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The Nonlinear Distortion of Propagation Cones of Lower Hybrid Wave in an Inhomogeneous Plasma

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Abstract

Nonlinear propagation of externally driven waves in the lower hybrid frequency range in an inhomogeneous plasma are investigated. The results of finite temperature, inhomogeneity of the plasma and density depression due to the ponderomotive force are emphasized since these effects are responsible for the propagation characteristics of the waves. The results shows taht the waves are localized in a spatial wave packet that propagates into the plasma center along the conical trajectory which makes a small angle with respect to the confining magnetic field.

§1. Introduction

Problems associated with propagation of lower hybrid wave from the outer edge of the plasma to the lower hybrid resonance layer are contemporary of interest in connection with the studies of effective heating methods in magnetically confined plasmas.

Cold plasma analysis of the propagation of waves near the lower hybrid frequency in an inhomogeneous plasma indicates that this frequency range may allow for very effective transport of rf power from an antenna, located just outside of a plasma column, to the lower hybrid layer[1).

Recently, the experimental measurements[2∿5) and numerical analysis[7∿11) showed that the waves are localized in a spatial wave packet propagating into the plasma along a conical trajectory which makes a small angle with respect to the confining magnetic field.[6)

Linear mode conversion in the presence of plasma inhomogeneity has an effect on the propagation[10∿13) and the density depression due to the ponderomotive force has a profound effect on the wave propagation[14∿16). The futher evolution of the waves near the resonance layer is also influenced by finite temperature effects.[11,13,17∿19)

In the previous paper[15) we investigated the wave propagation in a homogeneous plasma under the influence of two effects, namely the density depression due to ponderomotive force and the finite temperature effects. The result insures that the wave energy is carried into plasma center through the propagation cones, which takes a form of the stable soliton structures.

Some general characteristics of the conversion and absorption of the high frequency pump wave within the nonuniform plasma may be determined by the compitition of some effects, such as the plasma inhomogeneity, density depression and finite temperature effects etc. Such an analysis is the subject of this paper.

The plan of this paper is as follows:

In Section 2, a nonlinear wave equation will be derived within the theory of fluid equations including the finite temperatures. In Section 3 we derive the approximate equation describing the lower hybrid wave propagation in the WKB sense. Some numelical results are illustrated on the basis of the above equation in Sec.4. The last section will be devoted to the concluding discussions.

§2. Basic Equations

The basic systems of equations relevants to the present problems is written as follows[15]:

$$\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \vec{\nabla})\vec{v}_e = -\frac{e}{m}\vec{E} - \frac{e}{mc}\vec{v}_e \times \vec{B} - \frac{T_e}{m}\frac{1}{n_e}\vec{\nabla} n_e \ , \qquad (1)$$

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}\cdot(n_e \vec{v}_e) = 0 \ , \qquad (2)$$

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \vec{\nabla})\vec{v}_i = \frac{e}{M}\vec{E} + \frac{e}{Mc}\vec{v}_i \times \vec{B} - \frac{T_i}{M}\frac{1}{n_i}\vec{\nabla} n_i \ , \qquad (3)$$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}\cdot(n_i \vec{v}_i) = 0 \ , \qquad (4)$$

$$\vec{\nabla} \cdot \frac{\partial \vec{E}}{\partial t} = 4\pi e \, \vec{\nabla} \cdot (n_e \vec{v}_e - n_i \vec{v}_i) \tag{5}$$

In these equations, v_j is the j-th species fluid velocity, n_j is the density, $\vec{E}$ is the wave electric fluid, $\vec{B}$ is the magnetic field and $T_e(T_i)$ denotes the electron (ion) temperatures respectively.

The geometry of the problem is taken as two dimensional with the z-axis along the direction of the magnetic field B_0 and x-axis perpendicular to it. Also the plasma density varies in the x-direction. Thus the inhomogeneity is introduced through the density $n_0(x)$. We assume the spatial dependence of the steady state potential oscillations $\phi = \phi(x,z)\exp(i\omega t)$ + c.c. with $\Omega_i << \omega << \Omega_e$. Here $\Omega_e(\Omega_i)$ means the electrons (ions) cyclotron frequency.

