INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Equilibrium of REB-Ring with Soft-Core

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IPPJ-260 October 1976

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Abstract

A equilibrium configuration of a charge-neutralized toroidal relativistic electron beam (REB ring) with a current-carrying plasma column to provide the toroidal field is calculated analytically. This plasma column is called soft-core. There exists an equilibrium of REB ring with a stable soft-core provided that the length of the core is shorter than a critical length.

By the result of recent experimental studies made on the REB ring for the plasma confinement by using a series of the devices [1], it is confirmed that it is possible to construct the stable toroidal equilibrium with safety factor q_b smaller than unity. It is of great interest to take the theoretical study of the equilibrium in REB ring as far toward the reactor regime as possible by the major radius compression of the toroidal plasma [2] confined by the REB ring.

In the start-up phase the presence of the strong toroidal field is empirically inevitable in order to form a REB ring and it is usual that the toroidal field is supplied by flowing the current along a center conductor. To increase the density of the confined plasma the strong compression should be supplied by increasing the axial magnetic field to the REB ring like the hard-core θ pinch proposed by Yoshikawa [3]. It is advisable from the point of achieving the strong compression that the center conductor should be replaced by the plasma column in order to keep the hot plasma in the REB ring out of touch with the cold center-conductor (See Fig.1). Therefore, it is of importance to investigate the stability condition of the replaced plasma column (we hereafter call this as *soft-core*) in a tractable case.

In the present paper we show the existence of the toroidal equilibrium of REB ring with a soft-core, and we investigate the stability condition of the soft-core for the kink mode.

In case the relativistic electron-beam current exceeds the Alfvén limit by a large factor, the equilibrium of the charge-neutralized REB ring can be described by the well-known force-free condition [4]. The equilibrium of the current-carrying plasma with a negligible plasma pressure can also be described by the same condition. Thus, the equilibrium of the REB ring with soft-core can be investigated by solving the single equation

$$\nabla \times \vec{B} = \alpha \vec{B} \quad , \tag{1}$$

where α is a constant and the vector $\vec{B}$ represents the magnetic field, i.e. $\nabla \cdot \vec{B} = 0$. In the cylindrical coordinate system (r, θ, z) the stream function ψ is introduced by

$$B_r = -\frac{1}{r}\frac{\partial \psi}{\partial z} \quad , \tag{2}$$

$$B_z = \frac{1}{r}\frac{\partial \psi}{\partial r} \; .$$

Then, Eq.(1) is reduced to

$$r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} + \alpha^2 \psi = 0 \quad . \tag{3}$$

To simplify the stability analysis later, we obtain the solution of Eq.(3) for the thin REB ring with circular cross section. We define the dimensionless coordinates (x, y):

$$r = R(1 + \varepsilon x) \; ,$$

$$z = R\varepsilon y \quad , \tag{4}$$

where R is the major radius of the REB ring and ε is the ratio of the minor radius to the major radius. Then, Eq.(3) can be rewritten by

$$(1 + \varepsilon x)\frac{\partial}{\partial x}\frac{1}{1+\varepsilon x}\frac{\partial\psi}{\partial x} + \frac{\partial^2\psi}{\partial y^2} + \gamma^2\psi = 0 \quad , \tag{5}$$

where $\gamma = \alpha R\varepsilon$.

Taking $\varepsilon << 1$ and expanding ψ in the form

$$\psi = \psi_0(x,y) + \psi_1(x,y) + \cdots \tag{6}$$

we can solve Eq.(5) order by order. Choosing ψ_0 to be a constant then the leading order equation is automatically satisfied. The first order equation to be solved is

$$\frac{\partial^2\psi_1}{\partial x^2} + \frac{\partial^2\psi_1}{\partial y^2} + \gamma^2\psi_0 = 0 \tag{5)'$$

with the appropriate solution

$$\psi_1 = \frac{\gamma^2\psi_0}{2}(1 - x^2 - y^2). \tag{7}$$

Let the total current of the REB ring be I. Then we have $\gamma^2\psi_0 = \mu_0 IR/2\pi$, where μ_0 is the magnetic permeability of vacuum.

We now consider the vacuum region among the soft-core and the REB ring. Expanding the function ψ as before, the solution to first order is

$$\psi_1 = C_1 \ell n(x^2+y^2) + C_2 x(1 - \frac{1}{x^2+y^2}) \quad , \tag{8}$$

where C_1 and C_2 are constants to be determined. The second term of R.H.S. of Eq.(8) is of fundamental importance to investigate the magnetic configuration in the soft-core.

