INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

End Pluggings of a Linear Theta Pinch
by Hot High Z Plasmas of High Mass Number

Kei-ichi HIRANO and Masahiro WAKATANI

IPPJ-262 October 1976

RESEARCH REPORT

NAGOYA, JAPAN

End Pluggings of a Linear Theta Pinch by Hot High Z Plasmas of High Mass Number

Kei-ichi HIRANO and Masahiro WAKATANI

IPPJ-262 October 1976

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

Permanent Address : Institute of Plasma Physics,
Nagoya University, Nagoya, Japan.

Abstract

End plugging of a θ-pinch by high temperature plasma of high mass number is proposed. Using two different simplified models, thermal insulation mode (TIM) and free expansion mode (FXM), characteristic parameters interested in CTR reactor are calculated. The TIM claims that the energy is lost through the electron thermal conduction, whereas in the FXM the plasma energy is carried out with the plasma flowing along the magnetic field line. In the TIM the thermal insulation is improved due to the high Z number of plugging plasmas. In the FXM inertia of the high mass ions contribute to reduce the plasma expansion speed along it. It is estimated that the plugging proposed in this paper will save the system length by factor 3 $\sim$ 5 compared with the one without any plugging. A typical example of the length of the system will be 68 m for the Lawson criterion experiment under the operation of 200 kG. The energy required for the plugging is calculated to be $4.9 \times 10^{-2}\ Z_{eff}^{-1/2}(n\tau_E)^{1/2}T^{9/4}(\pi b^2)$ for TIM and $8.8 \times 10^{-6} \cdot \eta\mu^{-1/2}T^{-3/2}(n\tau_E)(\pi b^2)$ for FXM, where η is the throttling coefficient of about 2.5 and b is inner radius of the coil. For Lawson experiment magnetic energy of 92 MJ will be enough if the coil and the plasma radius is 6 and 3 cm, respectively.

§1. Introduction

Even at present a linear θ-pinch is very attractive and has many advantages over toroidal machine such as tokamak or scyllac because of its simplicity and its ease of heating the plasma to thermonuclear temperature. The most serious difficulty of a θ = pinch is in the free streaming out of a hot plasma from the ends, which usually makes the θ-pinch reactor enormously long. Ellis[1] estimated major parameters of a linear θ-pinch reactor system. He claims the length of the reactor to be 4.9 km under the operation of 200 kG.

Here, we propose a new scheme of end plugging of a linear θ-pinch: the plasma of DT mixture is plugged by hot plasmas of high Z atoms. The plugging plasma may be Xe's or some heavy metals'. Initially cold plugging gases or vapors are puffed from the both ends of the apparatus. Then the θ-pinch is applied to both DT mixture and plugging gas at the same time. Consequently we can obtain a high temperature DT plasma caulked by the high Z atoms'. The plugging is effective because it reduces both expansion speed of DT plasmas and electron thermal conductivity along the magnetic field lines. In order to simplify the physical model of the plug, we studied two extreme cases shown in Fig. 1. One is the thermal insulation mode (TIM) under which the plasma is terminated by cold end plates and the pressure balance along the magnetic field is assumed. Since the macroscopic flow of particle does not exist in this mode the major energy loss from the plasma is the electron thermal conduction along the field line. The other model is the free expansion mode (FXM). We consider the plugging plasma from the end of θ-pinch continuously expanding into the long dc solenoidal magnetic channels so as to isolate the plasma from the wall. Therefore the thermal conduction is not a problem in FXM but only the particle flowing out from the θ-pinch system should be taken into account. It is worthwhile to note that the radiation loss from the plug is not so important compared with the one coming from direct plasma flow out or electron thermal conduction. In this paper we give several parameters of the system which satisfy the condition of so called Lawson criterion experiment and of fusion reactor.

