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Trapping and Detrapping in a Single Wave and Particle Acceleration in a Magnetoplasma

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ABSTRACT

Particle motion within one bounce period $2\pi/\omega_B$ is considered when the cyclotron frequency ω_C is comparable with ω_B. The equation of motion with an approximation is solved analytically for certain initial values. The analytical solution thus obtained agrees quite well with numerically computed results over a wide range of ω_C and ω_B. It is shown that under some conditions the particle is accelerated quite efficiently as compared with that in an unmagnetized plasma.

In an unmagnetized plasma a charged particle trapped in an electrostatic wave makes an adiabatic motion provided the amplitude of the wave is unchanged. In a magnetoplasma the behavior of the particle strongly depends on the angle θ between the wave vector $\vec{k}$ and the magnetic field $\vec{B}$. For $\theta = 0$ the particle bounces along $\vec{B}$ in the potential well while for $\theta = \pi/2$ no particle is trapped even for one bounce period[1]. For θ between the two limits the motion is quite complicated as expected.

In this letter we examine particle motion in a coherent wave during a bounce period essentially when the cyclotron frequency ω_c ~ the bounce frequency ω_B[2]. A Cherenkov type resonance $\vec{k}\cdot\vec{v} - \omega = 0$ is considered. We show that under some conditions the particle is accelerated quite efficiently. It will be also shown how a trapping and a detrapping of a particle depend on θ, ω_c and ω_B.

We choose $B = B(0, 0, 1)$ and $k = k(0, \sin\theta, \cos\theta)$, and the time and lengths are scaled by ω^{-1} and k^{-1}, respectively. The motion of the particle is governed by

$$\dot{\vec{v}} = \omega_c \vec{v} \times \vec{B}/B - (\vec{k}/k)\omega_B^2 \sin(y\sin\theta + z\cos\theta - t) , \qquad (1)$$

where $\omega_c = eB/(mc\omega)$, and $\omega_B^2 = keE/(m\omega^2)$.

Expansion of the force term in the governing equation by means of Bessel function identity constitutes an elegant method for investigation of the particle motion in a magnetoplasma[3]. This method is effective when ω_c is larger than ω_B. On the contrary, when $\omega_c \sim \omega_B$, the expanded equation may no more be profitable since the path of resonant particle significantly deviates from the circular orbit. Here we will try to integrate the equation of motion directly.

The equation (1) gives a momentum conservation along x, i.e., $v_x - \omega_c y = C_1$. Using this and putting $y + C_1/\omega = \bar{y}$, and $z - (C_1/\omega_c)\tan\theta = \bar{z}$, we rewrite Eq.(1) in the form

$$v_x = \omega_c \bar{y}, \tag{2}$$

$$\dot{v}_y = -\omega_c^2 \bar{y} - \sin\theta\, \omega_B^2 \sin\psi, \tag{3}$$

$$\dot{v}_z = -\cos\theta\, \omega_B^2 \sin\psi, \tag{4}$$

$$\psi = \bar{y}\sin\theta + \bar{z}\cos\theta - t . \tag{5}$$

For simplicity hereafter we get rid of ' - ' over y and z. The set of Eqs.(2) - (5) yields the following conservation relation

$$v^2/2 - v_z/\cos\theta - \omega_B^2 \cos\psi = C_2, \tag{6}$$

or

$$\{v_x^2 + v_y^2 + (v_z - 1/\cos\theta)^2\}/2 - \omega_B^2 \cos\psi = C_2'. \tag{7}$$

The equation (7) is the energy conservation relation in the wave frame. The kinetic energy of a particle in the laboratory frame is obtained from (6) as $W = v^2/2 = v_o^2/2 + W_1 + W_2$,

$$W_1 = (v_z - v_{zo})/\cos\theta \ , \tag{8}$$

$$W_2 = \omega_B^2(\cos\psi - \cos\psi_o) \ . \tag{9}$$

As seen in Eq.(7), W_2 is the potential energy in the wave frame as well as in the laboratory frame. The quantity W_1 characterizes the energy gain of the particle in the electrostatic wave in the magnetized plasma.

Since it is very difficult to solve Eqs.(3) and (4) analytically we make an approximation of replacing $\sin\psi$ by $4\psi/\pi^2$ which gives the same potential depth as the real one at $\psi = 0$ and $\psi = \pm\pi$. It will be seen later on that the resultant set of equations gives a good approximation to the real motion of trapped particles.

