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EXPERIMENTAL STUDY ON SELF-MODULATION OF ION PLASMA OSCILLATIONS

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Abstract

Experimental study on the self-modulation of externally amplitude-modulated ion plasma oscillations is described. Oscillations were excited by applying an rf voltage with a frequency near the ion plasma frequency to a grid being a wall of the plasma container. The envelopes of such oscillations were observed to break up into several spikes if their amplitudes were above a value. Individual spikes thus formed were found to have similar properties to those of envelope solitons, although the spikes were observed not to move in the laboratory frame. The number of spikes per group was observed to depend strongly on various parameters. Frequency spectra of excited oscillations were studied over various amplitudes of the applied rf voltage. Comparison of these experimental results with theoretical ones indicates that the self-modulation of oscillations observed here occurs due to the modulational instability.

§1. Introduction

General theories on the modulational instability of waves in nonlinear dispersive media have been presented by Karpman and Krushkal[1], and Taniuti and Yajima[2]. Possibilities of the modulational instability in ion plasma modes have been theoretically investigated by Shimizu and Ichikawa[3], Kakutani and Sugimoto[4], Chan and Seshadri[5], and Ichikawa and Taniuti[6]. Chan et al.[5] have indicated, taking account of finite ion temperature and nonvanishing inertia of electrons, that the ion acoustic waves are modulationally stable, but the ion plasma oscillations can be unstable if the wavenumber of the oscillation is larger than a critical one. In this case, however, unstable oscillations can be usually realized for wavenumbers larger than the Debye wavenumber ($k > k_D$), in the region of which the linear Landau damping is expected to be very strong. On the other hand, Ichikawa et al.[6] have shown that the modulational instability can occur even in the region of ion waves corresponding to $k < k_D$ because of contributions of the resonant particles at the group velocity, even if the linear Landau damping of the waves is a higher order effect. Now we have two similar experiments on the self-modulation of ion wave packets made by Ikezi and Schwarzenegger[7], and Watanabe[8]. The former have described that the dominant process causing self-modulation is not the modulational instability but the phase velocity shift due to trapped ions. The latter has explained his experimental results in terms of the modulational instability.

In this paper we want to describe our experimental results on the self-modulation of the ion plasma oscillations. In the previous report of our preliminary experiment[9], which was made using a double-plasma device (DP device), we showed that envelope solitons could be formed in the ion plasma modes. However, a subsequent study let us to know that a density-modulated ion beam, whose modulation was made in its source region (corresponding to the driver plasma region), caused pseudo envelope solitons with the same propagating speed as that of the beam. Therefore, in the present experiment we use not a double-plasma but a single-plasma for studying the self-modulation of oscillations in a plasma. Many data on the properties of the self-modulation will be presented here.

§2. Apparatus and Experimental Method

Experiments were carried out by the use of a simple device as shown in Fig.1, which consists of a small chamber (anode), many filaments and a mesh grid G. This system was put in a large space chamber of 50 cm in diameter and 80 cm long, which was grounded. A plasma was produced by a discharge initiated by energetic electrons injected from filaments and had typical parameters as follows; electron temperature $T_e \simeq 0.5\sim1.5$ eV, ion temperature $T_i \simeq 0.2\sim0.4$ eV and plasma density $n_e \simeq 1\sim30 \times 10^8$ cm^{-3}. The space potential of the plasma was also observed to be about 0 V, when both the anode and the grid were grounded. The plasma density could be easily controlled by varying the surface-temperature of the filaments. In most cases argon gas was used at a pressure around 3×10^{-4} Torr. Under such a condition the mean free paths for ion-neutral collisions, which are predominant, are estimated to be comparable to the size of the chamber.

In order to excite ion plasma oscillations in such a plasma we applied to the grid G a sinusoidal rf voltage with a maximum amplitude up to 20 V and a frequency from 400 to 1000 kHz, while a dc voltage from - 30 to + 30 V was applied to the anode. As seen in our preliminary experiment[9], we could also excite the same type of oscillations by applying an rf voltage to the anode and a negative dc voltage to the grid. The externally applied rf voltage was amplitude-modulated with a frequency in the range from 0.5 to 30 kHz and a rate of modulation up to 85 %. Detection of the excited oscillations was made from electron saturation current received by an axially movable Langmuir probe. As the electron and ion plasma frequencies for the plasma used here are estimated to be about 100 MHz and 0.5 MHz respectively, all the excited oscillations are thought to belong to the ion plasma modes.

§3. Experimental Results

Although we externally applied to the grid G rf voltages with amplitude as large as 10 V for exciting ion plasma oscillations, the amplitudes of oscillations actually excited in a plasma were observed to be of the order of $\tilde{n}/n_o \lesssim 10$ %. Of course, no oscillation could be observed without a plasma. Phase differences between two different points in the plasma or a remarkable spatial change in amplitude of the oscillation could be hardly measured. These measurements show that the oscillation has a too slow velocity to be detected. From this result and the fact that the frequency of oscillation is comparable with the ion plasma frequency we conclude that observed oscillations belong to the ion plasma oscillations.

A. Self-modulation of oscillations

Under a suitable condition the envelope of an ion plasma oscillation was observed to break up into several spikes because of some nonlinear effect, for example, the modulational instability. The breaking up of the envelope (self-modulation) always occurred in the vicinity of the position, where the amplitude of the oscillation should be maximum unless the self-modulation did occur. The spikes were formed in a group. (see Fig.2) A way for the envelope of an oscillation to break up into spikes depends on the amplitude of the applied rf voltage. (see Fig.11) If the amplitude of an oscillation is small enough, then the breaking up of its envelope can not occur even if the rate of amplitude-modulation of the oscillation is as large as 80 %. On the other hand, if the amplitude of the oscillation is large enough, the breaking up is possible even if the rate of modulation is small.

