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Rotational Transform Angle of the Stellarator Field with Twisted Coils

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Abstract

The stellarator field produced by the system of twisted coils in the linear configuration is studied analytically. The current of this system can be represented by the superposition of the azimuthal components and the helical components. The rotational transform angle of this system is analyzed. It is found that the fundamental harmonic of the helical components of the current distribution gives the major contribution to the rotational transform angle. The numerical results in the case of the toroidal configuration are compared with the analytical results.

§1. Introduction

Stellarator fields are usually produced by helical windings. However if the rotational transform angle is fixed, stellarator fields can be created by a system of planer elliptic coils[1] or a system of twisted coils[2,3]. In this paper, the relations of rotational transform angle with the geometrical parameters of the system of twisted coils are analyzed.

The current of this system is the superposition of the azimuthal current components and the helical current components with various sets of pole numbers and periodic mode numbers. However the major contribution to the magnetic field configuration comes from the component of uniform azimuthal current and the component of the helical current with the equal pole number and periodic mode number to those of the twisted coil system. Thus the magnetic field configuration of this system is similar to that of usual stellarator fields with helical windings. The numerical results in the case of the toroidal configuration are compared with analytical results in the case of linear configuration.

§2. Current Distribution of the System of Twisted Coils

In this section the system of twisted coils in the linear configuration is studied analytically. The following geometrical parameters are introduced to clarify the relations between the toroidal and linear configuration of twisted coils.

$$L = 2\pi R \ , \tag{2.1}$$

$$m = \alpha \ell R \ , \tag{2.2}$$

where R is the major radius of the toroidal system of twisted coils, L is the length of linear configuration corresponding to the toroidal one, ℓ and m are the pole number and toroidal mode number of the system of twisted coils, and α is the pitch parameter of the corresponding linear system. The system of twisted coils is shown in Fig.1 with the geometrical parameter.

The position of coils of this system is represented as follows,

$$x = a\cos\phi \ , \tag{2.3}$$

$$y = a\sin\phi \ , \tag{2.4}$$

$$z = d\cos(\ell\phi + \phi_i) + z_i \ , \tag{2.5}$$

where $\phi_i = \dfrac{2\pi m i}{N}$ and $z_i = \frac{L}{N}(i-1)$, $(i = 0, \pm 1, \pm 2, \cdots)$. In these equations, d is the width of twisted coils.

It is assumed that the currents of this system are composed of filaments. The current density is given by the following formula,

$$j_r = 0 \ , \tag{2.6}$$

$$j_\theta = \frac{N}{L} \sum_i \delta\left(\frac{1}{L}(z - z_i - d\sin(\ell\phi + \phi_i))\right), \tag{2.7}$$

$$j_z = \frac{d\ell}{aL} \sum_i \cos(\ell\phi + \phi_i)\, \delta\left(\frac{1}{L}(z - z_i - d\sin(\ell\phi + \phi_i))\right) \ , \tag{2.8}$$

and can be expanded into the Fourier components as follows,

$$j = \vec{j}^T + \vec{j}^B + \vec{j}^H \ , \tag{2.9}$$

where

$$\begin{aligned} j_r^T &= 0 \ , \\ j_\theta^T &= \frac{NJ}{L} \ , \\ j_z^T &= 0 \ , \end{aligned} \tag{2.10}$$

$$j_r^B = 0 \,,$$
$$j_\theta^B = \frac{2NJ}{L} \sum_{i=1}^{\infty} J_o(kid)\cos(kiz) \,, \qquad (2.11)$$
$$j_z^B = 0 \,,$$

and

$$j_r^H = 0 \,,$$
$$j_\theta^H = \frac{2NJ}{L} \sum_{i=-\infty}^{\infty} \sum_{h=1}^{\infty} J_h^*(pd)\cos(h\ell\phi - pz) \,, \qquad (2.12)$$
$$j_z^H = \frac{2NJ}{L} \frac{\ell d}{a} \sum_{i=-\infty}^{\infty} \sum_{h=1}^{\infty} h \frac{J_h^*(pd)}{pd} \cos(h\ell\phi - pz) \,,$$

