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Landing of REB Ring on Equilibrium Orbit

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ABSTRACT

It is analytically shown that the decrease of the return current of plasma forces an injected relativistic electron beam to land on an equilibrium orbit inside a conductive toroidal chamber. The mechanism is similar to the adiabatic trapping of electrons in betatron.

1. INTRODUCTION

Successful injection of high current relativistic electron beam (REB) into toroidal systems will offer a new possibility for plasma confinement and heating. The poloidal magnetic field necessary for plasma confinement can be produced by toroidal REB currents,[1~3] and effective heating of plasma caused by the energy transfer from REB to plasma is also expected.[4~6] One of the problems which we have to solve in this scheme is how to inject REB into a toroidal chamber from the outside and to form an REB ring in an equilibrium state. Several contrivances for this purpose have been tried in several laboratories. In the case of ASTRON[7] and RECE-Berta[8], REB's were injected through snorts, which were set oblique to the symmetric axis of the magnetic mirrors to avoid the collision of the beams with the snorts. Resistors installed inside the chamber wall functioned to damp the axial oscillations of the beam. The pulsive extraction of a toroidal magnetic field towards the beam gun (field shaping cathode)[9] and the use of the toroidal drift of electron beam[10] have been applied to inject REB into toroidal magnetic field. It has been experimentally shown that the change of the relativistic factor γ during the injection also helps the REB to launch on the equilibrium orbit in an applied vertical magnetic field.[11]

Electron beam rings formed just after the injection deviate from the equilibrium position. This means that the beam ring has a potential of energy to the equilibrium state. If there is no dissipation of the energy, the ring oscillates or circulates around the equilibrium position with a constant amplitude.

In this paper, it is shown that the growth of the net current, which is caused by the decrease of the return current in the plasma, makes the oscillation shrink in amplitude adiabatically and forces the beam ring to land on the equilibrium orbit. This mechanism is formally similar to the adiabatic trapping of seeded electrons in betatron accelerators.[12)]

2. LANDING OF REB RING ON EQUILIBRIUM ORBIT

Let's consider the following case. An electron beam is injected into a toroidal conductive chamber from the outside. Both a toroidal magnetic field and a vertical magnetic field are externally applied. A neutral gas or a plasma of an appropriate density is present inside the chamber. At the injection the beam is directed parallel to the minor axis of the torus. The electron beam goes forwards along the toroidal chamber (ionizing the gas in the case of the injection into the neutral gas). The fast change of the magnetic field induced by the beam current gives rise to a return current inside the plasma, and the magnetic field of the beam current is shielded by the return current. If the return current decays faster than the beam current, the resultant net current increases with time. This net current induces a Foucalt current (image current) in the conductive toroidal chamber and, as a result, the motion of the beam is subject to the magnetic field due to the Foucalt current. This reaction increases with time during the decay of the return current. When the vertical field is set to make the beam ring settle on the axis of chamber in the final equilibrium state, a beam ring, which is initially formed eccentric to the axis, will oscillate about the equilibrium position changing its amplitude and frequency.

We shall consider a cold monoenergetic REB ring which is formed eccentric to the axis of the toroidal chamber. The basic equation of motion is

$$m_o \gamma \frac{D\vec{v}}{Dt} = -e[\vec{E} + \vec{v}\times\vec{B} - \frac{\vec{v}}{c^2}\vec{v}\cdot\vec{E}], \qquad (1)$$

where m_o and e are the rest mass of electron and the positive unit charge, and γ the relativistic factor. We employ cylindrical coordinates (r,ϕ,z) as shown in Fig.1. The toroidal conductive chamber has a circular cross section of the radius a and its major axis is R. The cross section of the ring is assumed to be negligibly small compared with that of the toroidal chamber. We confine ourselves to treat the axisymmetric motion of the ring within the ordering

$$\frac{v_r}{v_\phi} \sim \frac{v_z}{v_\phi} \simeq \varepsilon << 1,$$

and

$$I_A = \frac{4\pi m_o \gamma v_\phi}{e\mu_o} >> I_n ,$$

where I_A is the Alfvén limit[13)] and I_n is the net current. The latter ordering means that we are considering the case in which only a small part of the kinetic energy of the beam is transferred to the magnetic energy. The equation (1) is rewritten by

$$2m_o\gamma \frac{\partial v_r}{\partial t} = m_o\gamma \frac{v_\phi^2}{r} - e(v_\phi B_z - v_z B_\phi), \tag{2a}$$

$$2m_o\gamma \frac{\partial v_z}{\partial t} = -e(v_r B_\phi - v_\phi B_r). \tag{2b}$$

The toroidal magnetic field B_t and the vertical magnetic field B_v are externally applied so that

$$\begin{aligned} \vec{B}_t &= B_o \frac{R}{r} \vec{e}_\phi , \\ \vec{B}_r &= \frac{m_o\gamma v_\phi}{eR} \vec{e}_z \quad . \end{aligned} \tag{3}$$