Using a frequency analyzer, we could obtain spectra as shown in Fig.3, in which only the amplitude A of the main oscillation (frequency f_{RF} = 700 kHz) was changed by changing the one of the applied rf voltage. In this figure it is noted that, although there exists only the main peak (f_{RF} = 700 kHz) for small values of A (Fig.3(a)), a new broad peak appears near $f_{SP} \simeq$ 600 kHz for larger values of A (Fig.3(b) or (c)). The frequency f_{SP} gives the mean frequency of the spikes. From such data we can know that the frequency difference $\Delta f = f_{RF} - f_{SP}$ changes as a function of A, as demonstrated in Fig.4. Here, if we put $\Delta f \propto A^n$, then the exponent n is nearly equal to unity for small A and to two for larger A.

B. Properties of individual spikes

A group of the spikes was observed to include individual ones of various shapes with different amplitude and width. For these individual spikes we can say that the larger amplitude has a spike, the narrower is its width. Relation between the amplitudes and widths of these individual spikes is shown in Fig.5. This indicates that the reciprocal of the width of a spike is nearly proportional to its amplitude. The average interval between adjoining individual spikes was found to change with the change of several parameters. Relation between the average interval and the rf frequency is shown in Fig.6. It was also observed that the average interval between spikes strongly depended on the change in amplitude of the applied rf voltage.(see Fig.7) However, the average interval was found to be insensitive to the change of the modulation frequency.

C. Dependences of the number of spikes on various parameters

The number of spikes per group was found to depend on the frequency and amplitude of the rf voltage, the modulation frequency, plasma parameters and so forth. If we select suitable parameters, a large number of spikes, as seen in Fig.8, can be formed in a group. Observed relation between the number of spikes and the rf frequency for various values of the modulation frequency are demonstrated in Fig.9. This figure tells us that a larger number of spikes are formed for a lower modulation frequency and that for a fixed modulation frequency more spikes are obtained for a higher rf frequency. Observation also indicated that the number of spikes increased roughly in proportion to the period of the modulation for a fixed rf frequency. (Fig.10) By varying the amplitude of the externally applied rf voltage it was found that the number of spikes changed roughly in proportion to the amplitude of the rf voltage. (Fig.11) Moreover, we could observe that, even if the frequencies and amplitudes of the rf voltage and of the modulation were kept constant, the spike number could be changed by the changes of plasma parameters such as plasma density, electron temperature and plasma potential.

§4. Discussions

I. Self-modulation due to the modulational instability

According to the theory by Ichikawa and Taniuti[6], who take into account the contribution of resonant particles having the group velocity V_g, the evolution of the envelope of a plane wave in a collisionless plasma can be treated by using a modified nonlinear Schrödinger equation such as

$$i\frac{\partial\phi}{\partial\tau} + p\frac{\partial^2\phi}{\partial\xi^2} + q|\phi|^2\phi + s\,\frac{P}{\pi}\int\frac{|\phi(\xi',\tau)|^2}{\xi-\xi'}\,d\xi'\phi = 0, \qquad (1)$$

where $\phi(\xi,\tau)$ is the small but finite complex amplitude, and P denotes a Cauchy principal part. ξ and τ are stretched variables in the reductive perturbation theory[10] and, when a small parameter ε is introduced, expressed as a function of distance x and time t as follows;

$$\xi = \varepsilon(x - V_g t) \quad\text{and}\quad \tau = \varepsilon^2 t. \qquad (2)$$

Using the expression for the complex amplitude ϕ (Taniuti and Yajima[2])

$$\phi(\xi,\tau) = \rho^{1/2}\exp\{i\textstyle\int\sigma d\xi/2p\}, \qquad (3)$$

we have a set of coupled equations for the real functions ρ and σ instead of eq.(1). These coupled equations are linearized by the use of perturbation expansions

$$\begin{aligned}\rho &= \rho_0 + \rho_1 \exp\{i(K\xi-\Omega\tau)\}, \\ \sigma &= \sigma_0 + \sigma_1 \exp\{i(K\xi-\Omega\tau)\},\end{aligned} \tag{4}$$

and give the dispersion relation for perturbation as follows, (Ichikawa[11])

$$\{(\Omega-K\sigma_0)^2 + 2pq\rho_0 K^2 - p^2K^4\}^2 = -(2ps\rho_0K^2)^2. \tag{5}$$

In order to have a solution of eq.(5), we put the frequency Ω as

$$\Omega = \Omega_r + i\Gamma . \tag{6}$$

We substitute eq.(6) for Ω in eq.(5) and obtain the following expressions

$$\Omega_r = K\sigma_0 \pm \frac{1}{\sqrt{2}}[\{(p^2K^2-2pq\rho_0)^2+(2ps\rho_0)^2\}^{1/2} + (p^2K^2-2pq\rho_0)]^{1/2}|K| , \tag{7}$$

$$\Gamma = \mp \frac{1}{\sqrt{2}}[\{(p^2K^2-2pq\rho_0)^2+ (2ps\rho_0)^2\}^{1/2} - (p^2K^2-2pq\rho_0)]^{1/2}|K| . \tag{8}$$

For a given value of ρ_0, if $pq > 0$, we can determine the wavenumber K_m as

$$K_m^2 = \frac{q^2 + s^2}{pq}\rho_0 , \tag{9}$$

which gives rise to the maximum growth rate for the low frequency side band. Hence, we have

$$\Omega_m = K_m\sigma_0 \pm \frac{s}{|q|}(q^2+s^2)^{1/2}\rho_0 , \tag{10}$$

$$\Gamma_m = \mp (q^2+s^2)^{1/2}\rho_0 , \tag{11}$$

for the wavenumber K_m given in eq.(9). These theoretical results indicate that the contribution of the resonant particles at the group velocity drives the carrier wave to modulational instability in both cases of $pq \gtrless 0$. Here, it should be noted that the low frequency side band grows ($\Gamma > 0$), while the high frequency side band damps ($\Gamma < 0$).