From equations (1)∿(5) with the above assumptions, one can obtain the following nonlinear equations for ϕ as

$$\begin{aligned}
&\vec{\nabla}^2\phi + i\,\frac{4\pi e}{\omega}\,\vec{\nabla}\cdot[N(x)\,(\overleftrightarrow{\nabla}_e - \overleftrightarrow{\nabla}_i)\cdot\vec{\nabla}\phi] \\
&- \frac{4\pi T_e e}{\omega^2}\vec{\nabla}\cdot\{N(x)\overleftrightarrow{\nabla}_e\cdot\vec{\nabla}\,[\frac{1}{N(x)}\vec{\nabla}\cdot(N(x)\overleftrightarrow{\nabla}_e\cdot\vec{\nabla}\phi)]\} \\
&- \frac{4\pi T_i i}{\omega^2}\vec{\nabla}\cdot\{N(x)\overleftrightarrow{\nabla}_i\cdot\vec{\nabla}[\frac{1}{N(x)}\,\vec{\nabla}\cdot(N(x)\overleftrightarrow{\nabla}_i\cdot\vec{\nabla}\phi)]\} \\
&+ i\,\frac{4\pi e}{\omega}\,\vec{\nabla}\cdot[\delta n(\overleftrightarrow{\nabla}_e - \overleftrightarrow{\nabla}_i)\cdot\nabla\phi] = 0
\end{aligned} \tag{6}$$

where $\vec{E}=-\vec{\nabla}\phi$, $\overleftrightarrow{\nabla}_i$ is the usual mobility tensor and δn ($\delta n_e \hat{=} \delta n_i = \delta n$) is the low frequency derivation of the plasma density from its average value $n_0(x)$, which occurs through the action of the ponderomotive force of the high frequency oscillation. Here δn is given as

$$<(\vec{v}_j\cdot\vec{\nabla})v_j> \ = -\frac{e_j}{m_j}\vec{\nabla}\phi_s - \frac{T_j}{m_j}\vec{\nabla}\frac{\delta n_j}{N} \tag{7}$$

which is obtained by averaging over the characteristic time $1/\omega$ (ω is the high pump frequency). Here ϕ_s is the self-consisten. ambipolar potential which may be eliminated by the quasi-neutrality condition.

The z-component of the ponderomotive force gives rise to density cavities along the magnetic field. This cavities modify the linear characteristics of the lower-hybrid oscillations. Thus we consider only the z-component of the ponderomotive force. This procedure gives the density derivation in the form

$$\delta n \simeq -\frac{1}{8\pi T}\{-(\frac{\omega_{pe}^2(x)}{\Omega_e^2} - \frac{\omega_{pi}^2(x)}{\omega^2})(\frac{\partial\phi}{\partial x})^2 + \frac{\omega_{pe}^2(x)+\omega_{pi}^2(x)}{\omega^2}(\frac{\partial\phi}{\partial z})^2\} \tag{8}$$

The equation (6) with eq.(7) is quite general and may be useful for future investigations of another electrostatic modes.

Substitution of eq.(8) into eq.(6) yields the following equation in the nondimensional form

$$\frac{\partial}{\partial\bar{x}}(K_\perp(\bar{x})\frac{\partial}{\partial\bar{x}}\bar{\phi}) + K_\parallel\frac{\partial^2}{\partial\bar{z}^2}\bar{\phi} + \frac{\partial}{\partial\bar{x}}(a_1(\bar{x})\frac{\partial^3}{\partial\bar{x}^3}\bar{\phi})$$
$$+ \frac{\partial}{\partial\bar{x}}(a_2(\bar{x})\frac{\partial^2}{\partial\bar{x}^2}\bar{\phi}) + \frac{\partial}{\partial\bar{x}}(a_3(\bar{x})\frac{\partial}{\partial\bar{x}}\bar{\phi}) - \frac{\partial}{\partial\bar{x}}(b(x)\frac{\partial^3}{\partial\bar{x}\partial\bar{z}^2}\bar{\phi})$$
$$+ c(\bar{x})\frac{\partial^4}{\partial\bar{z}^4}\bar{\phi} - \frac{\partial}{\partial\bar{x}}\{\alpha_1(\bar{x})(\frac{\partial\bar{\phi}}{\partial\bar{x}})^3\} - \frac{\partial}{\partial\bar{x}}\{\alpha_2(\bar{x})(\frac{\partial\bar{\phi}}{\partial\bar{x}})(\frac{\partial\bar{\phi}}{\partial\bar{z}})^2\}$$
$$+ \beta_1(\bar{x})\frac{\partial}{\partial\bar{z}}\{(\frac{\partial\bar{\phi}}{\partial\bar{x}})^2(\frac{\partial\bar{\phi}}{\partial\bar{z}})\} + \beta_2(\bar{x})\frac{\partial}{\partial\bar{z}}(\frac{\partial\bar{\phi}}{\partial\bar{z}})^3 = 0 \tag{9}$$