Replacing $\sin\psi$ by $4\psi/\pi^2$ and introducing ψ and $\eta \equiv z\cos\theta - t$ in place of y and z we have instead of (3) and (4)

$$\ddot{\psi}_a + (\omega_b^2 + \omega_c^2)\psi_a = \omega_c^2\eta_a \ , \tag{10}$$

$$\ddot{\eta}_a = -\cos^2\theta\ \omega_b^2\psi_a \ , \tag{11}$$

where the subscript "a" stands for the approximation and $\omega_b^2 = 4\omega_B^2/\pi^2$. Note that Eqs.(10) and (11) constitute the equation of motion of a coupled oscillator. Suppose a particle be in the bottom of the potential in the beginning and $y_o = 0$. This statement is equivalent to

$$\psi_{ao} = 0, \qquad \eta_{ao} = 0. \tag{12}$$

As for $\vec{v}_o$ we choose such direction which is parallel to $\vec{k}$ and $|\vec{v}_o| = 1$ since we are thinking of the Cherenkov type resonance $\vec{k}\cdot\vec{v}/k = 1$. This initial value is equivalent to

$$\dot{\psi}_o = 0, \qquad \dot{\eta}_o = -\sin^2\theta. \tag{13}$$

Under the initial condition (12) and (13), the set of Eqs.(10) and (11) yields

$$\psi_a = -\tan^2\theta(\omega_1^2/\omega_b^2)\gamma(\sin\omega_1 t/\omega_1 - \sin\omega_2 t/\omega_2), \tag{14}$$

$$W_{1a} = \tan^2\theta[1 - \gamma(\cos\omega_1 t - (\omega_1/\omega_2)^2\cos\omega_2 t)], \tag{15}$$

$$2\omega_{1,2}^{\ \ 2} = \Gamma^2 \mp [\Gamma^4 - 4\omega_b^2\omega_c^2\cos^2\theta]^{1/2}, \tag{16}$$

where $\gamma = (1 - \omega_1^2/\omega_2^2)^{-1}$ and $\Gamma^2 = \omega_b^2 + \omega_c^2$.

The quantity ω_1^2/ω_2^2 is always less than unity. Especially, when $\pi/4 < \theta < \pi/2$, the relation $\omega_1^2/\omega_2^2 \ll 1$ holds; for a while we consider this circumstance. Notice that the set of

Eqs.(14), (15) and (16) is meaningful as long as $|\psi| \leq \pi$, that is, for particles in the potential well. The expression (15) implies that if the particle is kept trapped until $\omega_1 t > \pi$, Max $W_{1a} = 2\tan^2\theta$, which becomes infinite as θ tends to $\pi/2$. Since, however, ψ_a is also proportional to $\tan^2\theta$ the detrapping will easily occur. If the relation $|\psi| \leq \pi$ holds for $t > t_1 \equiv \pi/\omega_1$ we call the particle a well-trapped one. The figure 1 shows critical curves for various θ, above which particles are well-trapped. The solid lines are obtained by solving the set of Eqs.(3) and (4) numerically and the dotted lines are given by (14) and (16). We see that the agreement between them is quite well, which implies that the replacement of $\sin\psi \Rightarrow 4\psi/\pi^2$ is excellent.

Let us examine orbits and W_1 for some parameters above and below a critical line. Orbits for $\omega_c = 0.01$ and $\omega_c = 5.0$ when $\theta = 0.35\pi$ are given in Fig.2-a and Fig.2-b, respectively. Orbits of a-1 and of b-1 have parameters above the critical line and parameters of other orbits lie below the critical line. The particle of a-2 gets a large acceleration not only along v_z axis but also in $v_x - v_y$ space, which results in a nonadiabatic motion shown in Fig.2-a. Moreover, for $\omega_c < 1$, we have always this type of nonadiabaticity, which is consistent with the condition of appearance of stochasticity of motion[3). On the contrary, the particle of b-2 moves almost one-dimensionally along the z-axis, the motion seems to be adiabatic and is very similar to the one just outside of a separatrix which separates the trapped region and the untrapped one in the case of an unmagnetized plasma.