§2. General Considerations

The escape of plasmas from the ends of a θ-pinch is not always quantitatively understood yet[2]. Qualitatively, the major energy loss comes from the direct plasma flow out with sound velocity or from electron thermal conduction along the field. The scale length of the pinch and the boundary condition of the ends determine which process is important. Here we think about the end loss of a hot high Z plasma of high mass number. The plugging plasma parameters interested in CTR are as follows: $T_e = T_i \simeq 1 \sim 10$ keV, $n_e = Zn_i \simeq 10^{16} \sim 10^{18}/\mathrm{cm}^3$ $Z \simeq 50$, μ(mass number) $\simeq 100 \sim 200$, τ_E(energy confinement time) $\simeq 0.1 \sim$ 10 msec, and $\beta = 1$. The energy equipartition time between heavy ions and electrons is given by

$$\tau_{eq} = 2.6 \times 10^7 \mu \frac{T_e^{\frac{3}{2}}}{n_e Z} \quad , \tag{1}$$

where T_e is measured by eV. Hereafter $\ell n\Lambda$ which appears in collision time is always chosen to be 12 and all the units in this paper are written in c.g.s., eV and Gauss. Then the τ_{eq} becomes $\sim$ 240μsec under the plasma condition of T_e = 5 keV, $n_e = 10^{17}/cm^3$, μ = 131(Xe) and Z = 50, so that we can safely put $T_i = T_e = T$ since $\tau_E > \tau_{eq}$ is held, where τ_E is energy confinement time. The energy loss flux by electron thermal conduction can be written as[3]

$$q_t = \kappa_{\parallel e} \frac{\partial (kT)}{\partial x} \simeq n_e v_e k T_e \left(\frac{\lambda e}{x} \right) , \tag{2}$$

where x is the scale length of the plugs, λ_e is the electron mean free path and k is Boltzmann constant. The energy flux carried by particles is simply expressed as

$$q_p \simeq v_i n_e kT \quad , \tag{3}$$

since $n_e \gg n_i$. If we claim the condition $q_t \gtrsim q_p$, the relation

$$\frac{x}{\lambda_e} \leq \left(\frac{m_i}{m_e} \right)^{\frac{1}{2}} \tag{4}$$

should be satisfield. The upper bound of x is denoted as x_t, the threshold scale length. The eq.(4) indicates that in short system electron thermal conduction is the major energy loss factor. In order to have a rough scale length of the plug, the threshold x_t for Xe is plotted in Fig.2 using the formula

$$\lambda_e = 1.7 \times 10^{12} \frac{T^2}{Zn_e} \quad . \tag{5}$$

The charge state z is a function of the plasma temperature. Here, we assume the expression,

$$\frac{1}{Z} = \frac{1}{A} + \left(\frac{T_a}{T} \right)^g , \qquad (6)$$

where A is the atomic number, T_a and g are constants which depend on the plugging plasma element. The Moseley's law is applied to estimate Z as a function of T. As a result, we obtain T_a = 17 eV and g = 1.2 for Xe. However, eq.(6) does not give correct Z for very low temperature. As can be seen in Fig.2, the threshold length x_t of the plug below which the thermal conduction loss becomes important is about 40 m for the plugging plasma of T = 5 keV and n = $10^{17}/cm^3$. A simple calculation shows that the necessary scale length of the plug for CTR becomes very close to the threshold length x_t. In order to avoid the complicated physical condition, two different models, TIM and FXM, are introduced.

§3. Thermal Insulation Mode (TIM)

As shown in Fig.1 TIM claims that the plasma is terminated by cold plate at the end and also that steady state pressure balance along the pinch is maintained. In such a system macroscopic plasma flow does not exist and the electron thermal conduction is the dominant factor of the energy loss. In the TIM calculation, radiation loss from the high Z plugging gas is assumed to be so small that it is taken into account as a perturbation. Since one dimensional steady state is assumed, a set of equations governing the heat loss is written as follows:

$$\frac{\partial}{\partial x} \left(\kappa_{e\parallel} \frac{\partial}{\partial x} kT \right) = 0 , \qquad (7)$$

$$\frac{B^2}{8\pi} = n_e kT . \qquad (8)$$

The eq.(8) is held since Z value of the plugging gas is very high ($Z >> 1$). For the thermal conductivity $\kappa_{e\parallel}$, we use the classical formula[4],

$$\kappa_{e\parallel} = \frac{n_e k T_e \tau_e}{m_e} \gamma_o \quad , \tag{9}$$

$$\tau_e = 2.9 \times 10^4 \frac{T_e^{\frac{3}{2}}}{z n_e} \quad , \tag{10}$$

where γ_o is a constant which is a fuction of Z. For $Z >> 1$, $\gamma_o = 12.5$. The thermal flux through the plug q_t becomes constant by the steady state assumption (eq.(7)),