with

$$K_\perp(\bar{x}) = 1 + \left(\frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2}\right) f(\bar{x}) \quad , \tag{10}$$

$$K_\parallel(\bar{x}) = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2} f(\bar{x}) \tag{11}$$

$$a_1(\bar{x}) = \left(\frac{\lambda_{De}^2/L^2 \omega_{pe}^4}{\Omega_e^4} + \frac{\lambda_{Di}^2/L^2 \omega_{pi}^4}{\omega^4}\right) f(\bar{x}) \quad , \tag{12}$$

$$a_2(\bar{x}) = \left(\frac{\lambda_{De}^2/L^2 \omega_{pe}^4}{\Omega_e^4} + \frac{\lambda_{Di}^2/L^2 \omega_{pi}^4}{\omega^4}\right) \frac{1}{f(\bar{x})} \frac{\partial f(\bar{x})}{\partial \bar{x}} \quad , \tag{13}$$

$$a_3(\bar{x}) = \left(\frac{\lambda_{De}^2/L^2 \omega_{pe}^4}{\Omega_e^4} + \frac{\lambda_{Di}^2/L^2 \omega_{pi}^4}{\omega^4}\right) \frac{\partial}{\partial \bar{x}}\left(\frac{1}{f(\bar{x})} \frac{\partial f(\bar{x})}{\partial \bar{x}}\right) \tag{14}$$

$$b(\bar{x}) = 2\left(\frac{\lambda_{De}^2/L^2 \omega_{pe}^4}{\Omega_e^2 \omega^2} - \frac{\lambda_{Di}^2/L^2 \omega_{pi}^4}{\omega^4}\right) f(\bar{x}) \quad , \tag{15}$$

$$c(\bar{x}) = \frac{\lambda_{De}^2/L^2 \omega_{pe}^4 + \lambda_{Di}^2/L^2 \omega_{pi}^4}{\omega^4} f(\bar{x}) \quad , \tag{16}$$

$$\alpha_1(\bar{x}) = -\frac{1}{2} \frac{\lambda_{De}^2}{L^2}\left(\frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2}\right)^2 f(\bar{x}) \quad , \tag{17}$$

$$\alpha_2(\bar{x}) = \frac{1}{2} \frac{\lambda_{De}^2}{L^2}\left(\frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2}\right) \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2} f(\bar{x}) \, , \tag{18}$$

$$\beta_1(\bar{x}) = -\frac{1}{2} \frac{\lambda_{De}^2}{L^2}\left(\frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2}\right) \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2} f(\bar{x}) \, , \tag{19}$$

$$\beta_2(\bar{x}) = \frac{1}{2} \frac{\lambda_{De}^2}{L^2}\left(\frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}\right)^2 f(\bar{x}) \tag{20}$$

where $\lambda_{De}(\lambda_{Di})$ denotes the Debye length for electrons (ions) and L is the density seale length.

Several parameters used in this presentation are defined as follows

$$\bar{x} = \frac{x}{L} \quad , \quad \bar{z} = \frac{z}{L} \quad , \quad \tilde{\phi} = \frac{e\phi}{T} \quad , \tag{21}$$

$$n_0(\bar{x}) = Nf(\bar{x}) \tag{22}$$

Equation (9) has the generalized form of the equation for homogeneous plasma which has been derived in the previous paper[15].