The figure 3 shows the maximum values of $W_1/(\frac{1}{2}v_o^2) = 2W_1$. For a well-trapped particle, the maximum value of v_z gives Max $2W_1$. For a detrapped one, v_z at the point of detrapping (D in Fig.2), where $|\psi|$ becomes π, gives Max $2W_1$. In this figure, ω_c is fixed to 0.05 and 5.0. The solid lines are obtained by solving Eqs.(3) and (4) numerically and the dotted lines are given by Max $2W_1 = 2\tan^2\theta \times [1 - \gamma[1 - \pi^2\omega_b^4(\gamma\omega_1 \tan^2\theta)^{-2}]^{1/2}]$. This expression is obtained by looking for $\omega_1 t$ for which $|\psi_a| = \pi$ holds in Eq.(14) and substituting it into Eq.(15). Note that the solid lines and the dotted lines are very close to each other. An ω_B at a kink of a curve specified by certain values of θ and ω_c in Fig.3 is identical with the value of ω_B on the critical line of the corresponding θ at the corresponding ω_c in Fig.1. It is important to note that Max $2W_1$ or the energy gain is remarkably large. For example, for $\theta = 0.35\pi$ the maximum of Max $2W_1$ exceeds 15 times the initial value.

The Max $2W_1$ is compared with Max $2W_2$. It is clear from Eq.(9) that Max $2W_2 < 4\omega_B^2$. When $\omega_c^2 < 1$, the lower limit of Max $2W_{1a}$ is nearly $16\omega_B^4/(\pi^2\omega_c^2\sin^2\theta)$ and taking into account $\omega_B^2 \gtrsim \omega_c^2$ we see Max $2W_1$ is dominant. When $\omega_c^2 > 1$, the lower limit of Max $2W_{1a}$ is $4\omega_B^2(1 + \omega_B^2/\omega_c^2)/\sin^2\theta$ and we can regard

Max $2W_1$ dominant in this case, too. It is worth while to note that in the absence of B we have Max $2W_1 = 2\omega_B$ which is smaller than Max $2W_1$ for $B \neq 0$.

Let us consider an acceleration of a low energy particle in the scheme of our theory. Suppose the initial velocity is zero, i.e., $v_o = 0$, and $\omega_B \sim \omega_c \sim 1$. Also suppose $|\cos\theta| \ll 1$ so that $\omega_1^2 \ll \omega_2^2$. Using the inital values (12) and, $\dot{\psi}_o = \dot{\eta}_o = -1$ instead of (13), we solve (10) and (11) and obtain $2W_{1a} = 2\cos^{-2}\theta(1 - \cos\omega_1 t)$, and $\psi_a = -\cos^{-2}\theta(\sin\omega_1 t/\omega_1) \times (\omega_1/\omega_b)^2$. Since $2W_{1a}$ and ψ_a thus obtained are almost equivalent to Eqs.(14) and (15), then the critical ω_B in both cases must be quite close, and the criterion in Fig.1 is approximately applicable in the present case. Referring to Fig.1 we see that the critical ω_B is of the order of 1 for $\omega_c \sim 1$, which is consistent with the initial assumption $\omega_c \sim \omega_B \sim 1$. Note that $2W_{1a}$ is scaled by $v_{ph}^2/2$. Hence,if $v_{ph}^2/2 \sim v_{tr}^2/2 \sim 100$ eV, a particle of a few 10 eV can be accelerated up to a few keV in only one bounce period, as seen in Fig.3, where v_{tr} is the trapping velocity. For example, in the MACH II device[4) in a turbulent heating experiment a low frequency oscillation with an amplitude of $(1 - 2) \times 100$ V was observed and protons of a few keV were detected simultaneously. The characteristic parameters are $\omega_B \sim \omega_c \sim (1 - 10) \times 10^7$ rad/sec, $T_e \sim$ a few 100 eV and $T_i \sim (1 - 2) \times 10$ eV.

It is important to note that the criterion in Fig.1 is applicable for such v_o as $|v_{ph} - v_o| < v_{tr}$; the example stated above is an extreme case in the sense that $v_o = 0$.

We have found a condition (Fig.1) under which the energy gain is appreciable even in a bounce period. This process is expected to bring about high heating efficiency of plasma when a large amplitude wave exists. This will also cause an anomalous initial damping of waves in the magnetoplasma. The corresponding experiments will fall in the areas of laser pellet fusion, laser plasma heating, shock heating and turbulent heating, in which the correlation of waves is not very good while the amplitude of waves is quite large. Actually, Cairns[5)] succeeded to reproduce qualitatively the result of shock heating[6)], using a model closely related to the present theory.