Next we apply the above theory to the nonlinear modulation of ion plasma modes in the case that electrons are isothermal and $T_e >> T_i$. In this case we have the coefficients in eq.(1) as

$$p = -\frac{\omega_i}{k_D^2}\left(\frac{k}{k_D}\right)\left\{1 + \left(\frac{k}{k_D}\right)^2\right\}^{-5/2} , \tag{12}$$

$$q = \omega_i\left(\frac{e}{\kappa T_e}\right)^2 Q\left(\frac{k}{k_D}, \frac{T_e}{T_i}\right) , \tag{13}$$

$$s = \omega_i\left(\frac{e}{\kappa T_e}\right)^2 S\left(\frac{k}{k_D}, \frac{T_e}{T_i}\right) , \tag{14}$$

where k is the wavenumber of the carrier wave, k_D the Debye wavenumber and $\omega_i = (4\pi e^2 n/M)^{1/2}$. Complicated functions Q in eq.(13) and S in eq.(14) have been numerically calculated as a function of wavenumber k/k_D in two cases of $T_e/T_i = 10$ and 20 by Ichikawa and Taniuti[6]. These results show that for a small wavenumber $k/k_D \lesssim 0.5$ the values of Q and S strongly depend on the temperature ratio T_e/T_i, while for $k/k_D \sim 1$ their values are little sensitive to T_e/T_i and

$$-Q \sim S \sim 20 \quad (\text{for } k/k_D \sim 1). \tag{15}$$

In our case as shown in Fig.3, where $T_e \simeq 1.2$ eV, $T_i \simeq 0.4$ eV (Ar) and $n_e \simeq 1.2 \times 10^9$ cm^{-3}, the Debye length λ_D is estimated to be about 2.3×10^{-2} cm (Debye wavenumber $k_D \simeq 43$ cm^{-1}), the ion plasma angular frequency $\omega_i \simeq 7.4 \times 10^6$ Hz and $T_e/T_i \simeq 3.0$. Therefore, in this case the linear Landau damping is expected to be very strong, in particular, for large wavenumbers

$k > k_D$. However, we assume that the above theory can be applied to the ion plasma oscillations ($k \sim k_D$), as observed in our experiment. For such an oscillation with a maximum amplitude A ($= \rho_0^{1/2}$), the coefficients in eq.(1) for $k \sim k_D$ are estimated from eqs.(12), (13), (14) and (15) as

$$p \simeq -7.0 \times 10^2 \quad [\mathrm{cm^2/s}], \qquad (16)$$

$$q \simeq -1.5 \times 10^8 \ (e/\kappa T_e)^2 \quad [\mathrm{1/V^2 s}], \qquad (17)$$

$$s \simeq 1.5 \times 10^8 \ (e/\kappa T_e)^2 \quad [\mathrm{1/V^2 s}]. \qquad (18)$$

Thus the wavenumber K_m is estimated as

$$K_m \simeq 6.5 \times 10^2 \ (eA/\kappa T_e)^2 \quad [\mathrm{cm^{-1}}], \qquad (19)$$

from eq.(9). The corresponding frequency difference $\Delta\Omega$ between the carrier wave and the modulational instability is

$$\Delta\Omega = |K_m \sigma_0 - \Omega_m| \simeq 2.1 \times 10^8 \ (eA/\kappa T_e)^2 \quad [\mathrm{Hz}] \qquad (20)$$

and the growth rate for the low frequency side band

$$\Gamma_m \simeq 2.1 \times 10^8 \ (eA/\kappa T_e)^2 \quad [\mathrm{1/s}]. \qquad (21)$$

As mentioned earlier, we experimentally observed the dependence of the frequency difference Δf on the amplitude A of the main oscillation (corresponding to the carrier wave in the above theoretical discussion), which indicated the relation $\Delta f \propto A^{2.05}$ for large values of A. This observed relation of Δf and A is approximately in agreement with the relation of $\Delta\Omega$ and A given by eq.(20). Further, we have an experimental result that $\Delta f \sim$ 130 kHz for $\tilde{n}/n_0 \simeq (eA/\kappa T_e) \sim 0.05$. (for example, see Fig.3(c)) Using eq.(20) we can estimate $\Delta\Omega/2\pi \simeq 83$ kHz for the same value of $\tilde{n}/n_0$. Since there is probably a considerable error in the observed value of $\tilde{n}/n_0$, this calculated value of $\Delta\Omega/2\pi$ may be regarded to be nearly equal to the observed value of Δf. It should be also noted that only the low frequency side band appears in observed frequency spectra, as expected from the above theory. Such agreements of experimental results with theoretical ones suggest that the self-modulation of oscillations observed here comes from the modulational instability, and that the wavenumbers of such oscillations are comparable to the Debye wavenumber, that is, $k \sim k_D$.

II. <u>On the properties of individual spikes</u>

From our experiment we know that the envelope of a modulationally unstable ion plasma oscillation is broken up into several spikes under a suitable condition. The individual spikes thus formed have similar properties to those of envelope solitons. The widths of envelope solitons are theoretically known to change in proportion to the reciprocal of their amplitudes[1]. Although individual spikes observed here do not propagate in the laboratory frame unlike such solitons, it is experimentally found that the temporal widths of spikes change approximately in proportion to the reciprocal of their amplitudes.(see Fig.5)

Next, we consider the average temporal interval between adjoining individual spikes. Fom the theory[6] it is expected that the average wavenumber of wave packets (envelope solitons) formed due to the modulational instability may be determined by K_m in eq.(9) for a given amplitude $\rho_0^{1/2}$. Here, K_m has a value smaller than the wavenumber k of the carrier wave. So we can say that the average wavenumber of the wave packets may be affected by the rf frequency and plasma parameters as well as by the amplitude $\rho_0^{1/2}$, because the rf frequency and plasma parameters have effects on the values of the coefficients p and q. In our experiment the self-modulation of ion plasma oscillations was observed to give rise to several spikes instead of envelope solitons. For these spikes we found that the average interval between spikes (corresponding to the average wavelength of solitons in the case of envelope solitons) strongly depended on the amplitude and frequency of the oscillation. Dependence of the average interval on the rf frequency is illustrated in Fig.6, which shows that the average interval becomes narrower with increasing frequency. Fig.7 indicates that the reciprocal of the average interval is roughly proportional to the amplitude of the applied rf voltage. If we can regard the amplitude A $(= \rho_0^{1/2})$ of the main oscillation to be proportional to that of the applied rf voltage, then this relation may be anticipated from eq.(9).