§2. Derivation of Approximation Equation

In this section we derive a nonlinear equation which describes the lower hybrid wave propagating along the propagation cone in plasma with density gradient. We assume that the unperturbed density $n_0(x)$ is a slowly varying function of $\bar{x}$. To do so let us first introduce the coordinate-streching defined by

$$\left.\begin{aligned} \xi &= \varepsilon^{1/2}\left\{\int^{\bar{x}} \frac{d\bar{x}'}{\lambda(\bar{x}')} - \bar{z}\right\} \\ \eta &= \varepsilon^{3/2}\,\bar{z} \end{aligned}\right\} \tag{23}$$

where $\lambda(\bar{x})$ designates the propagation angle in x-z plane, which will be determined later. Also we expand $\tilde{\phi}(\xi,\eta)$ as follows

$$\tilde{\phi} = \tilde{\phi}^{(0)} + \varepsilon\tilde{\phi}^{(1)} + \cdots \tag{24}$$

Under WKB approximation, namely

$$\left|\frac{\partial\tilde{\phi}}{\Phi\xi}\right| \gg \left|\frac{\partial K}{\partial\xi}\right| \; , \; \left|\frac{\partial\lambda}{\partial\xi}\right| \tag{25}$$

we have for the lowest order of ε

$$\left(\frac{K_\perp(\xi)}{\lambda^2(\xi)} + K_\parallel(\xi)\right)\frac{\partial^2}{\partial\xi^2}\phi^{(0)} = 0 \qquad (26)$$

Since $\frac{\partial^2\phi^{(0)}}{\partial\xi^2} \neq 0$, $\lambda(\xi)$ must satisfy the relation

$$\lambda^2(\xi) = -\frac{K_\perp(\xi)}{K_\parallel(\xi)} \qquad (27)$$

which characterizes the propagation cone for lower hybrid wave in an inhomogeneous plasma. We should notice that λ reduces to zero at the position of resonance layer because $K_\perp \simeq 0$ at this layer. This means that in an inhomogeneous plasma lower hybrid waves have a conical trajectory which bends relative to the magnetic field so that at the resonant layer it is nearly parallel to the magnetic field[6),11)]. Therefore eq.(28) shows that, in the presence of a density gradient in the x-direction, the x-component of wave number k_x increases as the wave propagates into regions of increasing plasma density. Near the resonance layer mode conversion can also be expected to occur.

Introducing a function $\psi(\xi,\eta)$ as

$$\frac{\partial}{\partial\xi}\phi^{(0)} = \psi(\xi,\eta) \quad , \qquad (28)$$

the next order equation takes the following form

$$\frac{\partial}{\partial\eta}\psi + A\psi^2\frac{\partial\psi}{\partial\xi} + B\frac{\partial^3\psi}{\partial\xi^2} + C\psi = 0 \qquad (29)$$

where

$$A(\xi) = \frac{3}{2}\frac{K_\parallel}{K_\perp^2}(\alpha_1 + \alpha_2\lambda^2 - \beta_1\lambda^2 - \beta_2\lambda^4)$$

$$= -\frac{3}{4}\frac{K_\parallel}{K_\perp^2}\frac{\lambda_D^2}{L^2}(1+\lambda^2)^2 f(\xi) > 0 \quad , \qquad (30)$$

$$B(\xi) = -\frac{1}{2}\frac{K_{\parallel}}{K_{\perp}^2}(a_1 - b\lambda^2 + c\lambda^4)$$

$$= -\frac{1}{2}\frac{K_{\parallel}}{K_{\perp}^2}\frac{v_e^2/L^2}{\omega^2}\frac{\omega_{pe}^2}{\omega^2}\left[(\lambda^2 - \frac{\omega^2}{\Omega_e^2})^2 + \frac{T_i}{T_e}(\frac{m}{M})^2(\lambda^2+1)^2\right]f(\xi) > 0, \qquad (31)$$

$$C(\xi) = \frac{1}{2}\frac{1}{K_{\perp}}\frac{\partial K_{\perp}}{\partial \xi} \qquad (32)$$

The first three terms of this equation constitute the modified Korteweg-de Vries equation, although the coefficients $A(\xi)$, $B(\xi)$ and $C(\xi)$ are not constant but functions of ξ through $n_0(\xi)$. If the unperturbed density is uniform, equation (29) reduces to the usual modified Korteweg-de Vries equation derived in the previous paper[15]. A similar equation (Korteweg-de Vries equation) has also been obtained for long gravity waves in an uneven bottom[20)~21)] and for weak nonlinear magneto-acoustic waves in an inhomogeneous plasma[22]. It was shown numerically that such waves exhibit peculiar behaviours such as damping, growing or splitting depending upon inhomogeneity profile. In view of this analogy one may expect similar interesting phenomena for eq. (29).

