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Parametric Decay of Lower Hybrid Wave

Into Drift Waves

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Abstract

A dispersion relation describing the parametric decay of a lower hybrid wave into an electrostatic drift wave and a drift Alfvén wave is derived for an inhomogeneous magnetized plasma. Particularly the stimulated scattering of a drift Alfvén wave in such a plasma was investigated in detail. The resonance backscattering instability is found to yield the minimum threshold.

§1. Introduction

There is experimental evidence that parametric processes play an important role in the R.F. heating of magnetically confined plasmas (Porkolab et al., 1976; Porkolab, 1976). One frequency range which has recently received a great deal of attention is the lower hybrid resonance frequency (Porkolab, 1976). This frequency range is particularly useful because it interacts directly with the ions.

In the previous paper we developed the theory of parametric instabilities driven by a finite wavenumber lower hybrid wave pump in the homogeneous magnetized plasma and we also discussed the resonance backscattering instabilities and the oscillating two stream instabilities (OTST) in such system (Sanuki and Schmidt, 1977, Sanuki et al., 1976).

Recently the parametric decay of a lower hybrid wave into an ion acoustic wave was investigated for a plasma with a density gradient (Wersinger et al., 1976). However, in an inhomogeneous magnetized plasma, low frequency perturbations with sufficiently long parallel wavelengths tend to be electrostatic-electromagnetic modes because of coupling to Alfvén waves. Parametric decay instabilities can result because of coupling to a drift wave (Sundaram and Kaw, 1973) or a drift Alfvén wave (Bujarbarua and Kaw, 1976) depending on the angle between the direction of propagation of the low frequency mode and the magnetic field.

In the experiments by Hooke and Bernabei, and Moresco and Zilli, they observed the generation of beat frequency

$\omega_0 \pm \omega_1$ (where ω_0 denotes the pump frequency and ω_1 is of the order of the drift frequency) which they attribute to a nonlinear mixing of lower hybrid waves and drift waves. These modes are potentially dangerous from the point of view of plasma confinement since they may lead to macroscopic motions of the plasma. Investigation of the couping between a lower hybrid wave and a drift Alfvén wave is the object of the present paper.

§2. Three Wave Interaction in an Inhomogeneous Magneto-Plasma

Let us consider a weakly inhomogeneous plasma with a uniform background magnetic field. In the simple slab geometry the density gradient is chosen along the x-axis ($\nabla N_0/N_0=(-\kappa,0,0)$) and the backgound magnetic field is oriented along the z-axis of the Cartesian co-ordinate of system ($\vec{B}_0=(0,0,B_0)$). A high-frequency oscillating electric field $\vec{E}_0$ is applied obliquely to the magnetic field. The pump wave couples with the density fluctuation in a low frequency mode (ω, $\vec{k}$) and produces currents and magnetic modes at the high frequency side band frequencies ($\omega_\pm=\omega\pm\omega_0$, $\vec{k}_\pm=\vec{k}\pm\vec{k}_0$). The side-hand modes interact with the pump mode to produce a ponderomotive force at frequency ω, which can enhance the original perturbation and thus lead to an instability.

<u>High Frequency Wave Equation</u>

The high frequency waves satisfy the wave equation

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = -\mu_0\frac{\partial}{\partial t}(\vec{J}_L + \vec{J}_{NL}) \quad , \qquad (1)$$

where $\vec{J}_L$ is the linear current density

$$\vec{J}_L = eN(x)(\vec{v}_i - \vec{v}_e) \quad , \tag{2}$$

with N(x) the equilibrium particle density $\vec{v}_i$ and $\vec{v}_e$ the first order ion and electron fluid velocities respectively. The nonlinear current density $\vec{J}_{NL}$ may be written approximately as

$$\vec{J}_{NL} = e(n_{si}\ \vec{v}_{fi} - n_{se}\ \vec{v}_{fe}) \quad , \tag{3}$$

where the index s and f stand for law frequency and high frequency wave quantities respectively. Note that $\omega_s/\omega_f << 1$ consequently $n_f\vec{v}_s$ terms are small and hence have been neglected.