Here, B_v corresponds to the betatron field of electrons having the circulating radius of R. When the beam ring locates at $(R+\delta,\ \phi, z)$, the resultant Foucalt current produces an additive magnetic field B_f in the position of the beam. This field is approximately expressed by

$$\vec{B}_f \simeq S(t)(-z\vec{e}_r + \delta\vec{e}_z), \tag{4}$$

$$S(t) \equiv \frac{\mu_o I_n(t)}{2\pi a^2} \quad , \tag{5}$$

for the case $(\delta^2 + z^2)/a^2 \ll 1$. Here, the inverse aspect ratio a/R has been assumed to be sufficiently smaller than ε, whence the first and the second terms of Eq.2(a) can be approximated as

$$m_o\gamma \frac{v_\phi^2}{r} - ev_\phi B_z \simeq -eSv_\phi\delta$$

because

$$m_o\gamma \frac{v_\phi^2}{R^2} / (Sev_\phi) = \frac{1}{2}\left(\frac{a}{R}\right)^2 \frac{I_A}{I_n} \ll \varepsilon \quad .$$

Then, Eq.(2) is reduced to

$$\frac{d^2\ell}{dt^2} + i\,\frac{\Omega}{2}\frac{d\ell}{dt} + \alpha^2 f(t)\ell = 0, \tag{6}$$

where

$$\ell = \delta + iz,$$
$$\Omega = \frac{eB_\phi}{m_o\gamma}, \tag{7}$$
$$\alpha^2 f(t) = \frac{ev_\phi S(t)}{2\gamma m_o}.$$

By using the form

$$\ell(t) = e^{-i\frac{\Omega}{4}t}\,F(t),$$

Eq.(6) is rewritten as

$$\frac{d^2F}{dt^2} + \omega^2\left[1 + \frac{\alpha^2}{\omega^2} f(t)\right]F = 0, \tag{8}$$

where $\omega = \Omega/4.$

The coefficient in the brakets is an increasing function of time since the net current I_n increases with time. Therefore, as time passes, the frequency of the oscillation becomes higher but the amplitude decreases. This behaviour is analogous to the oscillation of a pendulum under an increasing gravitational force. In betatron accelerators, the seeding of electrons from the outside is well operated by using a similar adiabatic shrink of the amplitude of the betatron oscillation[2], though the physical origin of the shrink is different.

For further understanding, let's have an analytical solution of Eq.(8) by choosing the form of f(t) as

$$\frac{\alpha^2}{\omega^2} f(t) = \xi t. \tag{9}$$

Then, Eq.(8) becomes

$$\frac{d^2F}{dt^2} + \omega^2[1 + \xi t]F = 0.$$

From the general solution of the above equation, we have the amplitude

$$\ell = e^{-i\omega t}\omega(1+\xi t)^{\frac{1}{2}}[AH_{\frac{1}{3}}^{(1)}(k(1+\xi t)^{\frac{3}{2}})+BH_{\frac{1}{3}}^{(2)}(k(1+\xi t)^{\frac{3}{2}})], \tag{10}$$

where $k = \frac{2}{3}\frac{\omega}{\xi}$, $H^{(1)}$ and $H^{(2)}$ are Hankel functions of the first kind and the second, respectively, and A and B are the constants to be determined by an initial condition. For the initial condition

$$\ell = \ell_o \text{ and } \frac{d\ell}{dt} = 0, \text{ at } t = 0,$$

the solution becomes

$$\ell = \ell_o \cdot ie^{-i\omega t}\frac{\pi k}{4}(1+\xi t)^{\frac{1}{2}}[H_{\frac{4}{3}}^{(1)}(k)H_{\frac{1}{3}}^{(2)}(k(1+\xi t)^{\frac{3}{2}})-H_{\frac{4}{3}}^{(2)}(k)H_{\frac{1}{3}}^{(1)}(k(1+\xi t)^{\frac{3}{2}})$$

$$+(\frac{1}{3k} + \frac{\xi}{2\omega} -i)\{H_{\frac{1}{3}}^{(2)}(k)H_{\frac{1}{3}}^{(1)}(k(1+\xi t)^{\frac{3}{2}})-H_{\frac{1}{3}}^{(1)}(k)H_{\frac{1}{3}}^{(2)}(k(1+ t)^{\frac{3}{2}})\}] \tag{11}$$

In order to have more insight of the change of the amplitude with time, let's find an approximate form of Eq.(11) under a reasonable ordering of the parameters. From Eqs.(5) and (7) we have

$$\frac{\omega^2}{\alpha^2} = \frac{1}{16}\frac{I_A}{I_o}\frac{\Omega^2}{(v_\phi/a)^2} , \tag{12}$$

where the form $I_n(t) = I_o f(t)$ has been used. In most of cases $\Omega > v_\phi/a$, whence ω^2/α^2 is a large quantity. If we use the rise time of the net current τ, k is expressed by

$$k = \frac{2}{3}\,\frac{\omega}{\xi} \sim \frac{2}{3}\,\frac{\omega^2}{\alpha^2}(\tau\omega).$$