III. Threshold in amplitude for the self-modulation

Relation between the number of spikes and the amplitude of the applied rf voltage (Fig.11) directly indicates the existence of a threshold in amplitude of the oscillation for the occurrence of the self-modulation, because the amplitude of the oscillation increases with the increase of the rf amplitude. An observed result that the number of spikes increases roughly in proportion to the modulation period seems to indirectly tell us that there is a threshold in amplitude of the oscillation for the self-modulation to occur. If the threshold is denoted by the voltage V_c, the oscillation would be modulationally unstable in the region, where the amplitude is above V_c. On the other hand, the width of the unstable region may change nearly in proportion to the modulation period, when the modulation frequency is changed. Therefore, considering that the average interval between spikes is hardly affected by the change of the modulation frequency, we come to the conclusion that the number of spikes may increase proportionally to the modulation period. Fig.11 also shows that the threshold value decreases with the increase of the oscillation frequency (equal to the rf frequency).

Such experimental results on the threshold in amplitude can be obtained, but they are not easy to be theoretically explained. So far as we use theories based on eq.(1), the estimation of the threshold value and the explanation of its dependence on the oscillation frequency seem to be not possible.

§5. Conclusion

In this paper we describe observational results on the self-modulation of ion plasma oscillations in a collisionless plasma. Under a suitable condition the envelope of such an oscillation was observed to break up into several spikes. Individual spikes thus formed were found to have similar properties to those of envelope solitons predicted from the theory, though the spikes did not move unlike the solitons. Observations showed that the number of spikes per group was determined by various parameters. We also observed frequency spectra of the excited oscillations, which indicated the relation between the frequency difference Δf ($=f_{RF} - f_{SP}$) and the oscillation amplitude A. Here, we compare our experimental results with theoretical ones, and come to the conclusion that the self-modulation of ion plasma oscillations is caused by the modulational instability.

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Figure Captions

Fig.1. Schematic picture of apparatus.

Fig.2. Typical photograghs showing that the envelope of an oscillation breaks up into various number of spikes depending on the frequency of the oscillation. Here the amplitude of the applied rf voltage and the modulation frequency are kept constant. In each photogragh the upper trace is the applied rf signal and the lower the signal of oscillation observed by a probe.

Fig.3. Frequency spectra of excited oscillations. Amplitude of the main oscillation at f_{RF} = 700 kHz becomes larger from bottom (a) to upper (c).

Fig.4. Relation between the frequency difference $\Delta f = f_{RF} - f_{SP}$ and the amplitude A of the main oscillation, obtained from such spectra as shown in Fig.3.

Fig.5. Amplitude versus width for individual spikes.

Fig.6. Average interval between spikes versus the rf frequency.

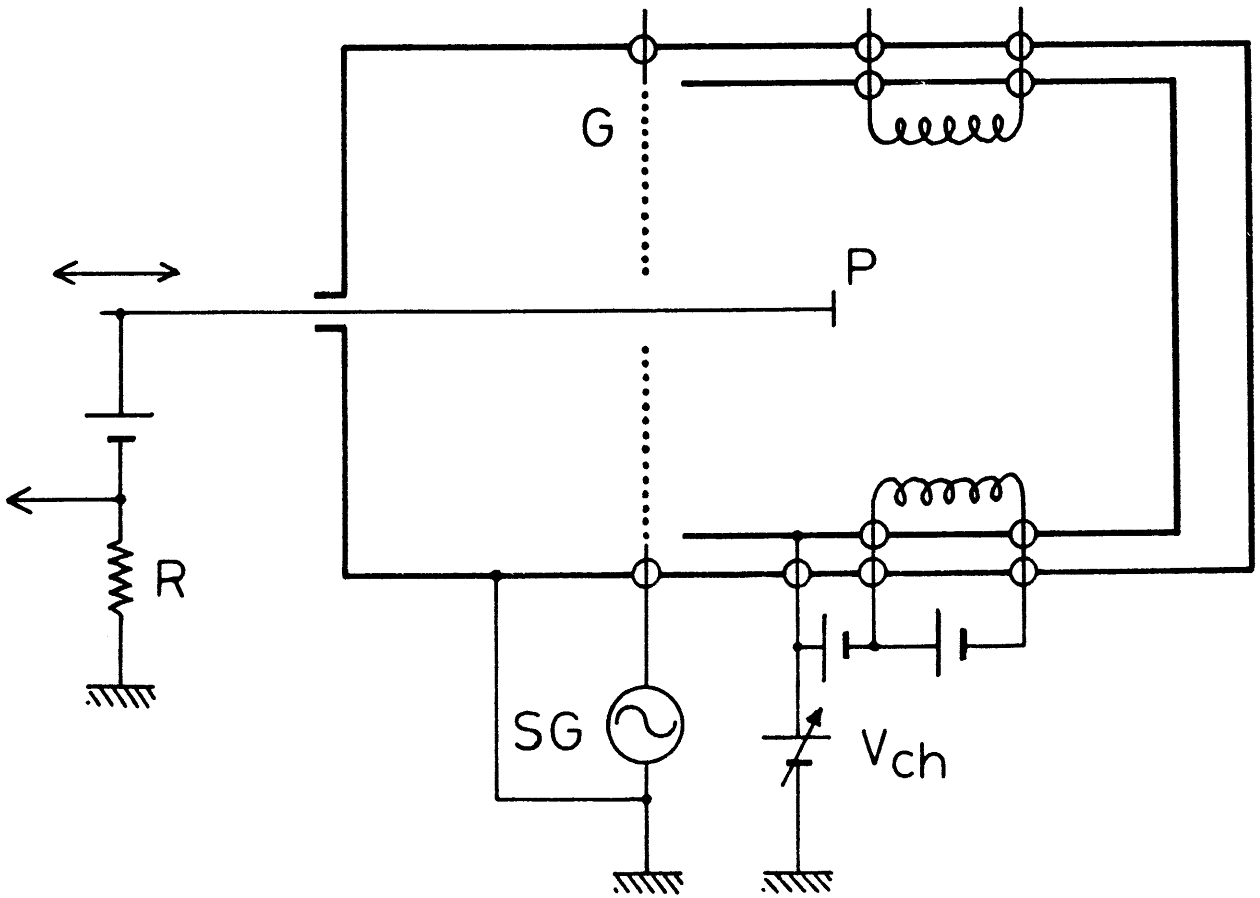
Fig.7. Average interval between spikes versus the amplitude of the rf voltage.