where
$$J_h^*(z) = \begin{cases} J_h(z) & \text{(when } z \geq 0) \,, \\ J_{-h}(-z) & \text{(when } z < 0) \,, \end{cases} \qquad (2.13)$$

$$k = \frac{2\pi N}{L} \quad \text{and} \quad p = \frac{2\pi}{L}(Ni + mh) \,.$$

Here $\vec{j}^T$ is the uniform azimuthal current producing the uniform toroidal field, $\vec{j}^B$ is the rippling azimuthal current producing the bumpy field due to the discreteness of coils and $\vec{j}^H$ is the helical component of the current.

The pole number of helical current is $h\ell$ $(h = 1,2,\cdots)$ and $h \geqq 2$ components correspond to the higher harmonics of the pole number of the twisted coil. The mode number of the helical current is given by the formula $Ni + hm$ $(i = 0, \pm 1, \pm 2, \cdots)$ and $i = 0$ components appear because of the discreteness of twisted coils. Thus the system of twisted coils can be regarded as the superposition of the infinite series of helical coils with various pole and mode numbers. It will be shown that only the fundamental component of helical current with the pole number ℓ and toroidal mode number m of the coils is mainly responsible to the field configuration. Therefore, this field system is very similar to the usual stellarator field with one kind of helical windings.

§3. Magnetic Scalar Potential, Averaged Magnetic Surface of The Twisted Coil System and Rotational Transform Angle

From the current distribution calculated in §2, the magnetic scalar potential is given by[4)]

$$\Phi = \Phi^T + \Phi^B + \Phi^H \quad , \tag{3.1}$$

where $\Phi^T = \mu_o \frac{NJ}{L} z$,

$$\Phi^B = -2\mu_o \; a \frac{NJ}{L} \sum_{i=1}^{\infty} J_o(kid) K_o^{'}(kia) I_o(kir) \sin(kiz) , \tag{3.2}$$

$$\Phi^H = 2\mu_o a \frac{NJ}{L} \sum_{h=1}^{\infty} \sum_{i=-\infty}^{\infty} J_h^{*}(pd) K_{h\ell}^{'}(|pa|) I_{h\ell}(|pr|) \times \sin(h\ell\phi - pz) \; , \tag{3.3}$$

where K'(z) is the modified Bessel function differentiated by z.

In the following calculation we neglect the bumpy components in calculating the z - component of magnetic field. This is a good approximation near the magnetic axis when the ratio L/Na is small. The averaged magnetic surface is represented as follows[4)],

$$\psi = - \frac{2\mu_o NJa^2}{L} \sum_{h=1}^{\infty} \sum_{i=-\infty}^{\infty} h \{J_h^{*}(pd) K_{h\ell}^{'}(|pa|)\}^2 I_{h\ell}(|pr|) \times I_{h\ell}^{'}(|pr|) = \text{const.} \tag{3.4}$$

In Fig. 2 the magnetic surface of toroidal twisted coil system is shown in the typical case of $\ell = 2$, $m = 4$, $N = 20$, $a/R = 0.35$ and $d/a = 0.34$.

With the use of average magnetic surface, the rotational transform angle of the system of twisted coils is given by[4)]

$$\iota = 2\pi R \sum_{h=1}^{\infty} \sum_{i=-\infty}^{\infty} \frac{p}{h\ell} \{ \frac{2ap}{h} J_h^{*}(pd) K_{h\ell}'(|pa|) \}^2$$

$$\times \{ \frac{2}{(h\ell)!} (\frac{h\ell}{2})^{h\ell+1} \}^2 \{(1-\frac{1}{h\ell}) (\frac{pr}{h\ell})^{2h\ell-4} + \frac{h\ell}{2}(\frac{pr}{h\ell})^{2h\ell-2}$$

$$+ \cdots\cdots \} . \qquad (3.5)$$

In the case of $\ell = 2$, eq.(3.5) is reduced to

$$\iota = 2\pi R \sum_{h=1}^{\infty} \sum_{i=-\infty}^{\infty} \frac{p}{2h} \{ \frac{ap}{h} J_h^{*}(pd) K_{2h}'(|pa|) \}^2$$

$$\times \{ \frac{2}{(2h)!} h^{2h+1} \}^2 \{(1-\frac{1}{2h}) (\frac{pr}{2h})^{4h-4} + h (\frac{pr}{2h})^{4h-2} + \cdots \} . \qquad (3.6)$$