§4. Numerical Analysis

We rewrite eq. (29) in the original coordinates, $(\bar{x},\bar{z})$ as

$$\frac{\partial\psi}{\partial\bar{z}} + \lambda\frac{\partial\psi}{\partial\bar{x}} + \lambda A\psi^2\frac{\partial\psi}{\partial\bar{x}} + \lambda^3 B\frac{\partial^3\psi}{\partial\bar{x}^3} + \frac{\lambda}{2K_{\perp}}\frac{\partial K_{\perp}}{\partial\bar{x}}\psi = 0 . \qquad (33)$$

On the basis of eq. (33), let us investigate numerically the propagation properties of the lower hybrid wave excited by a periodic source located near the plasma boundary as

$$\psi(\bar{x} = \bar{x}_0) = a \sin k_0 \bar{z} \quad ,$$

where $\bar{x}_0$, a and k_0 stand for the position of the source, an amplitude and a wave number for the periodicity of the source, respectively.

The parameters used in the numerical analysis are $m_i/m_e = 1836$, $T_e/T_i = 10$, $\Omega_e/\omega_{pe} = 1$ and $\omega = \omega_{LH} = \omega_{pi}/(1+\omega_{pe}^2/\Omega_e^2)^{1/2}$. In this case, the lower hybrid wave can propagate provided $m_e/2m_i \leq f(\bar{x}) \leq 1$ from eq.(27). The critical values, $f(\bar{x}) = m_e/2m_i$ and $f(\bar{x}) = 1$ correspond to the cut-off density and the resonance one for the lower hybrid wave, respectively. In the present calculation, the source is assumed to be located at the position of a density higher than the cut-off one.

Figures 1 and 2 show the spatial profiles of the lower hybrid wave and its propagation cones for a linear density profile, where Figs.1(a) and 2(a) correspond to a weak pump and Figs.1(b) and 2(b) to a high pump. In Fig.2, the dark parts denote large amplitude of ψ with $\psi > 0$.

Now, the resonance layer is at the position of $f(\bar{x})=1$ as is shown in Fig.1(a). In this case the resonance cone singularities become asymptotic to this layer. We see the conical wave packet propagation. We note that k_z is constant, and is fixed by the boundary condition imposed by the periodicity of the source. Note again the shortening of the wavelength, $2\pi/k_x$ toward the plasma center due to the increasing density. For axial density gradient, k_z will increase as the density decreases axially from eq.(11). These behaviour is the same as that found by Simonutti[13] and Bellan and Porkolab.[16]

For the case of high power pump shown in Figs.1(b) and 2(b), we also note that such waves exhibit peculiar behaviour such as growing and splitting due to the compitition of density inhomogeneity, density depression through the ponderomotive force and the finite temperature effects. As is seen in Figs.2, the large amplitude wave packet propagates into the plasma center with the large propagation angle than the small one in the ($\bar{x}$ - $\bar{z}$)plane, when it approaches the resonance layer.

Figures 3 and 4 show the spatial profiles of the lower hybrid wave and its propagation cones for a Gaussian density profile, where (a) corresponds to a weak pump and (b) to a high pump. The results are essentially same as those for the linear density profile shown in Figs. 1 and 2 except for the spatial growth rate due to inhomogeneity and the resonance cone trajectory.

Also we considered a paticular finite source with 8 periods distance. The results were qualitatively similar to the case imposed by the periodicity of the source shown in Figs. 1 and 2. However, the propagation properties will sensitively depend on the configulation of finite source.

§5. Conclusion

In this paper, we pointed out the general features of the effects of the finite temperature and the density depression due to ponderomotive force on driven waves in the lower hybrid frequency range in an inhomogeneous plasma.

Using a periodic source model, we have venified numerically the conical propagation of the spatial wave packet excited by this source. The results presented here are in good agreement with the recent experimental results of Bellan and Porkolab[6].

As was illustrated in Figs. 1 ∿ 4, the solutions insumes that the wave energy is indeed carried from the source into plasma center through the propagation cones. However the recent experimental observations suggests that the propagation cones are distorted by some another effects, nemely, the Landau damping,[6),23)] the collisional damping[4),23)] and the particle trapping. It seems to be necessary to extend the present studies to take account of these effects on the propagation characteristics.