Fig.8. Examples of the envelopes of oscillations showing many spikes (lower), and those of the applied rf voltages (upper).

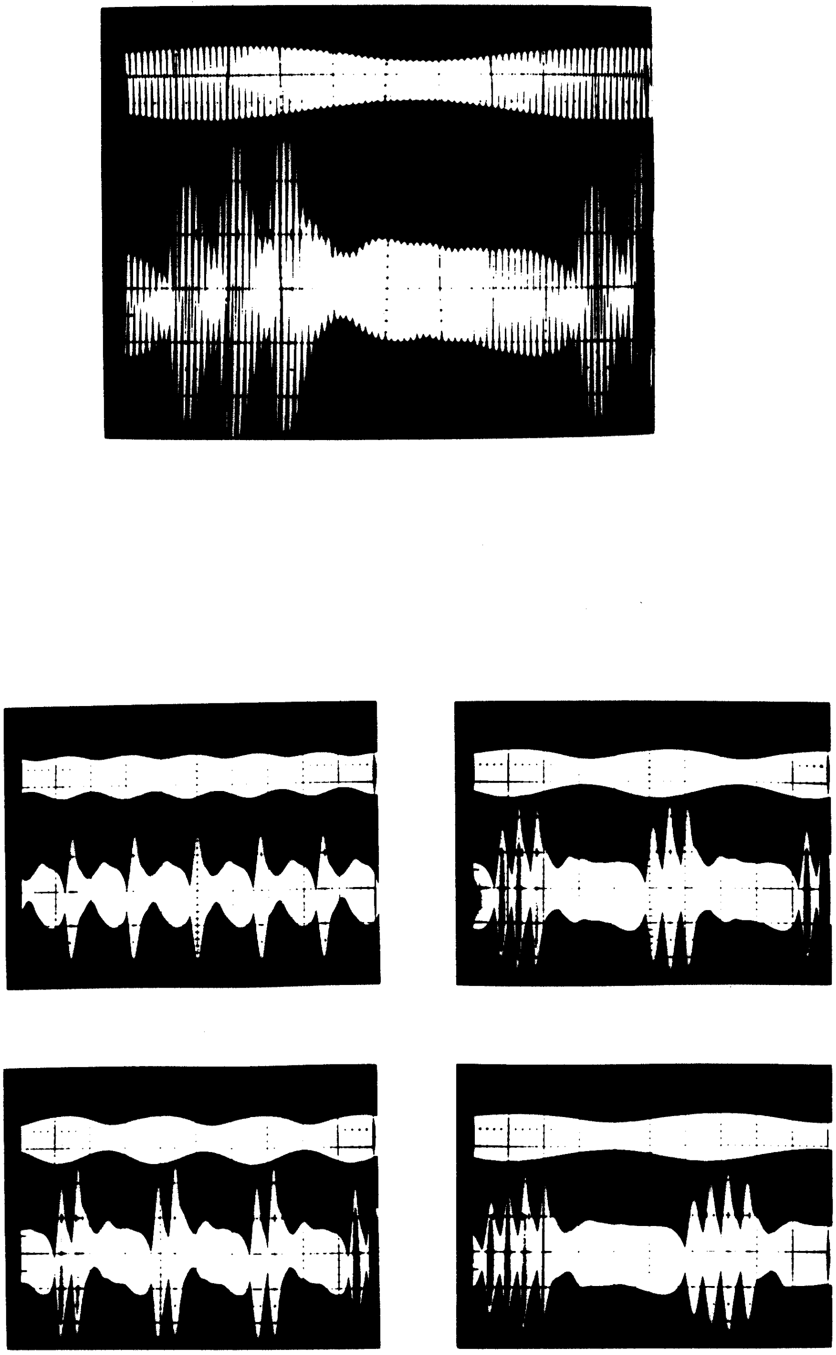
Fig.9. Number of spikes per group versus the rf frequency for various values of the modulation frequency.

Fig.10. Number of spikes versus the modulation period.