The fluid velocities can be expressed in terms of the electric fields. In particular if all quantities are Fourier decomposed in time

$$\vec{v}_{if} = \underset{\approx}{\nu}_{if}\cdot\vec{E}_f \quad , \tag{4a}$$

$$\vec{v}_{ef} = \underset{\approx}{\nu}_{ef}\cdot\vec{E}_f \quad , \tag{4b}$$

where $\underset{\approx}{\nu}$ is the mobility tensor in the magnetic field and we also assumed the Fourier transformation for all quantities in the form $\exp[i\omega t - i\vec{k}\cdot\vec{r}]$. So the space and time Fourier transformed eq. (1) becomes

$$\vec{k} \times (\vec{k} \times \vec{E}) + \frac{\omega^2}{c^2}\vec{E} - \mu_0 i\omega e N(x)(\underset{\approx}{\nu}_i - \underset{\approx}{\nu}_e)\cdot\vec{E}$$

$$= \mu_0 i\omega e(n_{si}\underset{\approx}{\nu}_i - n_{se}\underset{\approx}{\nu}_e)\cdot E \quad . \tag{5}$$

Hence the f index has been omitted since the only low frequency quantities are the particle densities. The $\vec{E}$-s on the left hand side are the $(\omega,\vec{k})$ Fourier components while the one on the right hand side is clearly a different Fourier component. Setting the ℓ.h.s. to zero gives the linear high frequency wave dispersion relation while the r.h.s. represents the coupling of the low frequency wave to the high frequency sidebands.

Low Frequency Wave Equation

Equation (5) shows how low frequency density fluctuations beating with the pump wave give rise to sidebands. On the other hand the beating of the pump with the sidebands produces the low frequency wave.

The nonlinear current density

$$\vec{J}_s = \Sigma\ q(N\vec{v}_s + \langle n_f \vec{v}_f \rangle) \quad , \tag{6}$$

where the summation is over electrons and ions, and the bracket $\langle\ \ \rangle$ indicates fast time scale averaging. The equation of motion for electrons is

$$\frac{\partial \vec{v}_{es}}{\partial t} = -\frac{e}{m}(\vec{v}_{es} \times \vec{B}_0 + E_s) - \frac{T_e}{m}\vec{\nabla}\left(\frac{n_s}{N(x)}\right) - \langle(\vec{v}_{ef}\cdot\vec{\nabla})\vec{v}_{ef}\rangle - \frac{e}{m}\langle\vec{v}_{ef} \times \vec{B}_f\rangle \quad , \tag{7}$$

where the last two terms are nonlinear, representing high frequency waves driving low frequency ones at the beat frequencies. The inertia term on the left hand side will be ignored as usual. Fourier decomposing this equation and the equation of continuity.

$$n_{es} = \frac{(\vec{k} - i\kappa\hat{e}_x)}{\omega_s} \cdot (N(x)\vec{v}_{es} + \langle n_{ef}\ \vec{v}_{ef}\rangle) \quad , \tag{8}$$

yields

$$\vec{E}_s + \langle\vec{v}_{ef} \times \vec{B}_f\rangle + \frac{m}{e}\langle(\vec{v}_{ef}\cdot\vec{\nabla})\vec{v}_{ef}\rangle + \vec{v}_{es} \times \vec{B}_0$$
$$= \frac{iT_e}{e\omega_s}\vec{k}\,(\vec{k} - i\kappa\hat{e}_x)\cdot(\vec{v}_{es} + \frac{1}{N(x)}\langle n_{ef}\,\vec{v}_{ef}\rangle). \tag{9}$$

In the absence of nonlinear terms one has

$$\vec{E}_s = (\vec{B}_0 \times + \frac{iT_e}{e\omega_s}\vec{k}\,(\vec{k} - i\kappa\hat{e}_x)\cdot)\vec{v}_{es}$$
$$\equiv \nu_{em}^{-1}\cdot\vec{v}_{es} \quad , \tag{10}$$

where ν_{em} is the modified mobility tensor, $\vec{v}_{es} = \underset{\approx}{\nu}_{em}\cdot\vec{E}_s$.