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Figure Captions

Fig.1 Critical lines for various θ. Above a line the corresponding particle motion becomes well-trapped Solid lines are obtained by a numerical calculation and dotted ones are done by the approximate solutions (14) and (16).

Fig.2 Orbits in v_z - z plane. $\theta = 0.35\pi$. Particles start at the point S. The a-1 ($\omega_B = 0.14$) and b-1 ($\omega_B = 1.0$) are well-trapped ones. The a-2 ($\omega_B = 0.12$) and b-2 ($\omega_B = 0.8$) detrap at D. The point A represents the position of the detrapped particle (a-2 in Fig.2-a, b-2 in Fig.2-b) at the end of one bounce period of the well-trapped particle (a-1 in Fig.2-a, b-1 in Fig.2-b).

Fig.3 Max $2W_1$ vs ω_B. The ω_c is fixed. Underlined quantities belong to $\omega_c = 5.0$ and others do to $\omega_c = 0.05$. Solid lines are given by a numerical calculation and dotted lines are done by the approximate solutions (14) - (16).

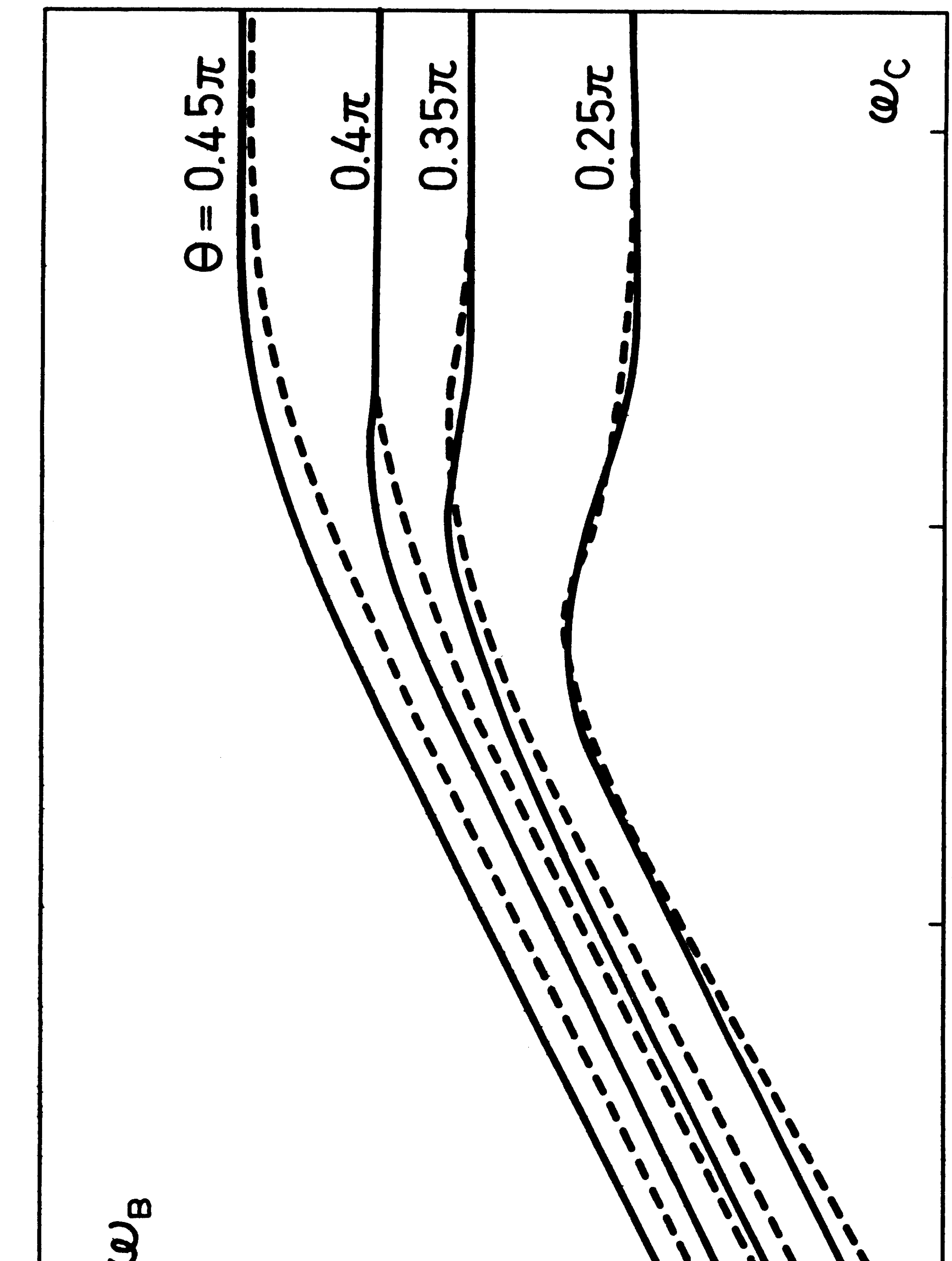


Fig. 1

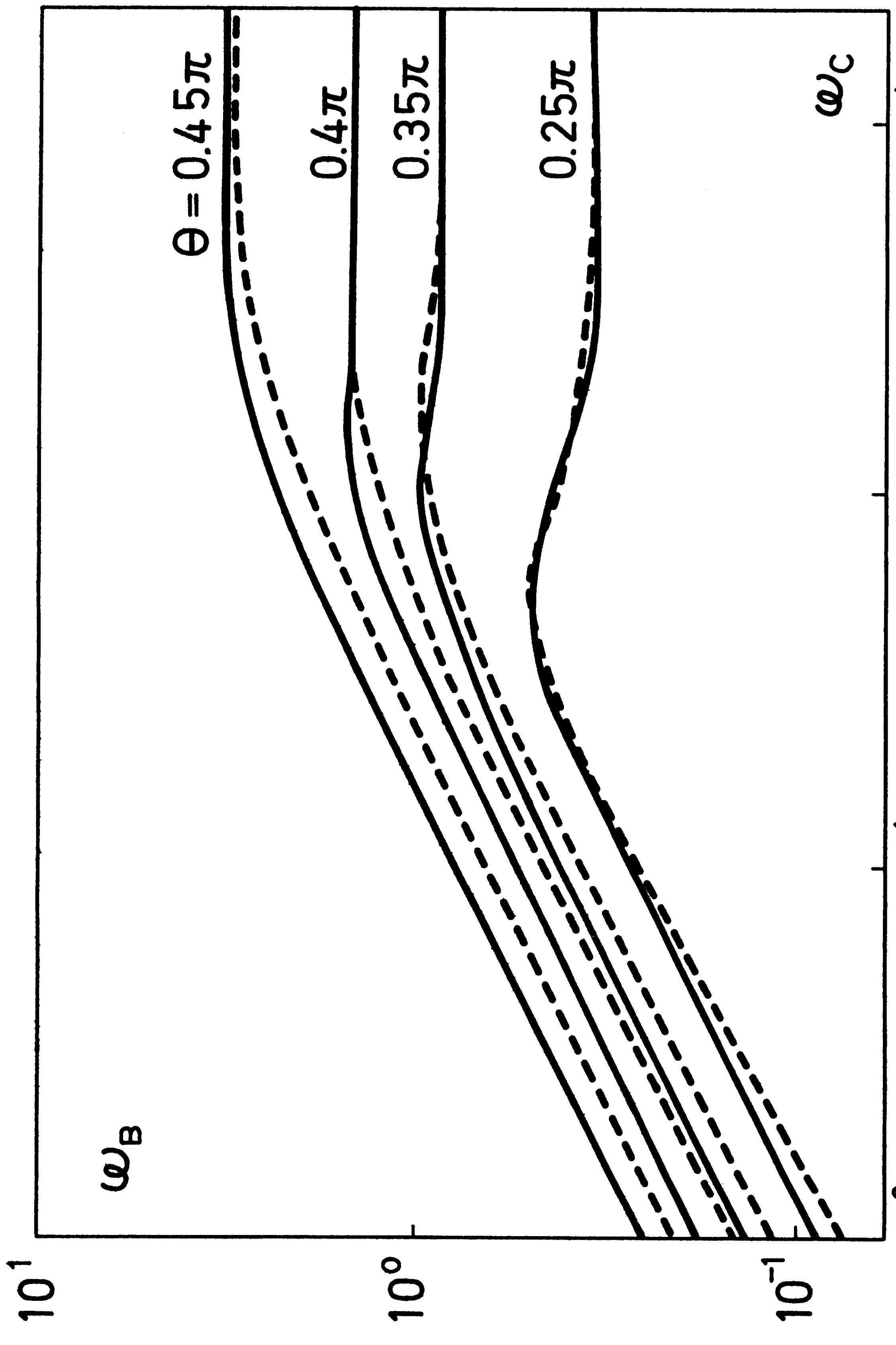
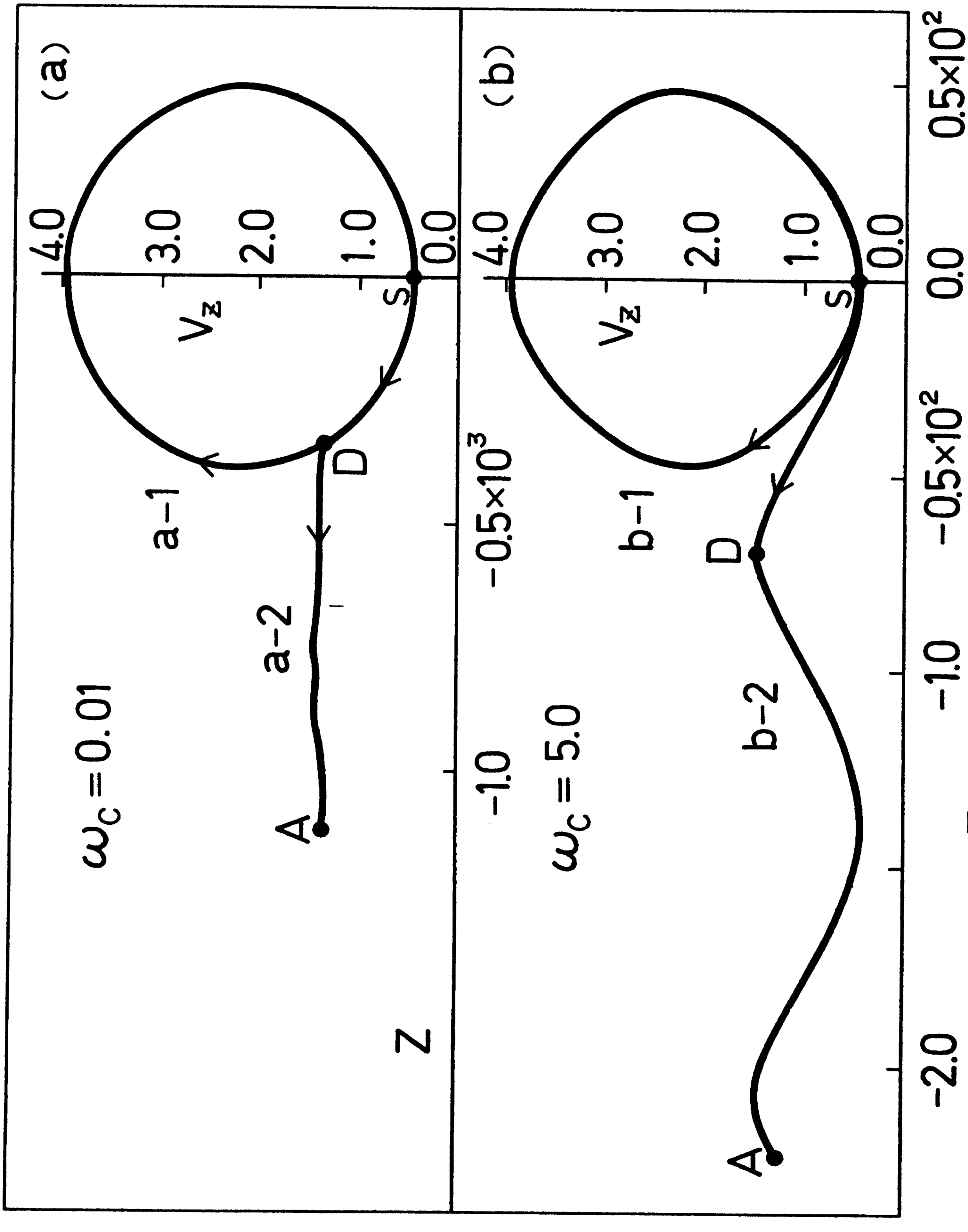