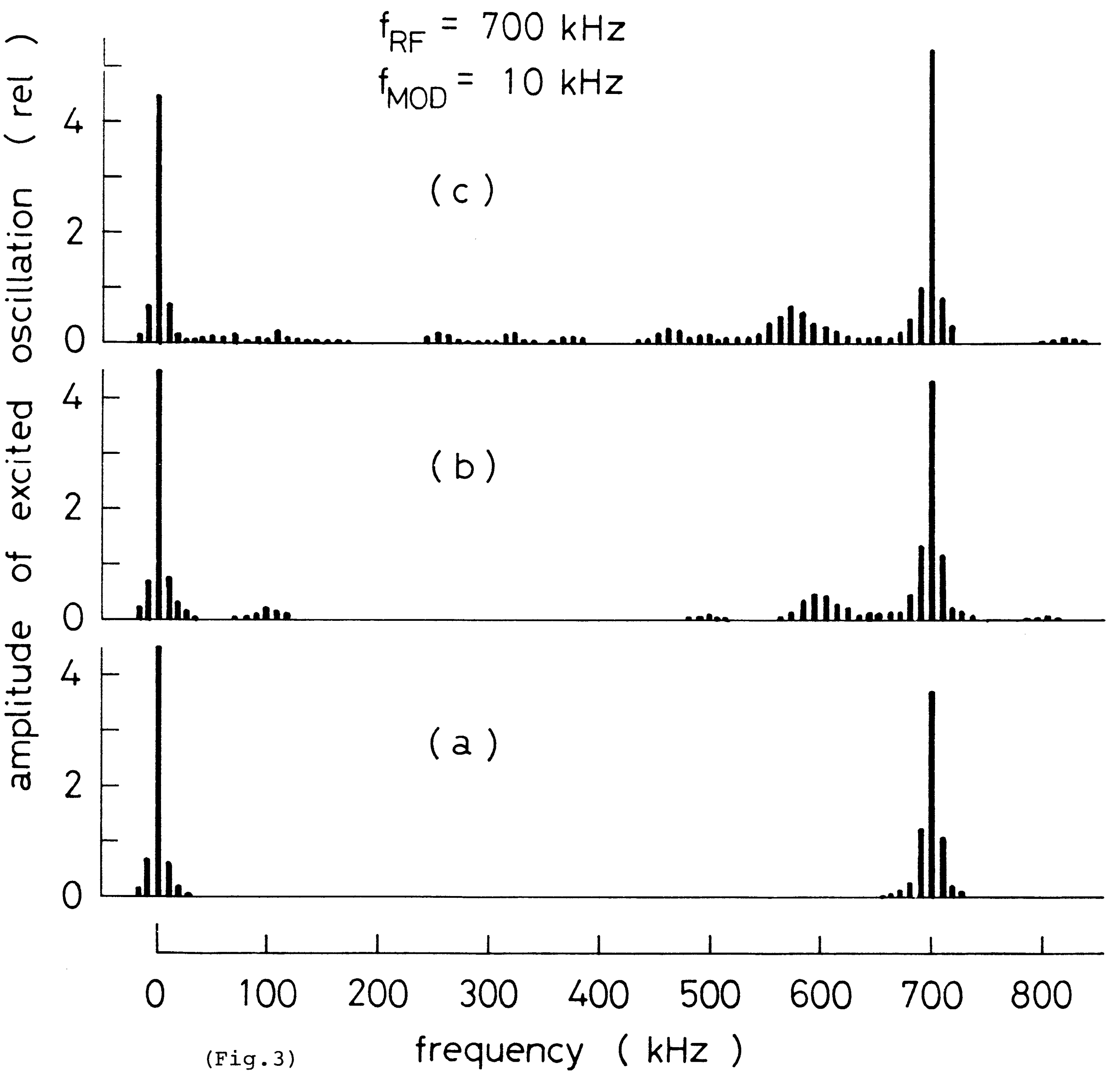
Fig.11. Number of spikes versus the amplitude of the rf voltage.



(Fig.1)

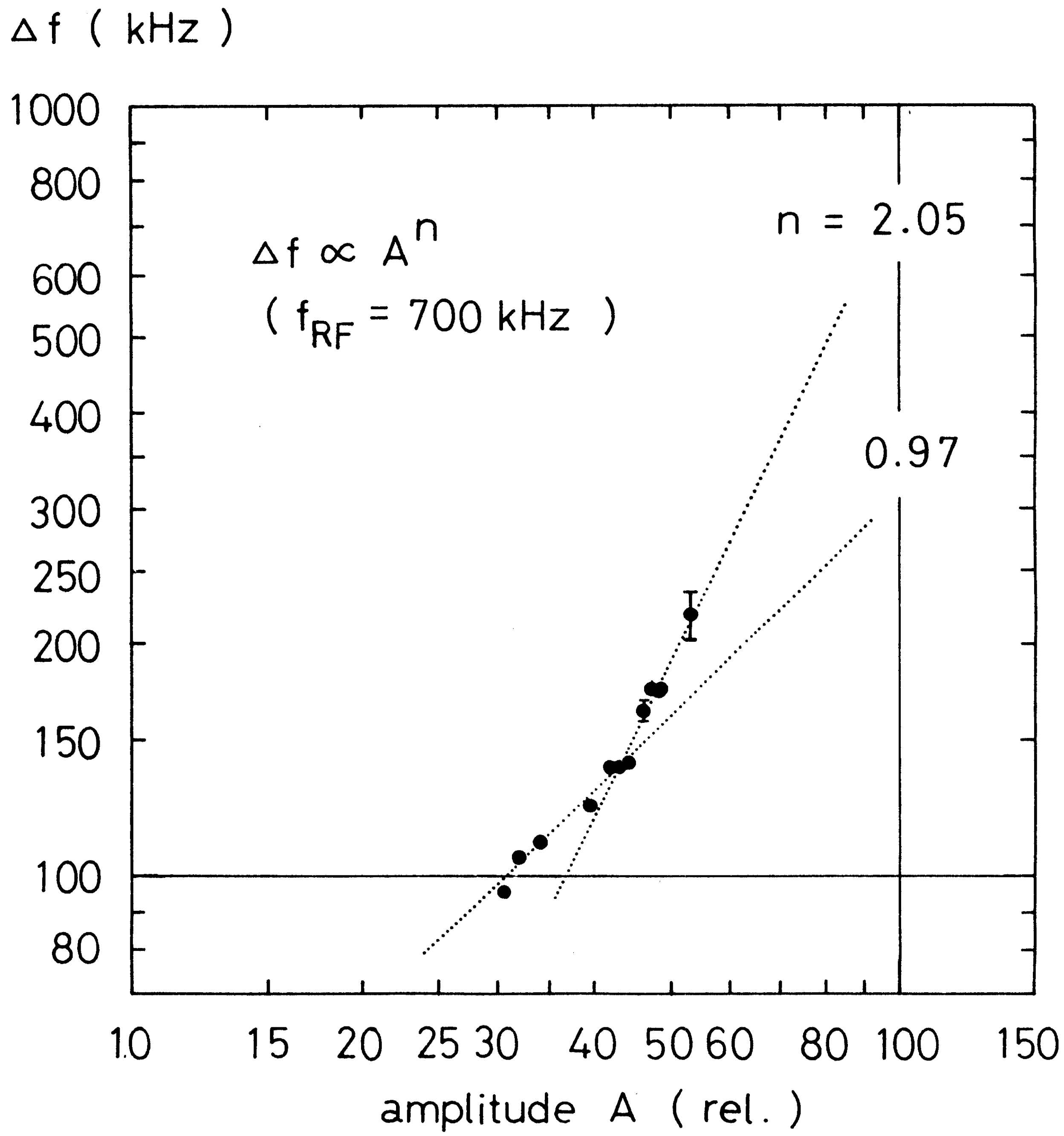


(Fig.2)