$$q_t = \kappa_{e\parallel} \frac{dkT}{dx} = \text{const.} \quad . \tag{11}$$

If the temperature of the end plate T_o is very low compared with that of contact surface T_c, the q_t is easily integrated by the aid of Eq.(6), (9), (10) and (11) as

$$q_t = 2.9 \times 10^8 \frac{T_c^{\frac{7}{2}}}{Z_{eff} x_c} \quad , \tag{12}$$

where suffix c denotes the quantities at the contact surface between the plug and the DT plasma. The Z_{eff} in Eq.(12) is defined by

$$Z_{eff} = \frac{1}{A} + \frac{1}{\frac{7}{2} - g} \left(\frac{T_a}{T_c} \right)^g \quad . \tag{13}$$

The energy confinement time τ_E is given by

$$\tau_E = \frac{3nkT_c \ell}{q_t} \quad , \tag{14}$$

where 2ℓ is equal to the length of DT plasmas and n is its density. Here, the bremsstrahlung is only considered for the radiation loss. It is estimated assuming that Z_{eff} is constant throughout the plug. The radiation loss rate

$$W_b = 1.6 \times 10^{-27} n_e^{\,2} Z_{eff} T^{\frac{1}{2}} \tag{15}$$

and pressure balance, Eq.(8), give the total radiation loss q_b from the plasmas,

$$q_b = \int_o^{x_c} W_b dx = 1.7 \times 10^{-6} B^4 Z_{eff} T_c^{-\frac{3}{2}} x_c \ . \tag{16}$$

Carring out the integration, we used the functional dependence of T on x obtained from Eq.(12), which is

$$T(x) = T_c (\frac{x}{x_c})^{\frac{2}{7}} \ . \tag{17}$$

The radiation loss is not so important, if the ratio

$$\frac{\tau_E}{\tau_b} = \frac{q_b}{q_t} = (\frac{x_c}{x_b})^2 \tag{18}$$

is small enough, where τ_b is the characteristic time defined by the radiation loss and x_b is a scale length written as

$$x_b = 1.3 \times 10^7 \frac{T^{\frac{5}{2}}}{B^2 Z_{eff}} \ . \tag{19}$$

It is apparent that if the plug length x_c is larger than x_b the model of the calculation breaks down since the radiation becomes much important. Therefore x_b is said to give the maximum length of the plug. The minimum value, on the other hand, comes from the blurring of the contact surface. Since we assume the pressure balance across the surface the diffusion length Δx_c of the plugging plasma into the DT's gives such a scale. It is well known that

$$\Delta x_c \simeq \sqrt{\frac{1}{2} v_i^2 \tau_i \tau_E}$$

$$= 1.3 \times 10^9 \frac{T^{\frac{5}{4}} (n\tau_E)^{\frac{1}{2}}}{Z^{\frac{3}{2}} \mu^{\frac{1}{4}} n} , \tag{20}$$

where τ_i is the ion collision time.

The value of $n\tau_E$ is calculated from the Eq.(14) and the pressure balance at the DT plasmas. The result is

$$n\tau_E = 2.6\, T^{-\frac{9}{2}} Z_{eff} B^4 x_c \ell . \tag{21}$$

If the total length of the pinch L is fixed constant, the $n\tau_E$ becomes maximum when

$$x_c = \ell = \frac{L}{4} . \tag{22}$$

Thus we obtain the plug length,

$$x_c = 0.63 \frac{(n\tau_E)^{\frac{1}{2}} T^{\frac{9}{4}}}{B^2 Z_{eff}^{\frac{1}{2}}} . \tag{23}$$

Using the Eq. (23) various parameters which are interested in CTR are calculated and listed in Table 1. As can be seen in Eq. (23) $x_c B^2$ is only a function of $n\tau_E$, T and Z_{eff}. We can introduce the magnetic field energy density W_{TIM} to sustain the TIM plugging plasma, where

$$W_{TIM} = \frac{B^2}{8\pi}(2x_c) = 4.9 \times 10^{-2} Z_{eff}^{-\frac{1}{2}} (n\tau_E)^{\frac{1}{2}} T^{\frac{9}{4}} . \qquad (24)$$

in Fig. 3 W_{TIM} is plotted as a function of $n\tau_E$ and T. For $Z_{eff} = 50$, $n\tau_E = 5 \quad 10^{13}$ and T = 5 keV, W_{TIM} is 1 MJ/cm^2 can be seen. If the system is operated with 200 kG under the above condition, x_c will be 33 m, while the radiation critical length x_b is 114 m and Δx_c is 4.5 cm. Therefore in this case the radiation loss is smaller than the thermal conduction loss by more than factor 10.