Fig. 1



(a)
(b)
$\omega_c = 0.01$
$\omega_c = 5.0$
a−1
a−2
b−1
b−2
A
D
S
V_z
Z
4.0
3.0
2.0
1.0
0.0
−0.5×10³
−1.0
0.5×10²
−0.5×10²
−2.0

Fig. 2

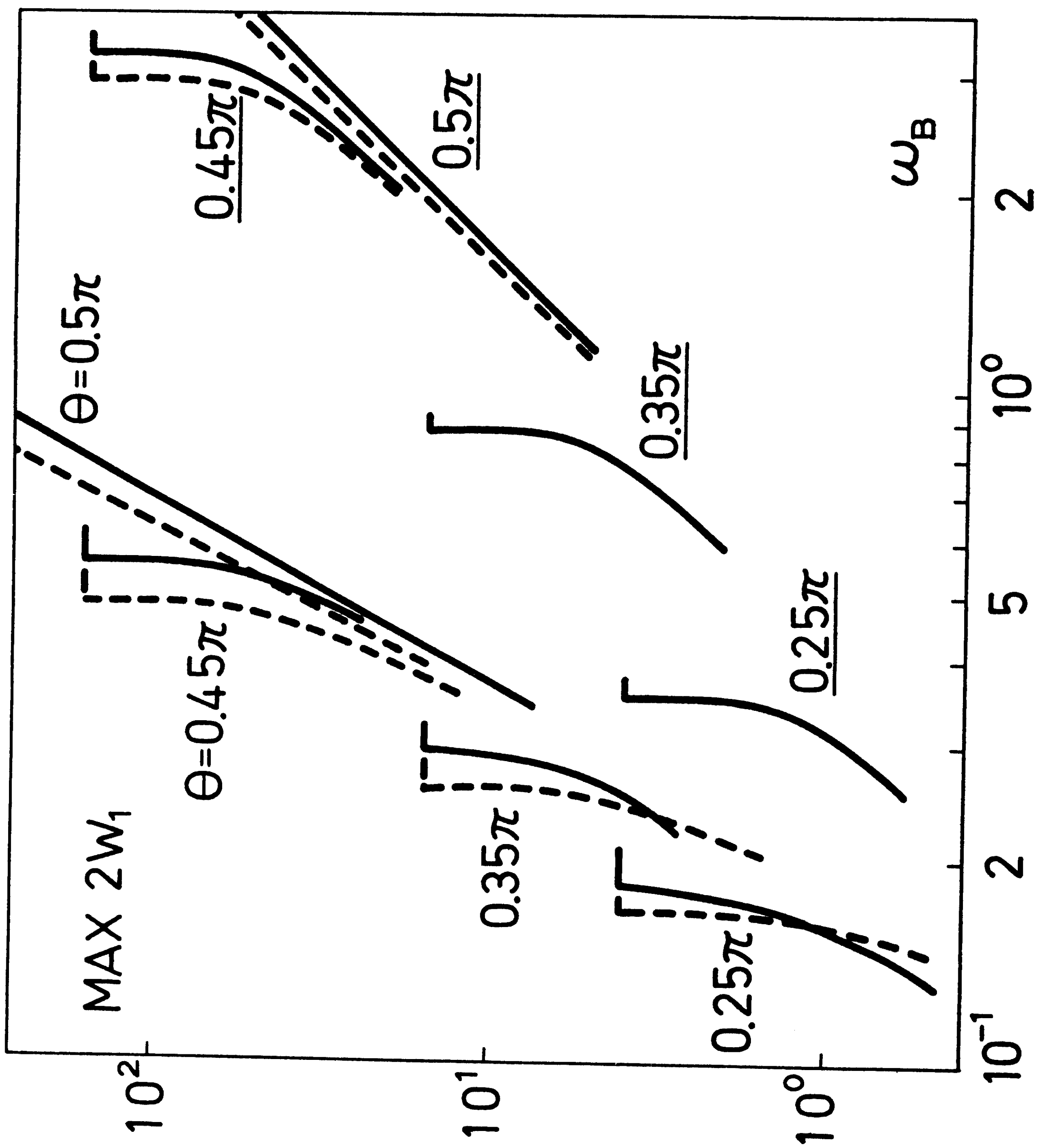

MAX 2W₁
θ=0.5π
θ=0.45π
0.45π
0.5π
0.35π
0.35π
0.25π
0.25π
ω_B
10⁻¹
2
5
10⁰
2
10²
10¹
10⁰

Fig. 3