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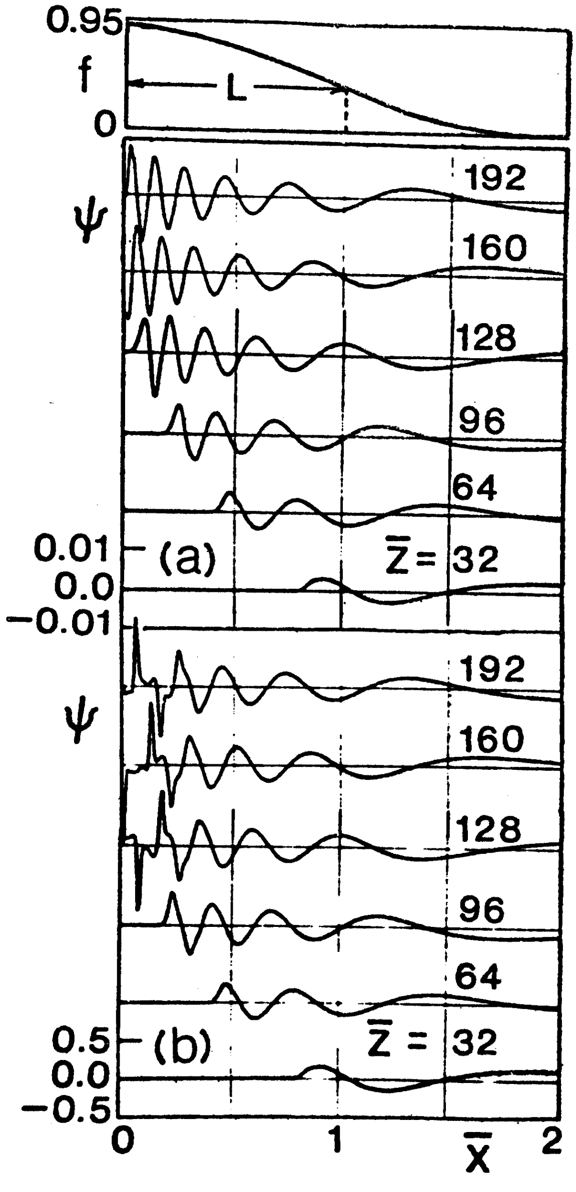
Figure Captions

Fig.1 Propagation of the lower hybrid wave in a linear density profile, $f(\bar{x})=0.95(1-0.95\bar{x})$ for (a) weak pump, $a=10^{-3}$ and for (b) high pump, $a=10^{-1}$, where $L/\lambda_{De}=64\pi$ and $k_0=4\pi/25$.

Fig. 2 Propagation cones of the lower hybrid wave under the same conditions as Fig.1.

Fig.3 Propagation of the lower hybrid wave in a Gaussian density profile, $f=0.95\exp(-\bar{x}^2)$ for (a) weak pump, $a=10^{-3}$ and for (b) high pump, $a=5\times10^{-2}$, where $L/\lambda_{De}=32\pi$ and $k_0=2\pi/25$.