§4. Free Expansion Mode (FXM)

This mode is just the opposit extreme to the TIM. As is shown in Fig.1, FXM is realized when the end of the θ-pinch is extended by dc solenoides of much weaker field strength. The configuration is quite analogous to the system of θ-pinch gun and plasma guiding field. If the extended magnetic channel is long enough, plugging plasma is well separated from the wall. In such a system thermal conduction does not come into the problem. In this case we define τ_E as

$$\begin{aligned} \tau_E &= x_c \left(\frac{m_i}{2kT} \right)^{\frac{1}{2}} \eta \\ &= 7.2 \times 10^{-7} x_c \mu^{\frac{1}{2}} T^{-\frac{1}{2}} \eta , \end{aligned} \qquad (25)$$

where m_i is the mass of the plugging plasma ion and η is throttling coefficient of the θ-pinch. Ellis[1] gave the θ-pinch reactor design with η = 7.1. However, recent numerical studies[5] show that $\eta = 2.8$, while the experiment gives the value of $\eta = 2.5$ Here, we use 2.5 for η so that fare comparisons with Ellis's results can be made when factor 2.8 is multiplied to the scale length of his system. The energy confinement time defined by Eq.(25) may be under estimated because it is just the time of the plugging plasma flowing out from the θ-pinch. We can expect that the plugging plasma still expands along the extended magnetic channel with its sound velocity, which may well choke the expansion of the hot DT plasma. In this paper detail physical processes of choking effect are not discussed but simple energy confinement time defined by Eq.(25) is used in order to show rough estimates of the merit of this plugging scheme. Since in FXM the plugging plasma does not contact with any cold heat sink, we have a good reason to assume that the temperature of the expanding plugging plasma is uniform. Therefore the bremsstrahlung loss q_b from the plug is simply

$$q_b = W_b x_c$$

$$= 9.9 \times 10^{-7} B^4 Z_{eff} T_c^{-\frac{3}{2}} x_c \quad . \qquad (26)$$

As in the case of TIM, effect of radiation loss is also estimated by the ratio τ_E/τ_b. For FXM we obtain the form

$$\frac{\tau_E}{\tau_b} = \frac{x_c}{x_b} , \qquad (27)$$

where

$$x_b = 8.4 \times 10^{10} \frac{T^2}{\mu^{\frac{1}{2}} \eta Z_{eff} B^2} \qquad (28)$$

The necessary scale length of the pinch for FXM is easily calculated from Eq.(25) as

$$x_c = 1.1 \times 10^{-4} \frac{T^{\frac{3}{2}}(n\tau_E)}{\mu^{\frac{1}{2}}\eta B^2} \quad . \tag{29}$$

The plugging magnetic energy density for FXM becomes

$$W_{FXM} = \frac{B^2}{8\pi}(2x_c) = 8.8 \times 10^{-6} \frac{T^{\frac{3}{2}}}{\mu^{\frac{1}{2}}\eta}(n\tau_E) \quad . \tag{30}$$

The value of W_{FXM} is also plotted in Fig.3. In Table 1 parameters necessary for the reactor and Lawson criterion experiment are given together with the ones of TIM. As can be seen, x_c is 17 m for $T_e = 5$ keV, $n\tau_E = 5 \times 10^{13}$, $\eta = 2.5$ and $\mu = 131$ (Xe). The present x_c is factor 2 smaller than the one of TIM. The value of x_b calculated under the same condition above becomes 350 m, so the radiation loss is very small in this case. The W_{FXM} is calculated to be 0.54 MJ/cm^2 which is also factor 2 smaller than the one of TIM.