Applying $\underset{\approx}{\nu}_{em}$ on eq.(9) and using eq.(10) yields

$$\vec{v}_{es} = \underset{\approx}{\nu}_{em}\cdot[\vec{E}_s + \langle\vec{v}_{ef} \times \vec{B}_f\rangle + \frac{m}{e}\langle(\vec{v}_{ef}\cdot\vec{\nabla})\vec{v}_{ef}\rangle$$
$$+ \frac{\vec{B}_0}{N(x)} \times\langle n_{ef}\,\vec{v}_{ef}\rangle] - \frac{\langle n_{ef}\vec{v}_{ef}\rangle}{N(x)} \quad . \tag{11}$$

Substitution of eq.(11) into eq.(6) gives

$$\vec{J}_{es} = -e\,N(x)\underset{\approx}{\nu}_{em}\cdot[\vec{E}_s + \vec{\psi}_{pe}] \quad , \tag{12}$$

where $\vec{\psi}_{pe}$ is the equivalent ponderomitive electric field

$$\vec{\psi}_{pe} = \langle\vec{v}_{ef} \times \vec{B}_f\rangle + \frac{m}{e}\langle(\vec{v}_{ef}\cdot\vec{\nabla})\vec{v}_{ef}\rangle$$
$$+ \frac{\vec{B}_0}{N(x)} \times \langle n_{ef}\,\vec{v}_{ef}\rangle \quad . \tag{13}$$

A similar expression holds for the ion current

$$\vec{J}_{is} = e\,N(x)\,\underset{\approx}{\nu}_{im}\cdot[\vec{E}_s + \vec{\psi}_{pi}] \quad , \tag{14}$$

where

$$\nu_{im}^{-1} = i\omega_s\frac{M}{e} + \vec{B}_0 \times - \frac{iT_i}{e\omega_s}\vec{k}\,(\vec{k} - i\kappa\hat{e}_x) \quad . \tag{15}$$

and

$$\vec{\psi}_{pi} = \langle\vec{v}_{if} \times \vec{B}_f\rangle - \frac{M}{e}\langle(\vec{v}_{if}\cdot\vec{\nabla})\vec{v}_{if}\rangle$$

$$+ \frac{\vec{B}_0}{N(x)} \times\langle n_{if}\vec{v}_{if}\rangle + \frac{i\omega_s M}{eN(x)} \langle n_{if}\vec{v}_{if}\rangle \quad . \tag{16}$$

Finally the electric field obeys the wave equation

$$[\vec{k} \times \vec{k} \times + \frac{\omega_s^2}{c^2}]\vec{E}_s$$

$$= ie\mu_0\omega_s N(x)[\underset{\approx}{\nu}_{im}\cdot(\vec{E}_s + \vec{\psi}_{pi}) - \underset{\approx}{\nu}_{em}\cdot(\vec{E}_s + \vec{\psi}_{pe})]. \tag{17}$$

The linear eigenmode satisfies the equation

$$[\vec{k}\times\vec{k} \times + \frac{\omega_s^2}{c^2} - ie\mu_0\omega_s N(x)(\underset{\approx}{\nu}_{im} - \underset{\approx}{\nu}_{em})\cdot]\vec{E}_s \equiv \underset{\approx}{\kappa}\cdot \vec{E}_s \quad , \tag{18}$$

so from eq.(17) one obtain

$$\vec{E}_s = ie\mu_0\omega_s N(x)\underset{\approx}{\kappa}^{-1}\cdot[\underset{\approx}{\nu}_{im}\cdot\vec{\psi}_{pi} - \underset{\approx}{\nu}_{em}\cdot\vec{\psi}_{pe}] \quad . \tag{19}$$