Fig.4 Propagation cones of the lower hybrid wave under the same condition as Fig.3.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$\bar{z}$ = 32
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64
0.5
(b)
$\bar{z}$ = 32
0.0
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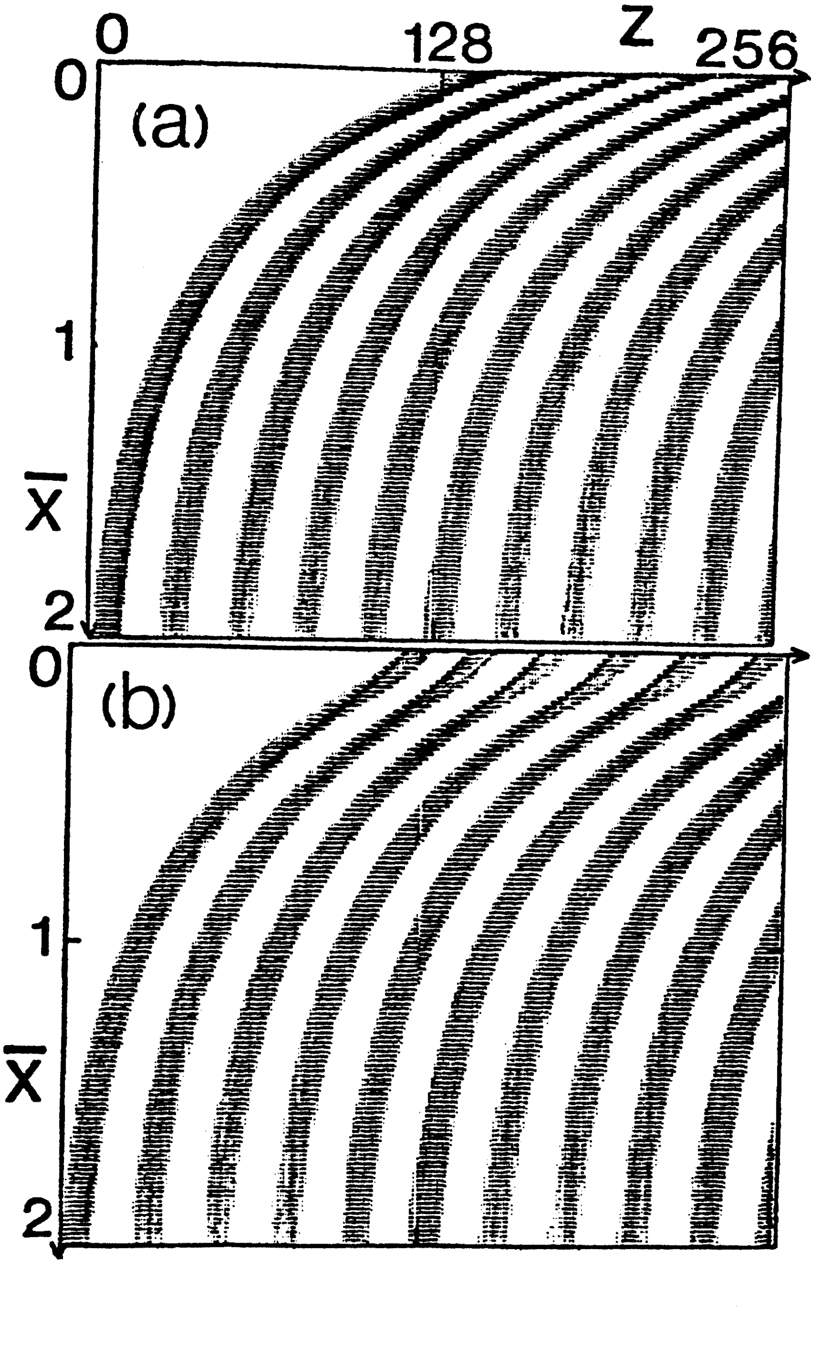