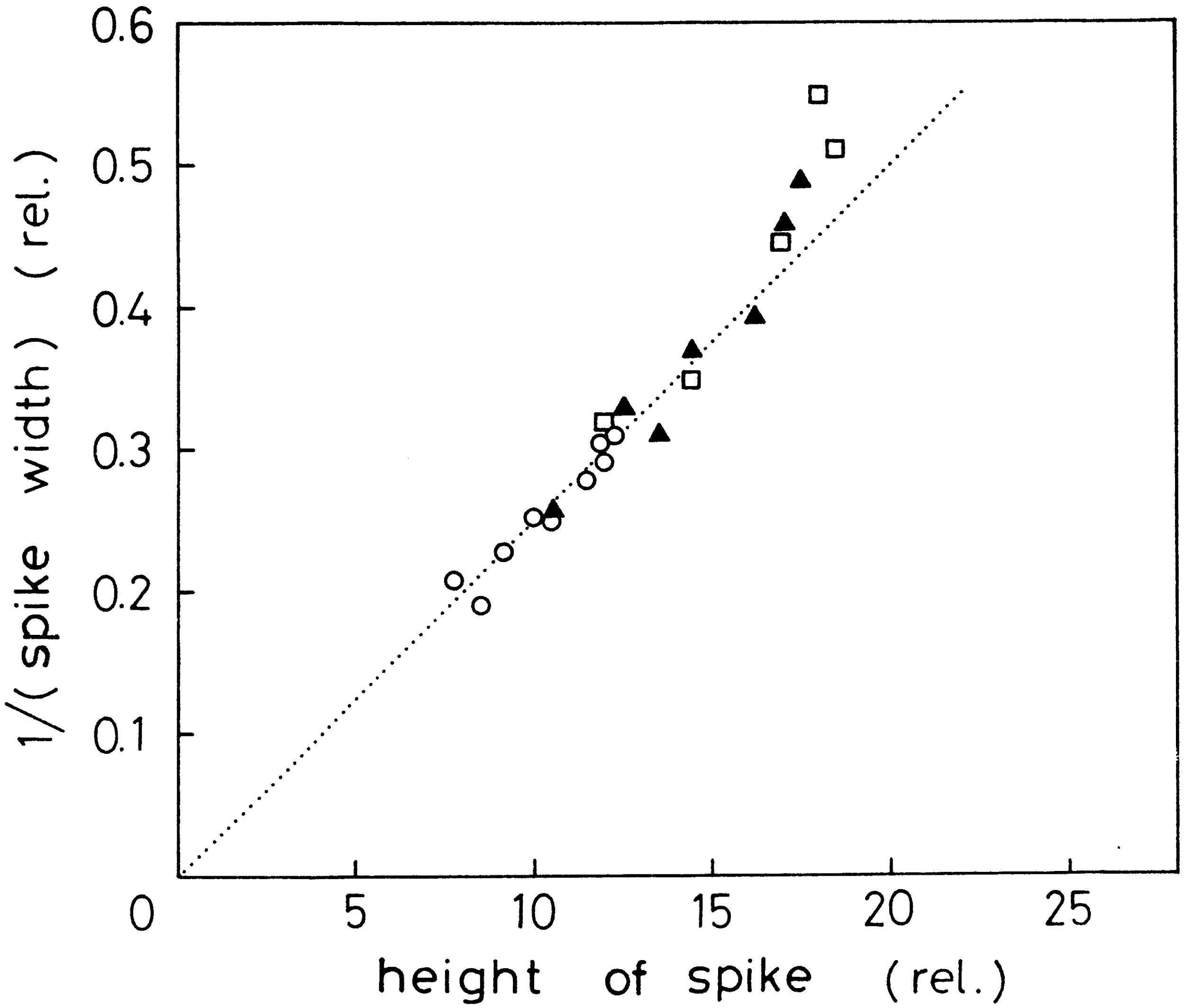


f_{RF} = 700 kHz
f_{MOD} = 10 kHz
(c)
(b)
(a)
amplitude of excited oscillation (rel)
0
2
4
0
100
200
300
400
500
600
700
800
frequency (kHz)

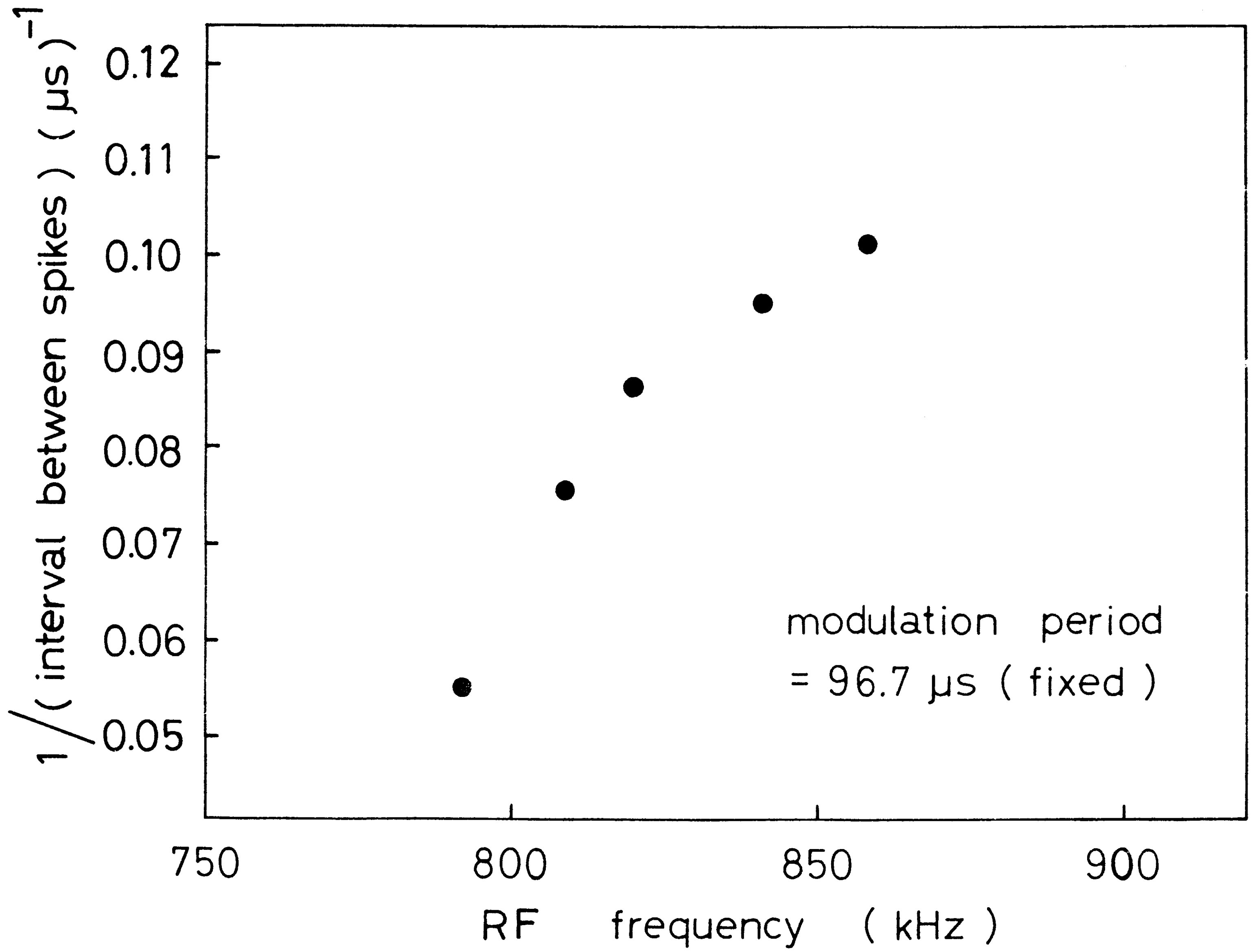
(Fig.3)