<u>Coupled Mode Equation</u>

The low frequency wave couples back to the high frequency waves (eq.(5)) via the low frequency particle density. From the equations of continuity we have

$$n_{es} = N(x)(\vec{k}-i\kappa\hat{e}_x)\cdot\underset{\approx}{\nu}_{em}\cdot[ie\mu_0 N(x)\underset{\approx}{\kappa}^{-1}\cdot(\underset{\approx}{\nu}_{im}\cdot\vec{\psi}_{pi}-\underset{\approx}{\nu}_{em}\cdot\vec{\psi}_{pe})+\frac{\vec{\psi}_{pe}}{\omega_s}] \quad , \tag{20a}$$

$$n_{is} = N(x)(\vec{k}-i\kappa\hat{e}_x)\cdot\underset{\approx}{\nu}_{im}\cdot[ie\mu_0 N(x)\underset{\approx}{\kappa}^{-1}\cdot(\underset{\approx}{\nu}_{im}\cdot\vec{\psi}_{pi}-\underset{\approx}{\nu}_{em}\cdot\vec{\psi}_{pe})+\frac{\vec{\psi}_{pi}}{\omega_s}] \quad . \tag{20b}$$

Here $\vec{\psi}_p$ are functions of the fast variables. The fast velocities are determined from eq.(4), while

$$n_f = N(x)(\vec{k} - i\kappa\hat{e}_x)\cdot\vec{v}_f \quad . \tag{21}$$

The equations (4), (5), (20) and (21) constitute a closed set of equations, yielding the dispersion relation for ω_s. Of course one must specify the high frequency and low frequency waves under consideration, so the mobility tensors can be calculated. The amplitude of one of the high frequency waves (the pump) must also be specified.

§3. Parametric Decay of Lower Hybrid Wave Into Drift Alfvén Wave

As an example we shall examine the decay of an electrostatic lower hybrid wave into lower hybrid sidebands and a drift Alfvén wave or an electrostatic drift wave.

Now we shall assume that the pump wave is in the x-z plane ($k_{0x} >> k_{0z}$) and we also restrict ourselves to low frequency perturbations in the y-z plane (k_x=0) with $k_y >> k_z$. Since the high frequency waves are electrostatic $\vec{B}_f$=0 eliminating the $<\vec{v}_f \times \vec{B}_f>$ terms from the ponderomotive force. Equation (5) is simplified by the elimination of the $\vec{k}\times(\vec{k}\times\vec{E})$ term. The low frequency waves will be in an electrostatic drift wave or a drift Alfvén wave ($\omega << \Omega_i$), so the displacement current in the low frequency wave equation may be ignored. It follows then from the low frequency wave equation that $\nabla\cdot\vec{J}_s$=0, consequently the low frequency wave is quasi neutral with $n_{si} \approx n_{se} \approx n_s$. Under these conditions eq.(5) becomes

$$\left[\frac{\omega^2}{c^2} - \mu_0 i\omega e N(x)(\underset{\approx}{\nu}_i - \underset{\approx}{\nu}_e)\cdot\right]\vec{E} = i\mu_0 e\omega n_s(\underset{\approx}{\nu}_i - \underset{\approx}{\nu}_e)\cdot\vec{E} \quad . \tag{22}$$

For the high frequency waves, the mobility tensors can be simplified to be

$$\underset{\approx}{\nu}_{if} = -\frac{ie}{M\omega_f}\underset{\approx}{\mathrm{II}} , \tag{23a}$$

$$\underset{\approx}{\nu}_{ef} = -i\,\frac{e}{m}\begin{bmatrix} \frac{\omega_f}{\Omega_i{}^2} & -\frac{i}{\Omega_e} & 0 \\ \frac{i}{\Omega_e} & \frac{\omega_f}{\Omega_e{}^2} & 0 \\ 0 & 0 & -\frac{1}{\omega_f} \end{bmatrix} , \tag{23b}$$

where $\vec{B}_0$ was chosen in the z-direction and $\underset{\approx}{\mathrm{II}}$ denotes the unit dyadic.