It is clear from the character of the function of K' that the major contribution comes only from the fundamental component $h = 1$ and $i = 0$ among the infinite series of harmonics due to the helical current components with various pole and mode numbers. Therefore the rotational transform angle is represented as follows,

$$\iota = \frac{\pi}{2} m^3 (\frac{a}{R})^2 J_1^2 (\frac{md}{R}) K_2'^2 (\frac{ma}{R}) \{ 1 + \frac{m^2}{2} (\frac{r}{R})^2 \} . \qquad (3.7)$$

The dependence of the rotational transform angle on the degree of twist of coils is shown in Fig. 3. The dependence of the rotational transform angle on the toroidal mode number is shown in Fig.4 where the degree of twist d/a is kept constant. The numerical results in the case of the corresponding toroidal configuration are also shown in dots in the figures.

§4. Conclusion

The character of the magnetic field produced by the system of twisted coils is studied both analytically and numerically. Although the magnetic field of this system is the superposition of the infinite series of helical components with various pole numbers and toroidal mode numbers, the fundamental component gives the major contribution to the characteristics of magnetic fields such as the rotational transform angle.

In this sense, the system of twisted coils and usual stellarator field with helical windings can be regarded to be equivalent with the assumption of the following relations,

$$I_h^{\ell} = \frac{2}{\pi} \frac{NJ}{m} J_1 \left(\frac{m}{R} d \right) , \qquad (4.1)$$

where I_h^{ℓ} is the helical current per one helical winding.

Figure Captions

Fig.1 The system of twisted coils of $\ell = 2$ in the linear configuration. Dotted line indicates the lower part of coils.

Fig.2 The system of twisted coils of $\ell = 2$ in the toroidal configuration. The geometrical parameters are $m = 4$, $N = 20$, $a/R = 0.35$ and $d/a = 0.34$. Magnetic surfaces of this system are also shown with the magnetic line of force projected on the poloidal section.

Fig.3 The dependence of the rotational transform angle $\iota/2\pi$ at the center on the degree of twist of coils d/a. The geometrical parameters are $\ell = 2$, $m = 4$, $N = 20$ and $a/R = 0.35$. Solid line shows the analytical results in the case of toroidal configuration and dots show the numerical results in the case of toroidal configuration.

Fig.4 The dependence of the rotational transform angle $\iota/2\pi$ at the center on the toroidal mode number m. The geometrical parameters are $\ell = 2$, $a/R = 0.35$, $N = 20$ and $d/a = 0.225$. Solid line shows the analytical results in the case of linear configuration and dots show the numerical results in the case of toroidal configuration.

References

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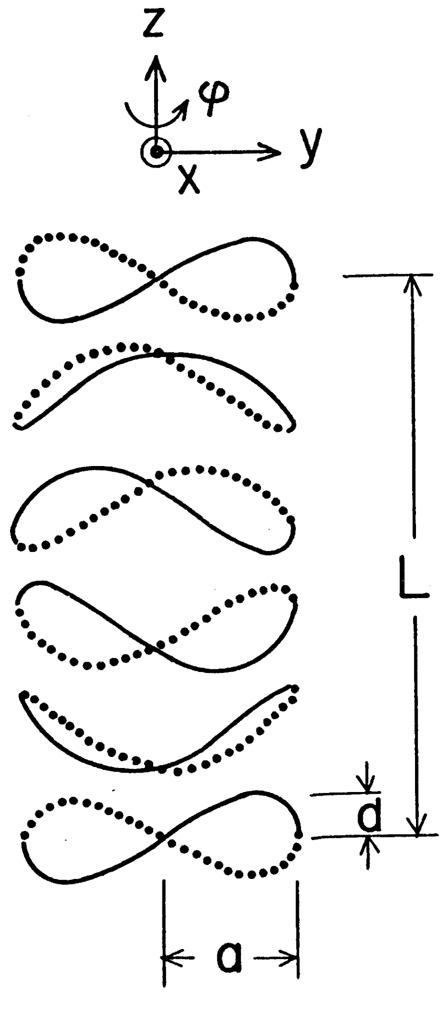