The rise time τ is longer than the electron cyclotron period $1/\Omega = 1/(4\omega)$ in usual cases, so that $\tau\omega > 1$. It is sufficient to consider the case $k >> 1$. For the same reason we can regard $\xi t = (\alpha^2/\omega^2)\cdot(t/\tau)$ as a small quantity. Hence, the approximate form of the solution (11) is found to be

$$\ell \simeq \ell_o \frac{1}{(1+\xi t)^{1/4}} e^{i\frac{\xi\omega}{4}t^2}[1+\frac{1}{2}(\frac{\xi}{\omega} - 1 - i)\{1 - e^{-2i\omega t(1+\frac{\xi}{4}t)}\}]. \tag{13}$$

This shows that the amplitude $|\ell|$ decreases with time in proportion to $(1+\xi t)^{-1/4}$, whilst the revolution of the beam around its equilibrium orbit becomes faster as $\exp(i\xi\omega t^2/4)$. From Eqs.(9) and (12) it is evident that the fast rise of the net current and/or the lower toroidal magnetic field are required to achieve the fast landing of the beam ring onto the equilibrium orbit.

If the toroidal magnetic field is not present, the problem becomes simpler and we can easily show the shrink of the amplitude of the oscillation which occurs around the equilibrium orbit. This mechanism helps the formation of a beam ring in an Astron device.

3. CONCLUSIVE REMARKS

A mechanism to form a ring of relativistic electron beam inside a toroidal conductive chamber has been discussed. It originates in the increase of the net current, caused by the fading of the return current. Other mechanisms due to the energy dissipation of the beam are also helpful to form the ring in equilibrium. It should be noted that, experimentally, the efficient trapping of injected beams in toroidal conductive chambers is well correlated with the smooth rise of the poloidal magnetic field.[14]

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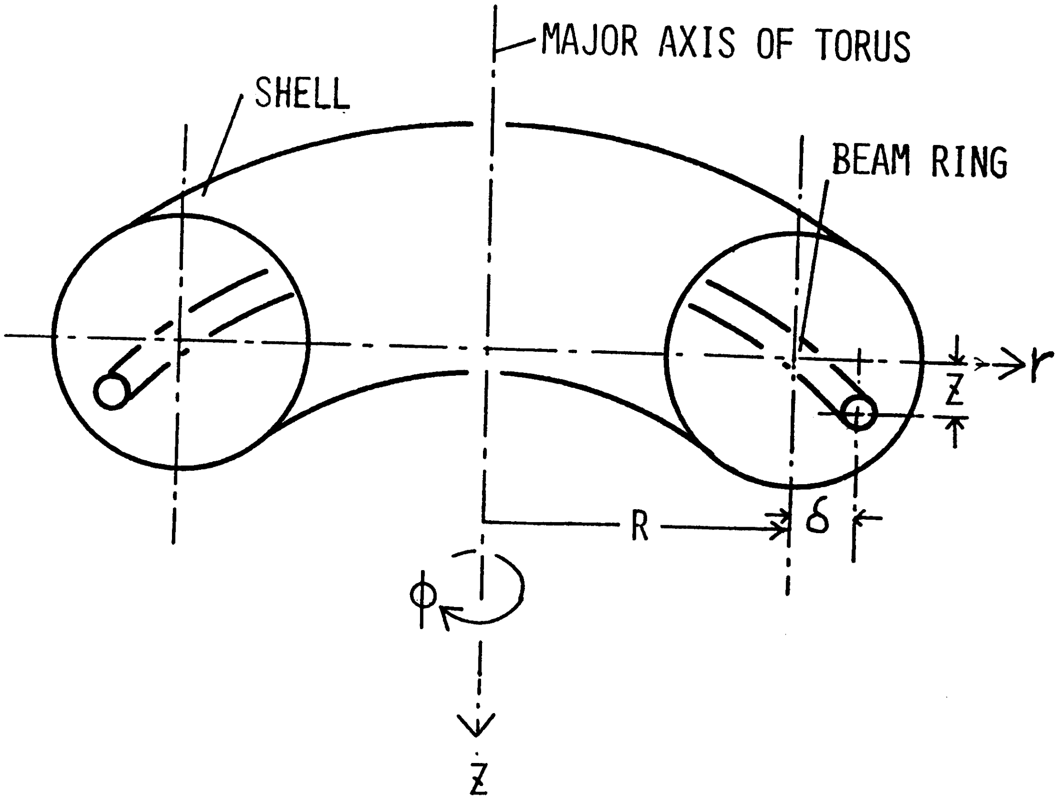

Fig.1 Cross-section of torus showing the coordinates (r, φ, z) and the position of the beam ring.