For the low frequency wave, the mobility tensors are

$$\underset{\approx}{\nu}_{im} = \begin{bmatrix} i\,\frac{\omega}{B_0\Omega_i}\,\frac{1-\frac{v_i{}^2 k_y{}^2}{\omega^2}}{(1+\frac{\omega_{*i}}{\omega})} & \frac{1}{B_0}\,\frac{1}{(1+\frac{\omega_{*i}}{\omega})} & \frac{1}{B_0}\,\frac{v_i{}^2 k_y k_z}{\omega^2}\,\frac{1}{(1+\frac{\omega_{*i}}{\omega})} \\ -\frac{1}{B_0} & i\frac{\omega}{B_0\Omega_i}\,\frac{1}{(1+\frac{\omega_{*i}}{\omega})} & \frac{i}{B_0}\,\frac{v_i{}^2 k_y k_z}{\Omega_i\omega}\,\frac{1}{(1+\frac{\omega_{*i}}{\omega})} \\ -\frac{1}{B_0}\,\frac{v_i{}^2 k_y k_z}{\omega^2}\,\frac{1}{(1+\frac{\omega_{*i}}{\omega})} & \frac{1}{B_0}\,\frac{v_i^2 k_y k_z}{\Omega_i\omega}\,\frac{(1-\frac{\Omega_i}{\omega}\frac{\kappa}{k_y})}{(1+\frac{\omega_{*i}}{\omega})} & -i\,\frac{e}{M\omega} \end{bmatrix} \tag{24a}$$

and

$$\underset{\approx}{\nu}_{em} = \begin{bmatrix} 0 & \frac{1}{B_0} & -\frac{1}{B_0}\frac{k_y}{k_z} \\ -\frac{1}{B_0} & 0 & 0 \\ \frac{1}{B_0}\frac{k_y}{k_z} & i\,\frac{\kappa}{B_0 k_z} & -i\,\frac{e}{T_e}\frac{\omega}{k_z{}^2}\,(1-\frac{\omega_{*e}}{\omega}) \end{bmatrix} \tag{24b}$$

Also the operator $\underset{\approx}{\kappa}$ becomes for the low frequency mode with $\omega > \omega_*$

$$\underset{\approx}{\kappa}=\begin{bmatrix} -k^2+\frac{\omega^2}{v_A{}^2}(1-\frac{\omega_{*i}}{\omega}) & i\,\frac{\Omega_i\omega_{*i}}{v_A{}^2}(1-\frac{\omega_{*i}}{\omega}) & -i\,\frac{\Omega_i\omega}{v_A{}^2}\frac{k_y}{k_z} \\ 0 & -k_z^2+\frac{\omega^2}{v_A{}^2}(1-\frac{\omega_{*i}}{\omega}) & k_yk_z \\ i\,\frac{\Omega_i\omega}{v_A{}^2}\frac{k_y}{k_z} & k_yk_z(1-\frac{\Omega_i\omega}{v_A{}^2k_z{}^2}\frac{\kappa}{k_y}) & -k_y{}^2\{1+\frac{\Omega_i{}^2}{v_A{}^2k_y{}^2}[1-\frac{\omega^2}{c_s^2k_z^2}(1-\frac{\omega_{*e}}{\omega})]\} \end{bmatrix} \tag{25}$$

where v_A and c_s are the Alfvén and the ion sound velocities, and $\omega_{*j}=T_j\kappa/|e|B\cdot k_y$ is the drift frequency for j-species of plasma particles respectively.

