic energy of the ion at the initial condition. As typical parameters of toroidal reactors, $R = 10$ m, $a = 3$ m, $B_o = 5$ Wb/m^2, $n_i = 0.5 \times 10^{20}$ m^{-3} are chosen. Then the velocity $V_\perp$ of the toroidal drift and the ion collision time τ_i are $V_\perp = 2 \times 10^3\ K_i$ m/s and $\tau_i = 0.44 Z^{-2} K_i^{3/2}$ (s) where K is expressed in the unit of 100 keV. The mean drift length is $\Delta_d = V_\perp\ \varepsilon_h \tau_i \sim 8.8 \times 10^2\ \varepsilon_h Z^{-2}\ K_i^{5/2}$m. Usually ε_h is the order of 0.1 at the plasma boundary and the average of $<\varepsilon_h>$ is the order of $<\varepsilon_h> \sim 0.05$. The effective Z number is assumed to be $Z = 3$. Accordingly the mean drift length is the order of $\Delta_d \sim 5\ K_i^{5/2}$ m. The energy of 80 $\sim$ 100 keV of the trapped ions is necessary to drift into the plasma center.

References

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Figure Captions

Fig.1. Coordinates system of torus

Fig.2. Dependence of the function $F(\kappa) = E(\kappa)/k(\kappa) - 0.5$ on κ^2

Fig.3. Examples of orbits of trapped particles when $\varepsilon_t^{o} = a/R = 0.2$, $\varepsilon_h^{o} = 0.1$, $\ell = 2$, $n = 4$, $\phi = \phi_o(1 - (r/a)^2)$, $e\phi_o/\kappa_i = -0.071$, and $\alpha_o = 0.06$, $n\varphi_o = -\pi$.

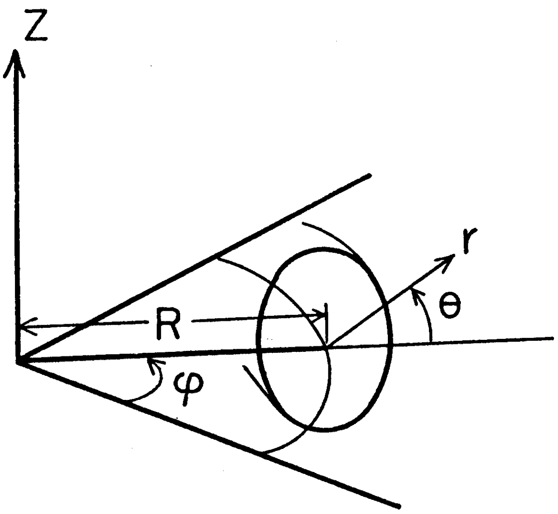

Fig. 1

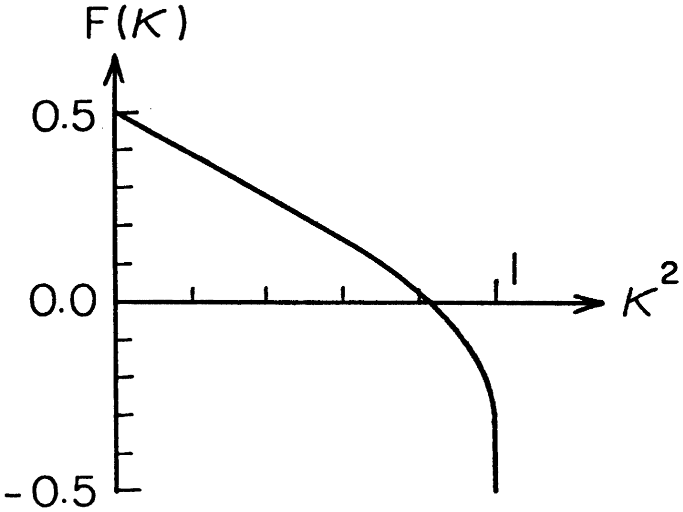

Fig.2

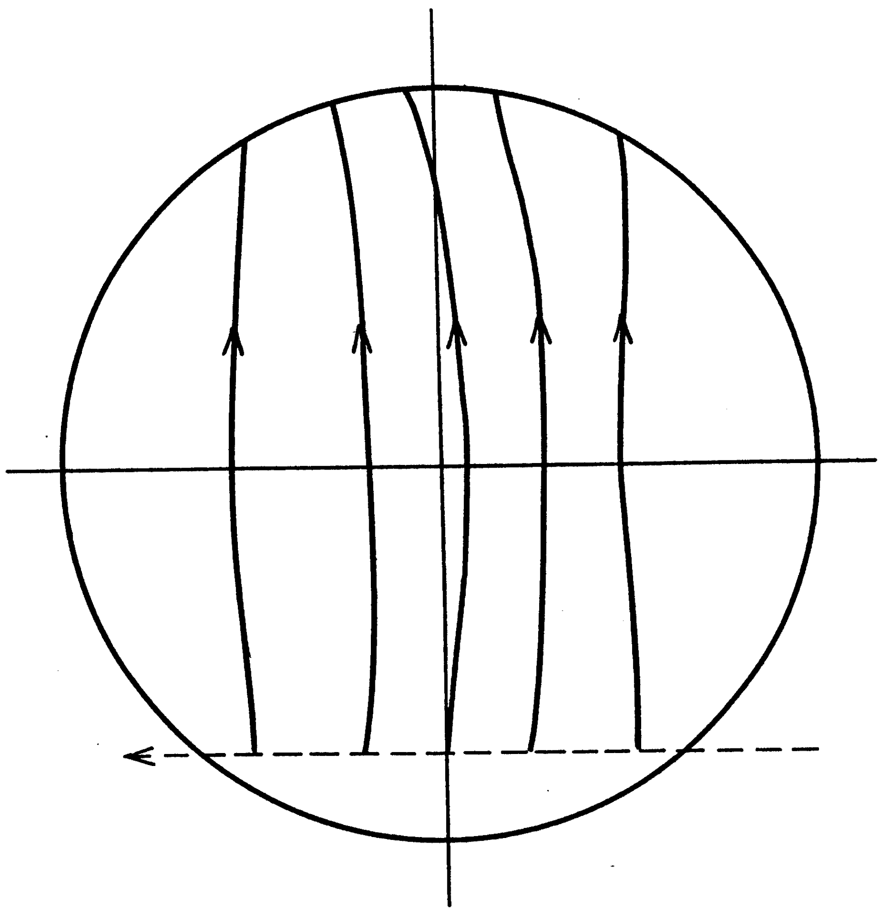

Fig. 3