Fig.1

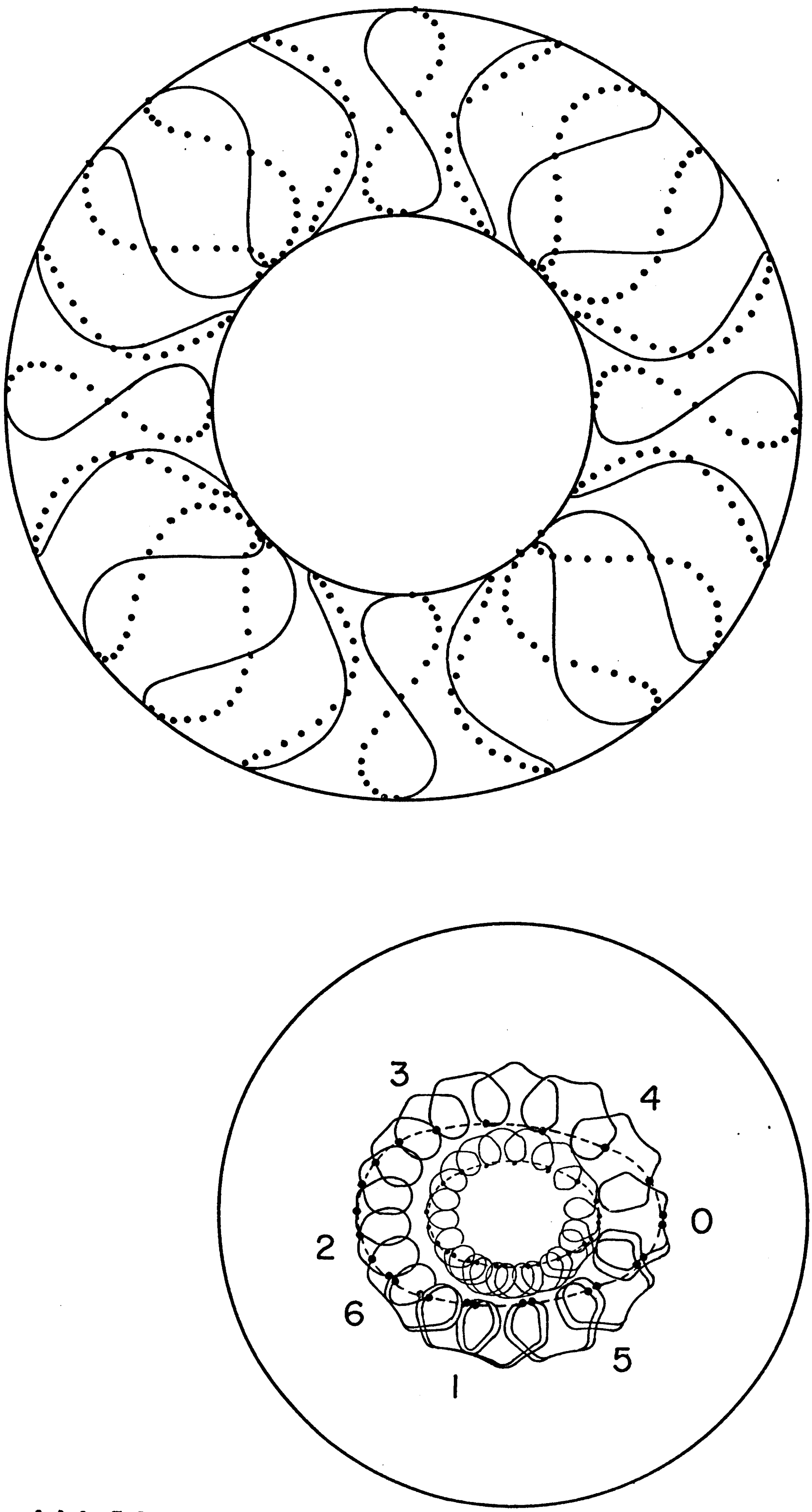