The linear dispersion relation for the low frequency mode follows from $|\vec{\kappa}| = 0$, to yield

$$|\underset{\approx}{\kappa}| = \frac{\Omega_i{}^2}{v_A{}^6k_z{}^2c_s{}^2}(\omega^2-\omega\omega_{*i}-v_A{}^2k_z{}^2)\{\omega^4-\omega^3(\omega_{*e}+\omega_{*i})$$
$$-\omega^2[(v_A^2+c_s{}^2)k^2-\omega_{*e}\omega_{*i}]+\omega(c_s{}^2k_z{}^2\omega_{*i}+v_A^2k^2\omega_{*e})+v_A^2c_s^2k_z^2k^2\}$$
$$+\text{(some small terms)}\ . \tag{26}$$

Now $\underset{\approx}{\kappa}$ has to be inverted and the matrix multiplications prescribed by one of eqs.(20), say eq.(20a) performed to yield n_s. The ponderomotive fields are calculated from the pump field E_0 and the sideband fields $E^{\pm}=E(\omega\pm\omega_0)$, where ω_0 is the pump frequency. The calculation is lengthy but straightforward and we omit the details.

For the parametric decay of a lower hybrid wave into a drift Alfvén and an electrostatic drift wave branch, one finds the dispersion relation as

$$(\omega^2 - \omega_{*i}\omega - v_A{}^2k_z{}^2)\{\omega^4 - \omega^3(\omega_{*e}+\omega_{*i}) - \omega^2[(v_A^2 + c_s^2)k^2 - \omega_{*e}\omega_{*i}]$$

$$+ \omega(c_s{}^2k_z{}^2\omega_{*i} + v_A{}^2k^2\omega_{*e}) + v_A{}^2c_s{}^2k^2k_z{}^2\}$$

$$= [\frac{\Gamma_+}{D_+} + \frac{\Gamma_-}{D_-}] \quad , \tag{27}$$

with

$$D_\pm = \omega_\pm{}^2 - \omega_{LH}{}^2(1 + \frac{M}{m}\frac{k_{\pm z}{}^2}{k_\pm{}^2}) \quad , \tag{28}$$

$$\Gamma_\pm = v_0{}^2\omega^2(\omega^2 - \omega\omega_{*i} - v_A{}^2k_z{}^2)\omega_{LH}{}^2\frac{k_{0x}^2}{k_0{}^2}\frac{k_y{}^4}{k_\pm{}^2}$$

$$- \frac{m}{M}v_0{}^2\omega\omega_{*i}(\omega-\omega_{*i})\omega_{LH}^2\omega_\pm\frac{k_{0x}{}^2k_y{}^2}{k_0{}^2k_\pm{}^2}[1 - \frac{\Omega_e{}^2}{\omega_\pm{}^2}\frac{k_z(k_z\pm 2k_{0z})}{k_y{}^2}]$$

$$+ (\omega^2-\omega\omega_{*i}-v_A{}^2k_z{}^2)(\omega^2-\omega\omega_{*i}-v_A{}^2k^2)\frac{v_0{}^2}{c_s{}^2}\omega_{LH}^2\,\omega(\omega-\omega_{*e})$$

$$\times \frac{k_{0x}{}^2}{k_0{}^2}\frac{k_y{}^2}{k_\pm{}^2}(i + \frac{m}{M}\frac{c_s{}^2k_y{}^2\omega_\pm}{\Omega_e{}^2(\omega-\omega_{*e})}) \quad , \tag{29}$$

where $v_0=E_0/B_0$, $\omega_{LH}{}^2=\omega_{pi}{}^2/(1+\frac{\omega_{pe}{}^2}{\Omega_e{}^2})$, $\omega_\pm = \omega \pm \omega_0$ and $\vec{k}_\pm=\vec{k}\pm\vec{k}_0$ respectively.