§5. Concluding Remarks

We presented brief calculations for the θ-pinch end plugging by high temperature plasma of high mass number under very simplified physical model. As is demonstrated the results are very encouraging. We can save the linear θ-pinch reactor length by factor 3 to 5 compared with the system with no plugging at all. The Lawson criterion experiment will be able to be done with the scale length 68 m under 200 kG or 30 m under 300 kG. If we assume the coil and plasma radius to be 6 cm and 3 cm, respectively, and the length of the plug and DT plasma to be the same ($x_c = \ell$), total magnetic energy required will be about 92 MJ. This value may well be compared with the Lawson criterion experiment by tokamak. Since the energy confinement time is order of $\sim$ 500 μsec, the staged θ-pinch concept[6] will reduce the cost of the power source. If we claim the field rise time of the 200 kG system be 200 μsec, the voltage required is about 1 kV. The power crowbar system developed at Nagoya[7] is available. The super conducting homopolar[8] will become very important for the reactor system. The further study of this plugging system will be worthwhile because of the present encouraging results.

Acknowledgement

The authors would like to appreciate the hospitality of Dr. F. L. Ribe who gave one of them (KH) a chance to think about the end loss problem. They are also grateful to Prof. K. Takayama and Prof. T. Taniuti for their stimulating discussions and interests in this problem.

References

(1) W. R. Ellis, Nuclear Fusion 15 225 (1975).

(2) J. P. Freidberg and H. Weitzner, Nuclear Fusion 15 217 (1975).

(3) R. L. Morse, Phys of Fluids 16 545 (1973).

(4) S. I. Braginskii, Rev. of Plasma Physics Vol 1.

(5) J. V. Brackbill, M. T. Menzel, and D. C. Barns, The Third Topical Conference on Pulsed High Beta Plasmas, Culham, England, Sept. 9-12, 1975.

(6) R. F. Gribble et al, in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 5th Int. Conf. Tokyo, 1974) 3 IAEA Vienna 381 (1975).

(7) S. Kitagawa and K. Hirano, Rev. Sci. Instrum. 45 962 (1974).

(8) K. I. Thomassen LA-UR-76-509 (1976).

Figure Captions

Fig.1. Schematic drawings of two different models of the plasma ends of a θ-pinch. In TIM (thermal insulation mode) the plugging plasmas are terminated by cold end plates whereas in FXM (free expansion mode) they continuously expand along the dc magnetic channel.

Fig.2. The threshold length x_t of a Xe θ-pinch plasma where the energy loss due to electron thermal conduction is equal to that due to direct plasma flowing out.

Fig.3. The magnetic energy density necessary to sustain the plugging plasma.