Fig.2

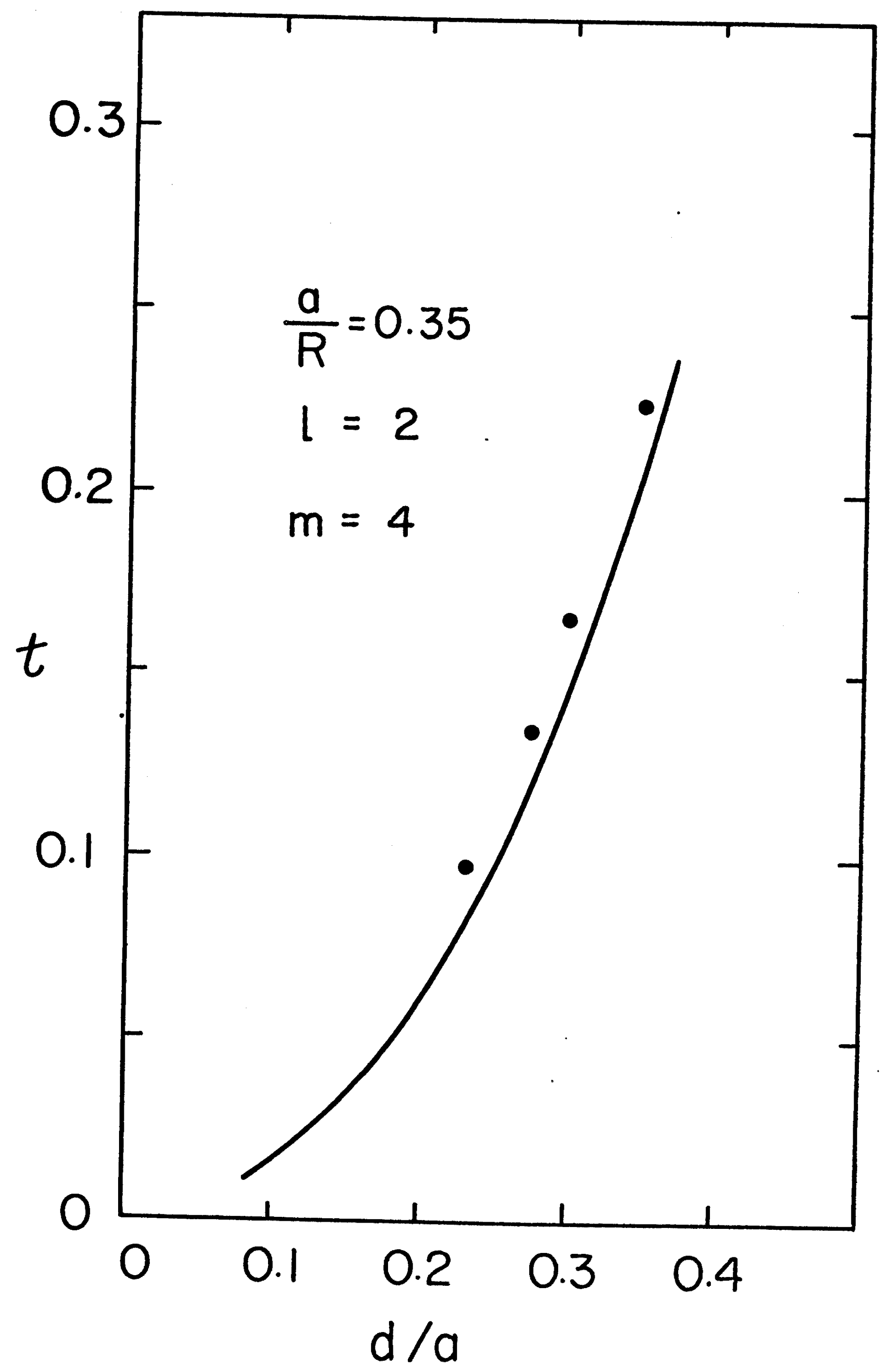