Let us now consider the parametric decay into a drift Alfvén wave. Retaining only the leading terms in eq.(27), we can approximate the dispersion relation (27) in the dimensionless form

$$\{\Omega(\Omega+i\gamma_1)-\Omega-\beta^2\}\{(\Omega + i\gamma_2 - \alpha)^2 - \delta^2\} = \Gamma(\Omega-\alpha) \quad , \tag{30}$$

with

$$\Gamma = 2\frac{m}{M}\frac{v_0{}^2}{v_A{}^2}\frac{\omega_{LH}^2}{\omega_*{}^2}\frac{k_y{}^2}{k^2}\frac{k_y{}^2}{k_0{}^2+k^2}(1 - \frac{\Omega_e{}^2}{\omega_0{}^2}\frac{k_z{}^2}{k_y{}^2}) \quad , \tag{31}$$

$$\alpha = \frac{v_gk_z}{\omega_*} = \frac{\omega_{LH}^2}{\omega_0\omega_*}\frac{M}{m}\frac{k_{0x}{}^2k_{0z}k_z}{k_0{}^2(k_0{}^2+k^2)} \quad , \tag{32}$$

$$\delta = \frac{1}{2}\frac{\omega_{LH}^2}{\omega_0\omega_*}\frac{M}{m}\frac{(k_{0x}{}^2k_z{}^2-k_{0z}{}^2k_y{}^2)}{k_0{}^2(k_0{}^2+k^2)} \quad , \tag{33}$$

where $\Omega=\omega/\omega_*$, $\beta^2=v_A{}^2k_z{}^2/\omega_*{}^2$. For simplisity, we assumed $T_e=T_i$ and we also introduced the natural damping of the slow wave γ_1 and the pump wave γ_2 in a phenomenological manner.

§4. Modulational Instabilities

Writing $\Omega=x+iy$ we can separate equation (30) into its real and imaginary parts

$$x^2 - y^2 - \gamma_1 y - x - \beta^2$$
$$= \Gamma(x-\alpha)\,\frac{[(x-\alpha)^2 + y^2 - \gamma_2{}^2 - \delta^2]}{F(x,y)}\quad , \tag{34}$$

$$2xy + \gamma_1 x - y$$
$$= -\,\Gamma\,\frac{\{2\gamma_2[(x-\alpha)^2+y^2]+(x-\alpha)^2 y + y^3+y\gamma_2{}^2+y\delta^2\}}{F(x,y)}, \tag{35}$$

where

$$F(x,y) = [(x-\alpha)^2-(y+\gamma_2)^2-\delta^2]^2 + 4(x-\alpha)^2(y+\gamma_2)^2 \quad . \tag{36}$$

It is straightforward to obtain an estimate for the threshold power ($\Gamma=\Gamma_e$, $y\to 0$) from the above equations. Then we get

$$\Gamma_c = -\frac{\gamma_1 x_c}{2\gamma_2(x_c-\alpha)^2}\{[(x_c-\alpha)^2 - \gamma_2{}^2 - \delta^2]^2 + 4(x_c-\alpha)^2\gamma_2{}^2\}, \tag{37}$$

with the critical frequency x_c given by the relation

$$x_c{}^2 - x_c - \beta^2 = -\frac{\gamma_1 x_c}{2\gamma_2(x_c-\alpha)}\{(x_c-\alpha)^2-\gamma_2{}^2 - \delta^2\} \quad . \tag{38}$$