As is well-known, the flux function of a current loop located in the plane z=0 is described by [5]

$$\psi_\ell = \frac{\mu_0 I_\ell}{\pi k}(rR_\ell)^{1/2}\{(1 - \frac{k^2}{2})K(k) - E(k)\} \quad , \qquad (9)$$

where $k^2=\frac{4rR_\ell}{(R_\ell+r)^2+z^2}$, and R_ℓ and I_ℓ are the major radius and the total current of the loop, while E(k) and K(k) are complete elliptic integrals. In the vicinity of the loop, $k \simeq 1$. Using expressions that apply for E and K when $k \simeq 1$ [6],

$$E(k) = 1 + \frac{1}{2}(\ln \frac{1}{\sqrt{1-k^2}} - \frac{1}{2})(1 - k^2) + \cdots ,$$

$$K(k) = \ln\frac{1}{\sqrt{1-k^2}} + \frac{1}{4}(\ln \frac{4}{\sqrt{1-k^2}} - 1)(1 - k^2)+\cdots \qquad (10)$$

the expression for ψ_ℓ can be reduced to

$$\psi_\ell = \frac{\mu_0 I_\ell}{2\pi} (\ln \frac{8R_\ell}{\rho} - 2) \ , \qquad (11)$$

where $\rho^2 = (r - R_\ell)^2 + z^2$.

For $R_\ell=R+\Delta$, (11) can be expanded as

$$\psi_\ell = \frac{\mu_0 I_\ell}{4\pi} [\ln\{(r-R)^2 + z^2\} - \frac{2(r-R)\Delta}{(r-R)^2+z^2}] \qquad (11)'$$
$$+ \text{(constant terms)}.$$

If the uniform field is superposed on the field of the loop, the equation for the magnetic surface becomes

$$\psi^* = \psi_\ell + \frac{B_0}{2} r^2 \ . \qquad (12)$$

Using the dimensionless coordinates (x,y) defined by (4), Eq.(12) is rewritten by

$$\psi^* = \frac{\mu_0 I_\ell R}{4\pi} [\ln(x^2+y^2) - \frac{2\Delta}{\varepsilon R} \frac{x}{x^2+y^2}] + \varepsilon R^2 B_0 x$$

$$+ O(\varepsilon^2) + \text{(constant terms)} \ . \qquad (13)$$

By comparing the expression (13) with (8), the constants C_1, C_2 and Δ have to be

$$C_1 = \frac{\mu_0 I_\ell}{4\pi} \quad ,$$

$$C_2 = \varepsilon R^2 B_0 \tag{15}$$

and

$$\Delta = 2\pi R^3 \varepsilon^2 B_0 / \mu_0 I_\ell \quad .$$

From the condition of continuity of the poloidal field at the interface we have

$$I = I_\ell$$

and

$$\frac{\mu_0 I}{2\pi} + \varepsilon R B_0 = 0 \quad . \tag{16}$$

This shows that $\Delta < 0$ since the product $B_0 I$ is negative. Thus, the vacuum region among the soft-core and the REB ring can be described by the flux function given by

$$\psi = \frac{B_0}{2} r^2 + \frac{\mu_0 I}{\pi k} (rR_\ell)^{1/2} \{(1 - \frac{k^2}{2})K(k) - E(k)\}. \tag{17}$$

From Eq.(17) the magnetic field B_z in the vicinity of the z axis can be written by

$$B_z = B_0 + \frac{\mu_0 I}{2} \frac{R_\ell{}^2}{(R_\ell{}^2 + z^2)^{3/2}} \quad . \tag{18}$$

In case the length of the soft-core, L, is sufficiently shorter than the major radius, R_ℓ, the magnetic field is approximately uniform in the soft-core.

$$B_z = B_0 + \frac{\mu_0 I}{2R_\ell} \quad . \tag{18'}$$

The ratio $|\mu_0 I/2R_\ell B_0|$ is called the field-reversing factor. In the present case the factor is approximately equal to $\pi\varepsilon$. Let the radius of the soft-core be a and the current along it be I_s. Then, the toroidal field in the vacuum region and on the surface of the REB ring becomes

$$B_\theta = \frac{\mu_0 I_s}{2\pi r} . \qquad (19)$$

Now it is possible to estimate the stability of the soft-core for the kink mode. To facilitate the analysis the following assumptions are made:

i) The soft-core consists of a sufficiently dense plasma so that there is no coupling of motion between the soft-core and the REB ring.

ii) The current along the soft-core is uniformly distributed in the plasma column.

Then, by using (19) the stability condition of the soft-core for the kink mode becomes

$$q_p \equiv |4\pi^2 a^2 B_z/\mu_0 I_s L| > 1 \quad \text{(Kruskal-Shafranov limit)}. \qquad (20)$$

By using (16) and (18)', the quantity q_p is reduced to

$$q_p = 2\pi a^2/\varepsilon R L \ . \qquad (21)$$

This means that the length of the soft-core has to be shorter than

$$L < 2\pi a^2/\varepsilon R \qquad (22)$$

for the stability.

While the safety factor q_b of the REB ring in the present approximation becomes

$$q_b = \varepsilon^2 I_s / I \qquad (23)$$

Since the quantity q_p is independent of the safety factor q_b, we may say that it is possible to have a stable soft-core which is not influenced by the motion of the REB ring.