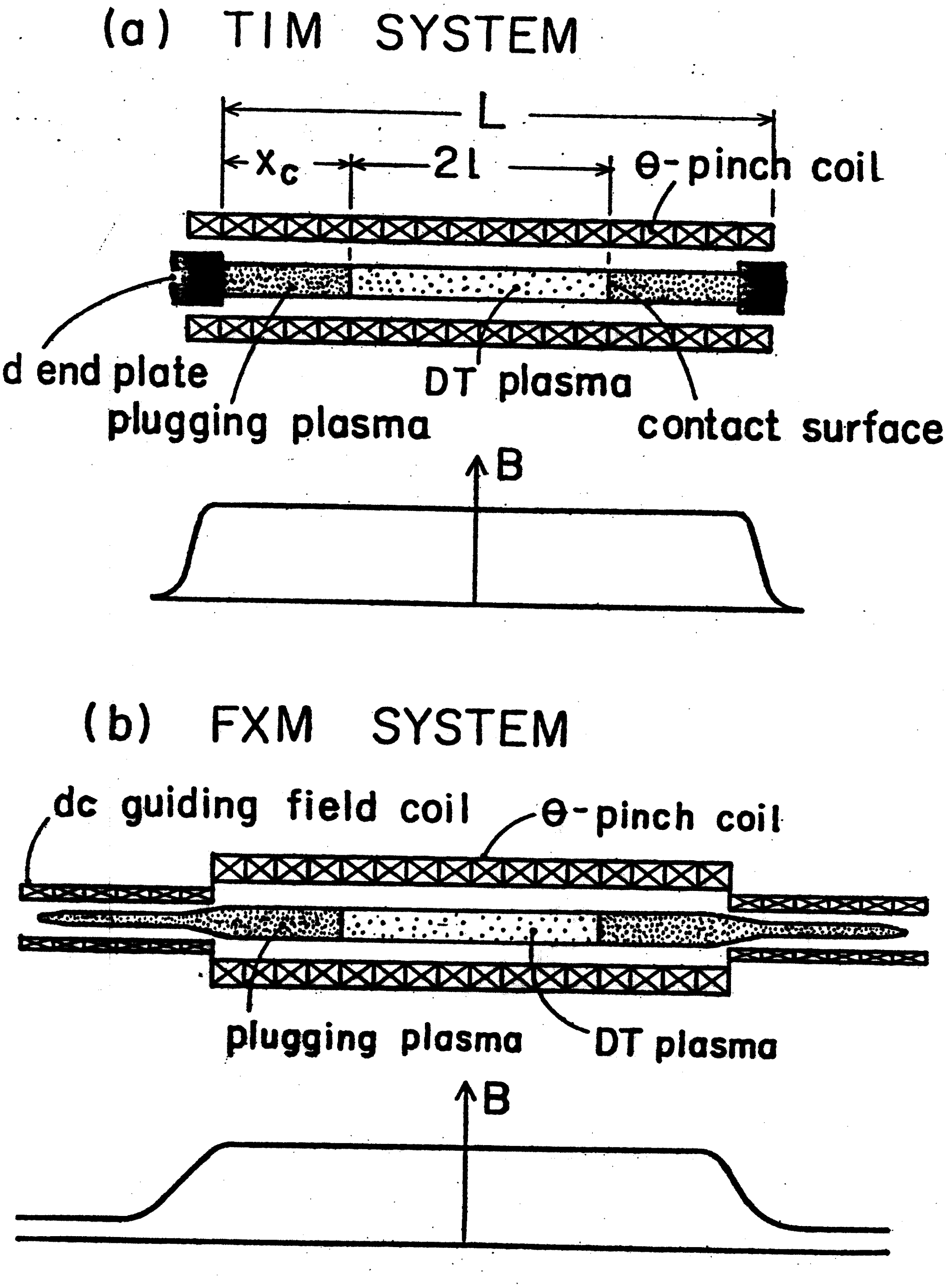
TABLE 7 SCALE LENGTHS OF A LINEAR THETA PINCH EXPERIMENT AND REACTOR

	B (kG)	n ($10^{17} cm^{-3}$)	τ_E (ms)	L = $4x_c$ (m) TIM	FXM	ELLIS × 2.84
Lawson Criterion Experiment, T = 5 keV, $n\tau_E = 5 \times 10^{13}$	100	0.25	2.01	5.2×10^2	2.7×10^2	1.0×10^3
	200	1.0	0.50	1.3×10^2	6.7×10	2.5×10^2
	300	2.2	0.22	5.8×10	3.0×10	1.1×10^2
	400	4.0	0.12	3.3×10	1.7×10	6.3×10
	500	6.2	0.08	2.1×10	1.1×10	4.0×10
θ-Pinch Reactor, T = 10 keV, $n\tau_E = 10^{15}$	100	0.12	81	1.1×10^4	1.5×10^4	5.6×10^4
	200	0.50	20	2.8×10^3	3.8×10^3	1.4×10^4
	300	1.1	8.9	1.2×10^3	1.7×10^3	6.3×10^3
	400	2.0	5.0	7.0×10^2	9.6×10^2	3.4×10^3
	500	3.1	3.2	4.5×10^2	6.1×10^2	2.2×10^3

N.B.:

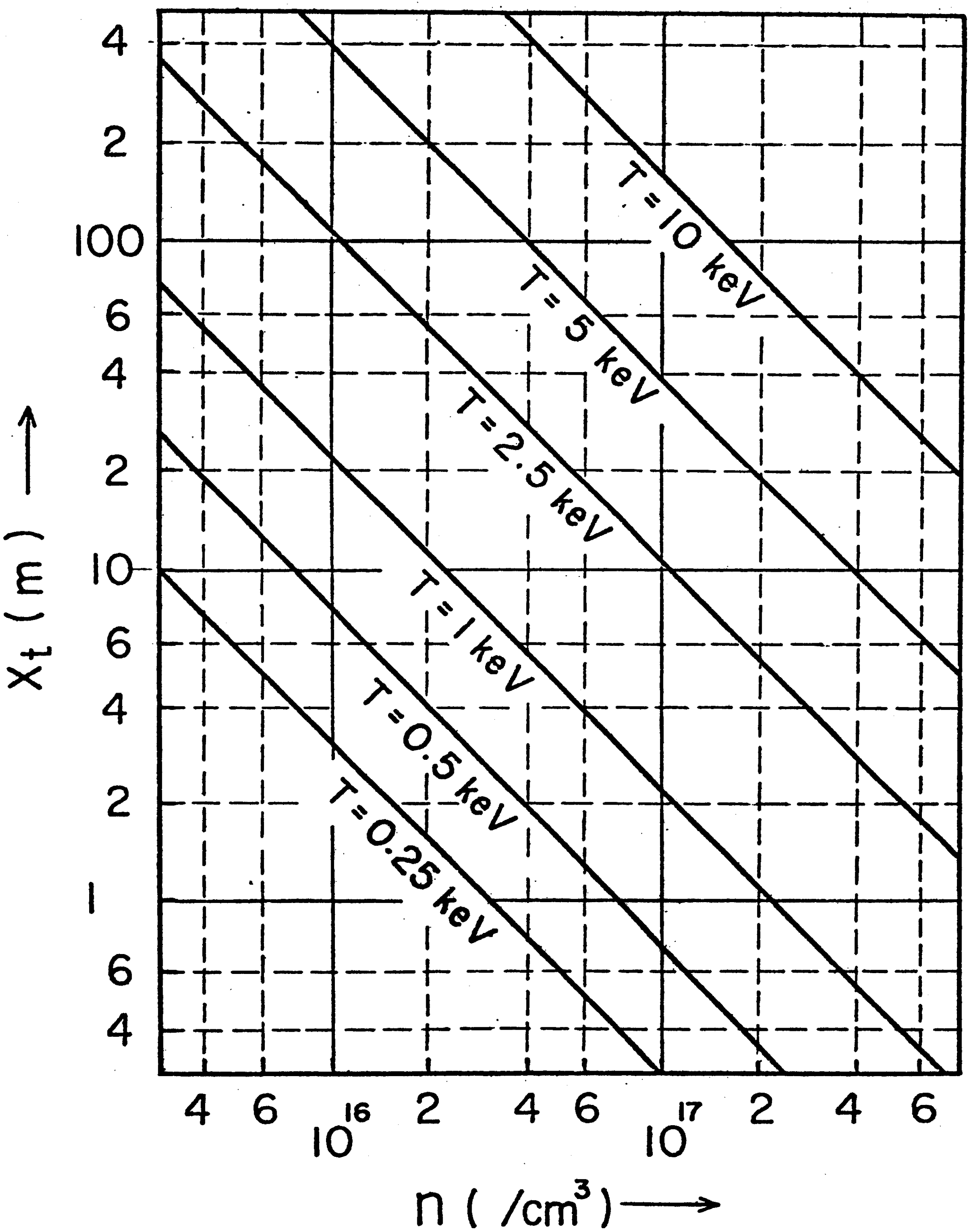
(1) For plugging plasma Xe is assumed to be used ($\mu = 131$, $Z_{eff} = 50$)

(2) The scale lengths of Ellis system are multiplied by 2.84 in order to make a fare comparison with the present proposal.



(a) TIM SYSTEM
L
x_c
2l
θ-pinch coil
cold end plate
plugging plasma
DT plasma
contact surface
B
(b) FXM SYSTEM
dc guiding field coil
θ-pinch coil
plugging plasma
DT plasma
B

Fig. 1



4
2
100
6
4
2
10
6
4
2
1
6
4
X_t (m)
4
6
10^{16}
2
4
6
10^{17}
2
4
6
n (/cm^3)
T = 10 keV
T = 5 keV
T = 2.5 keV
T = 1 keV
T = 0.5 keV
T = 0.25 keV