The threshold expression (37) can be further minimized with respect to δ to obtain the minimum power necessary for parametric instability. This occurs for $\partial\Gamma_c/\partial\delta = 0$, which yields

$$x_c \approx \alpha - \delta \quad . \tag{39}$$

This is the condition for the resonance backscattering instability to occur, which was discussed in the previous papers (Sanuki et al., 1977). Then the minimized threshold is given as

$$\Gamma_m = -\frac{\gamma_1\gamma_2 x_c}{2} \quad . \tag{40}$$

One can express eq.(40) in terms of a minimum field amplitude E_m as

$$\frac{E_m^{\ 2}}{4\pi N_0 T} \simeq \frac{1}{2}\,\frac{M}{m}\,\frac{\gamma_1\gamma_2}{\omega_{LH}^{\ 2}}\left(\frac{v_A}{c}\right)^2\left(\frac{v_A}{c_s}\right)^2 \tag{41}$$

where we used the approximate relation $x_c \simeq 1+\dfrac{v_A^{\ 2}k_z^{\ 2}}{\omega_*^{\ 2}}$ derived from eqs.(38) and (39).

Just above the minimum threshold equations (34) and (35) can be linearized to give in the case of $\omega>\omega_*$ and $k_z^{\ 2}/k_y^{\ 2} > \omega_0^{\ 2}/\Omega_e^{\ 2} \gtrsim m/M$

$$\Delta x \simeq \begin{cases} \dfrac{\gamma_1^{\ 2}}{16x_c}\,\dfrac{\Delta\Gamma}{|\Gamma_m|} & \text{for} \quad \gamma_1/\gamma_2 \ll 1 \,, \qquad (42) \\[2ex] \dfrac{4\gamma_2^{\ 2}}{x_c}\,\dfrac{\Delta\Gamma}{|\Gamma_m|} & \text{for} \quad \gamma_1/\gamma_2 \gg 1 \,, \qquad (43) \end{cases}$$

$$y \simeq \begin{cases} \dfrac{\gamma_1^{\ 2}}{64\gamma_2}\,\dfrac{\Delta\Gamma}{|\Gamma_m|} & \text{for} \quad \gamma_1/\gamma_2 \ll 1 \,, \qquad (44) \\[2ex] \gamma_2\,\dfrac{\Delta\Gamma}{|\Gamma_m|} & \text{for} \quad \gamma_1/\gamma_2 \gg 1 \,, \qquad (45) \end{cases}$$

where $\Delta x = x - x_c$ is the frequency shift in the real part of Ω, y the growth rate and $\Delta\Gamma = \Gamma - |\Gamma_m|$.

Thus we conclude that there is an upper limit of the electric pump field for driving a drift Alfvén wave unstable. The maximization arises because the growth rate y increases with $\Delta\Gamma$ and the real part of frequency x decreases with $\Delta\Gamma$, so that the damped nature of the drift Alfvén wave at the large Γ is a direct consequence of the change of the sign of $\Delta\Gamma$ at $\Gamma \approx \Gamma_m$. These results are qualitatively similar to the theoretical results obtained for modulational instabilities corresponding to the parametric decay of a plane electromagnetic wave into a drift Alfvén wave (Bujarbarua and Kaw, 1976) and for the parametric excitation of a drift wave by ion-ion hybrid wave (Satya et al., 1975).

§5. Conclusion

The treatment employed in the present paper is similar to the one used in the previous study of the parametric instabilities in the homogeneous magnetized plasma (Sanuki and Schmidt, 1977). We derived a dispersion relation which describes the parametric decay instability of a lower hybrid wave into an electrostatic drift wave and a drift Alfvén wave. Particularly the stimulated scattering of a drift Alfvén wave in an inhomogeneous magnetized plasma has been investigated in detail. We have also estimated the threshold and the growth rate for a modulational instabilities of this mode. It is shown that the minimum threshold is given for the resonance backscattering instability. It turns out that there is an upper limit of the pump wave for driving a drift Alfvén wave unstable. These results are qualitatively in agreement with theoretical ones obtained by Bujarbarua and Kaw and Satya et al.

As mentioned in the paper by Sundaram and Kaw, one important question in applying the parametric instabilities of heating confined fusion plasmas will be possible excitation of drift waves, which may, however, possibly lead to enhanced plasma loss. These problems may be governed by the process discussed in the present paper.

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