Finally it should be pointed out that the concept of the soft-core may be applicable for the stabilization of the precessional mode in the magnetic compression of intense ion rings [7,8]. Then, the plasma of the soft-core may become a target of the ion beam as a two component fusion reactor [9], if the radius of the soft-core is larger than the radius of the ion ring. In this case the choice of the plasma ion species of the soft-core may make it possible to examine into the use of the exotic CTR reactions [10] as a reactor.

ACKNOWLEDGMENTS

The author is grateful to Prof. A. Mohri for suggestive conversations.

Figure Caption

Schematic drawing of compression machine of REB ring with soft-core. After the current along the soft-core is driven by the transformer the poloidal field coil produces slowly-increasing vertical field followed by the formation of REB ring, and the major radius of the REB ring decreases slowly.

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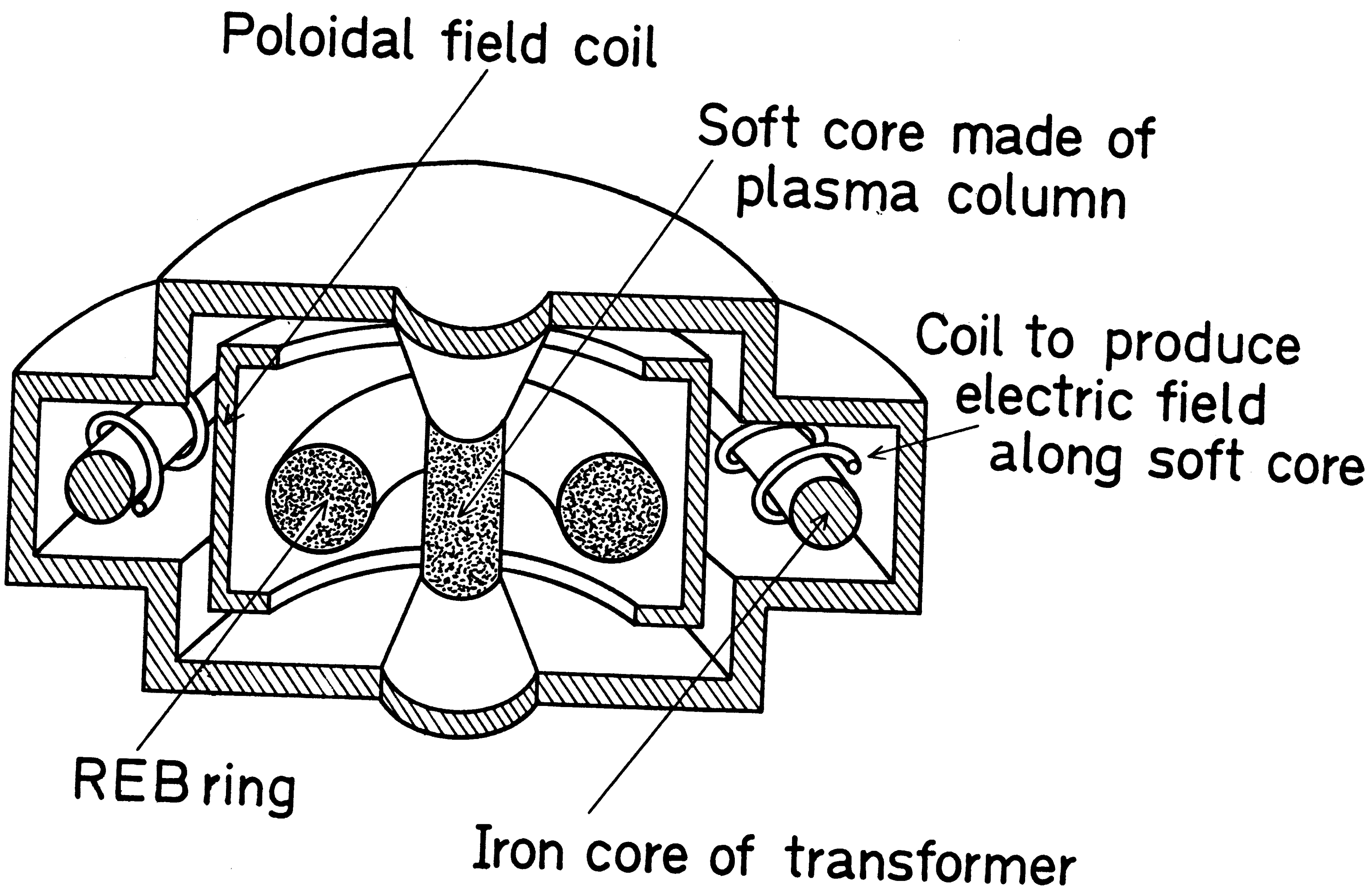
Poloidal field coil
Soft core made of
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