Fig. 2

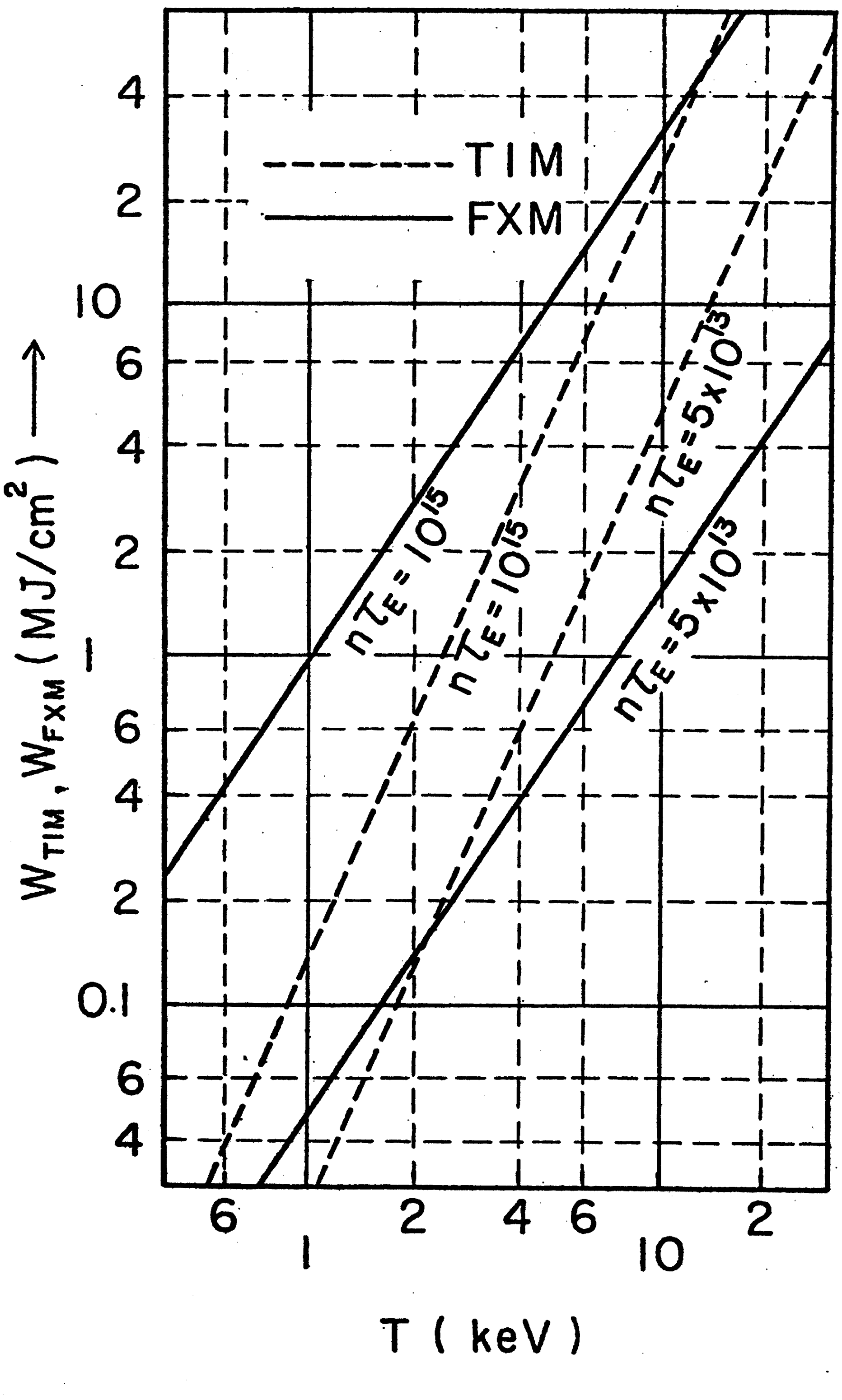

TIM
FXM
W_{TIM}, W_{FXM} (MJ/cm^2)
$n\tau_E = 10^{15}$
$n\tau_E = 10^{15}$
$n\tau_E = 5\times10^{13}$
$n\tau_E = 5\times10^{13}$
4
2
10
6
4
2
1
6
4
2
0.1
6
4
6
1
2
4
6
10
2
T (keV)

Fig. 3