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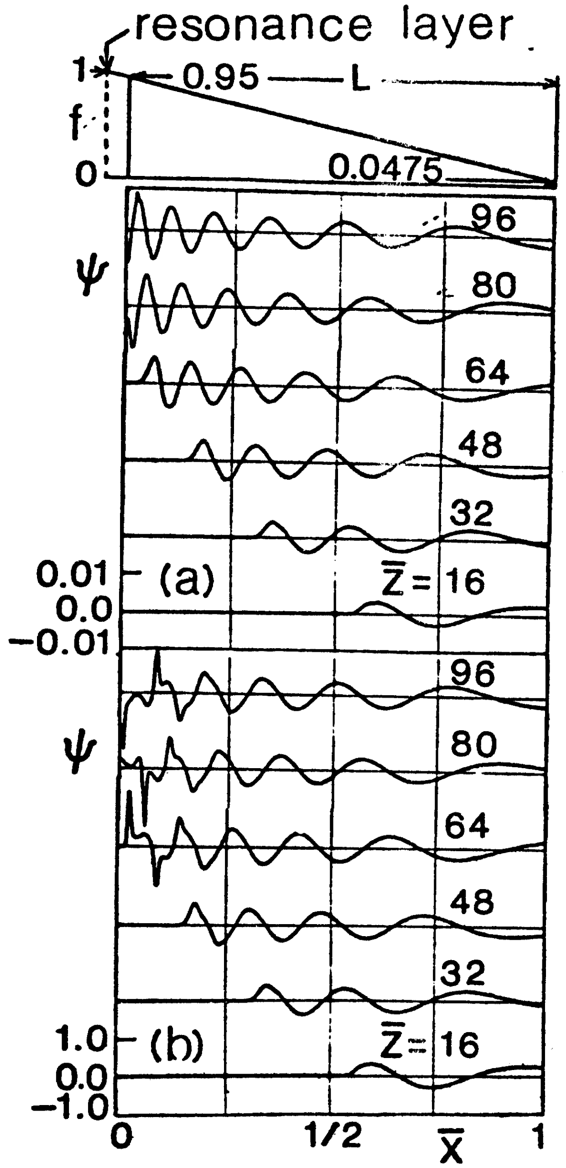
resonance layer
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Z̄ = 16
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(b)
Z̄ = 16
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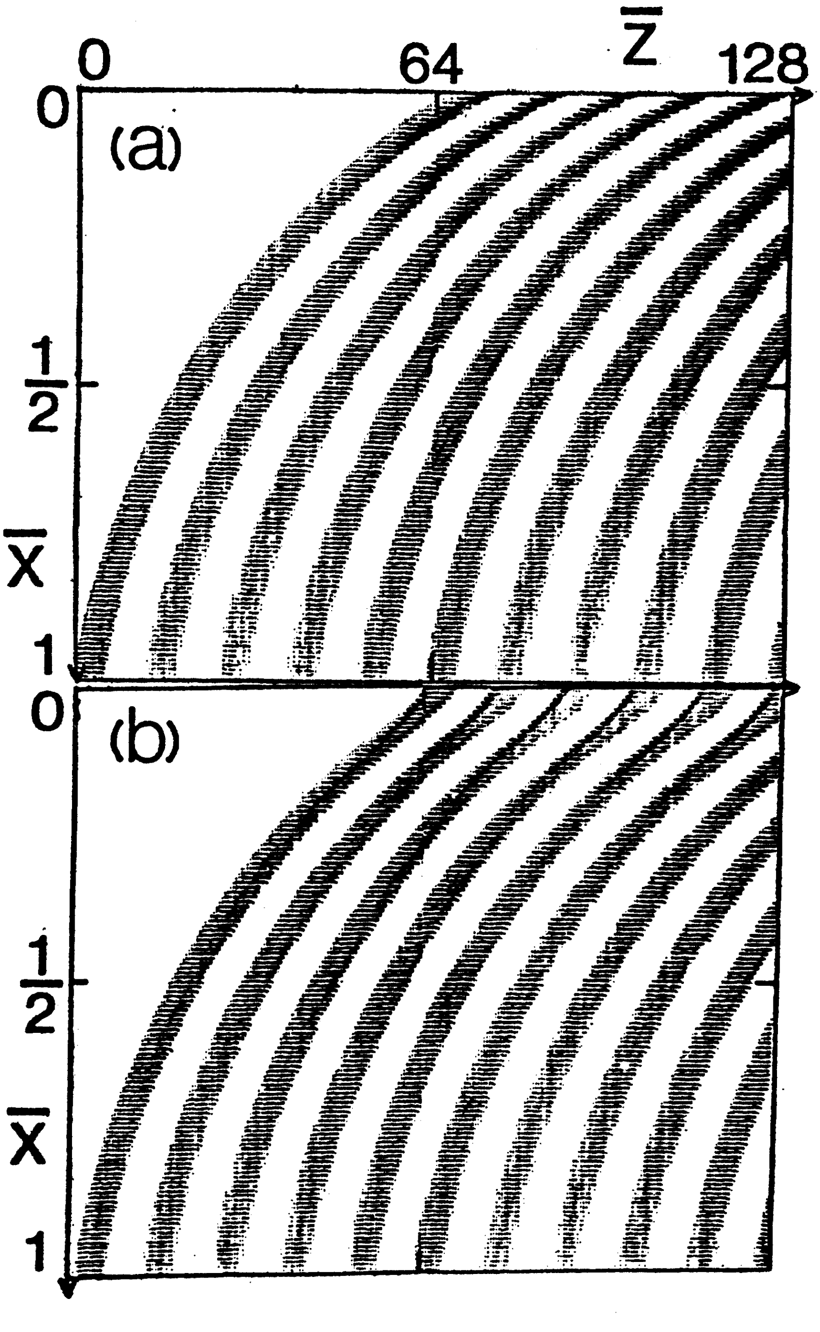
1/2
X̄
1



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