Fig.3

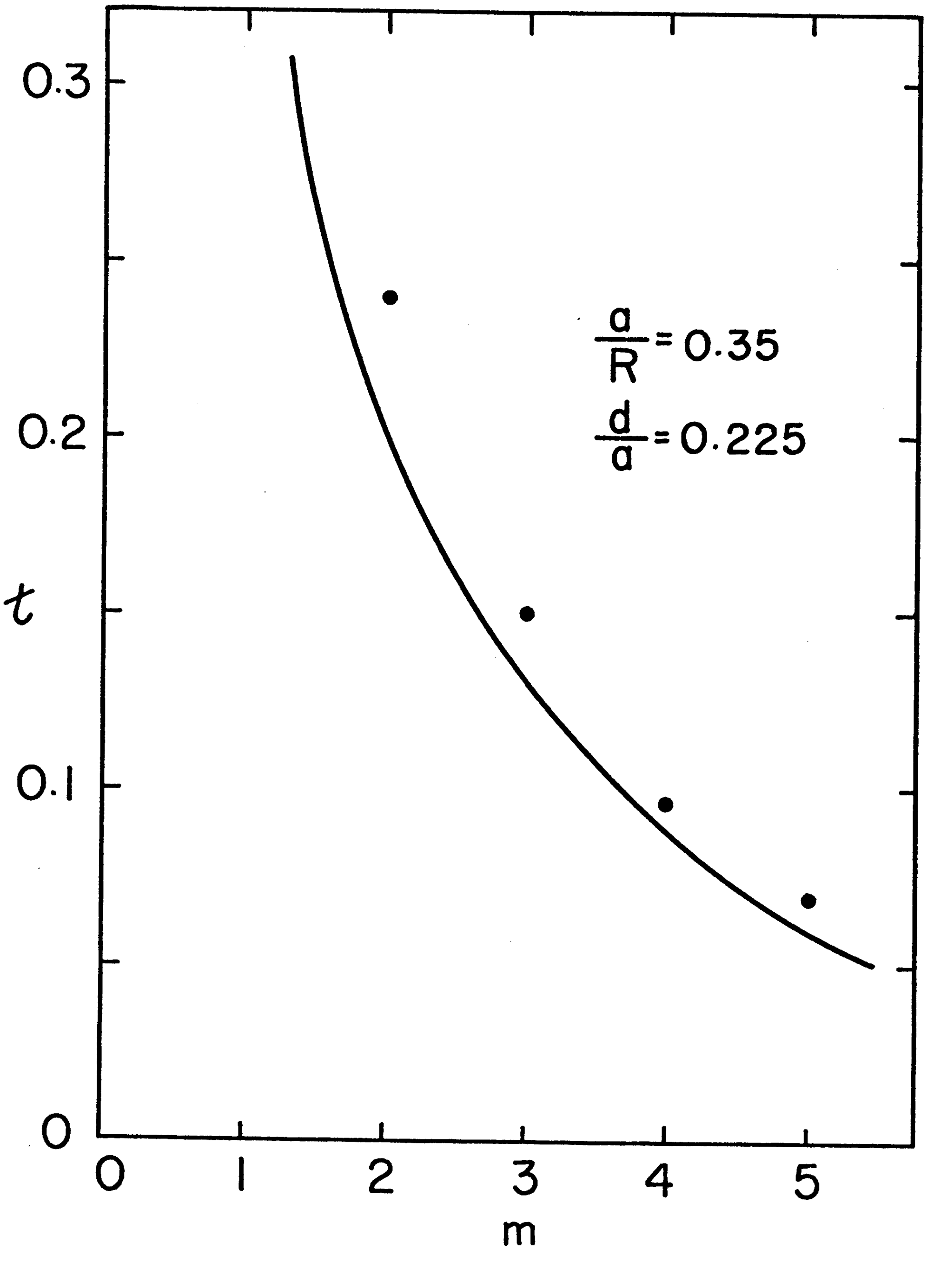

Fig.4