Δf (kHz)
1000
800
600
500
400
300
250
200
150
100
80
Δf ∝ A^n
(f_RF = 700 kHz)
n = 2.05
0.97
1.0
15
20
25
30
40
50
60
80
100
150
amplitude A (rel.)

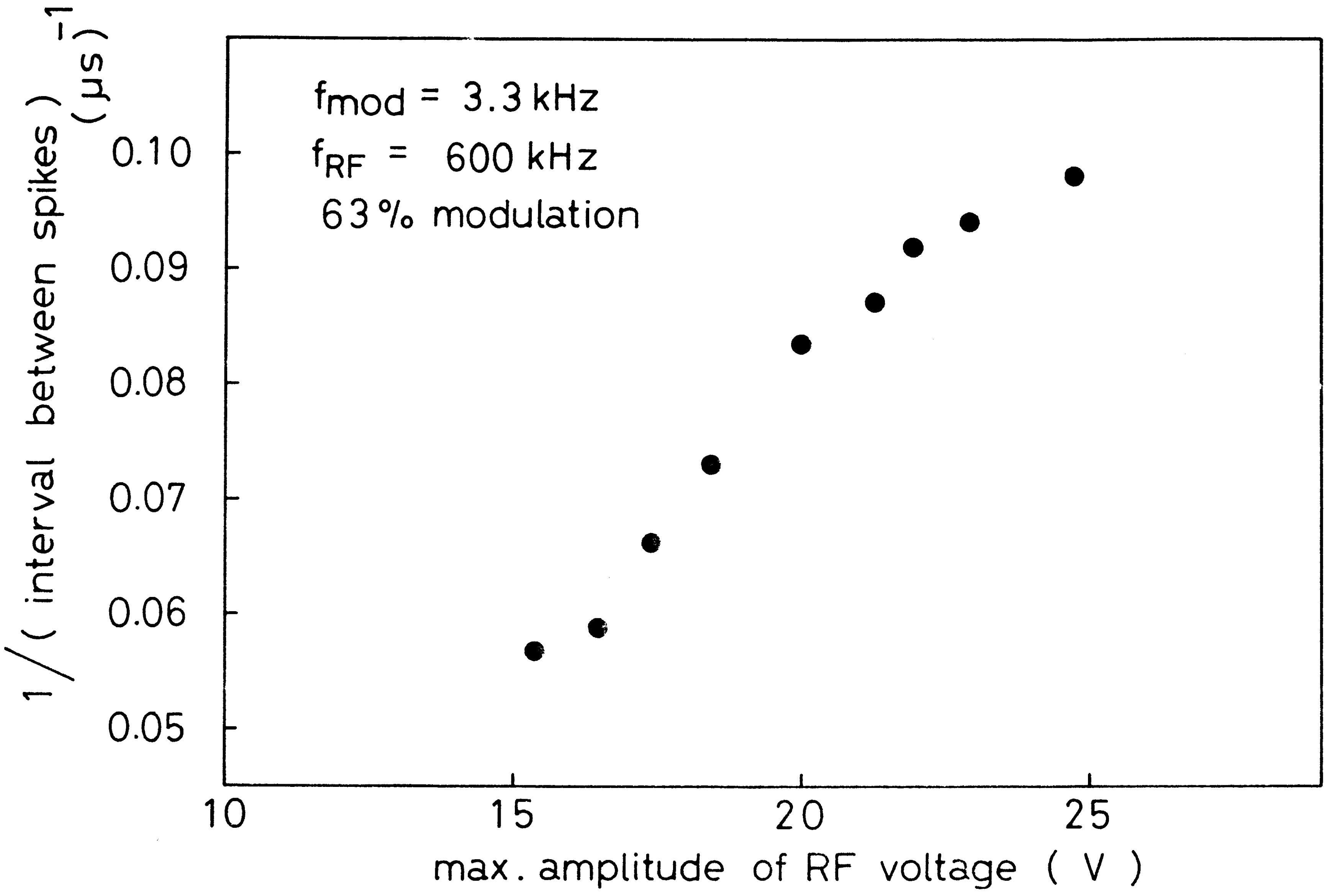
(Fig.4)


0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
1/(spike width) (rel.)
height of spike (rel.)

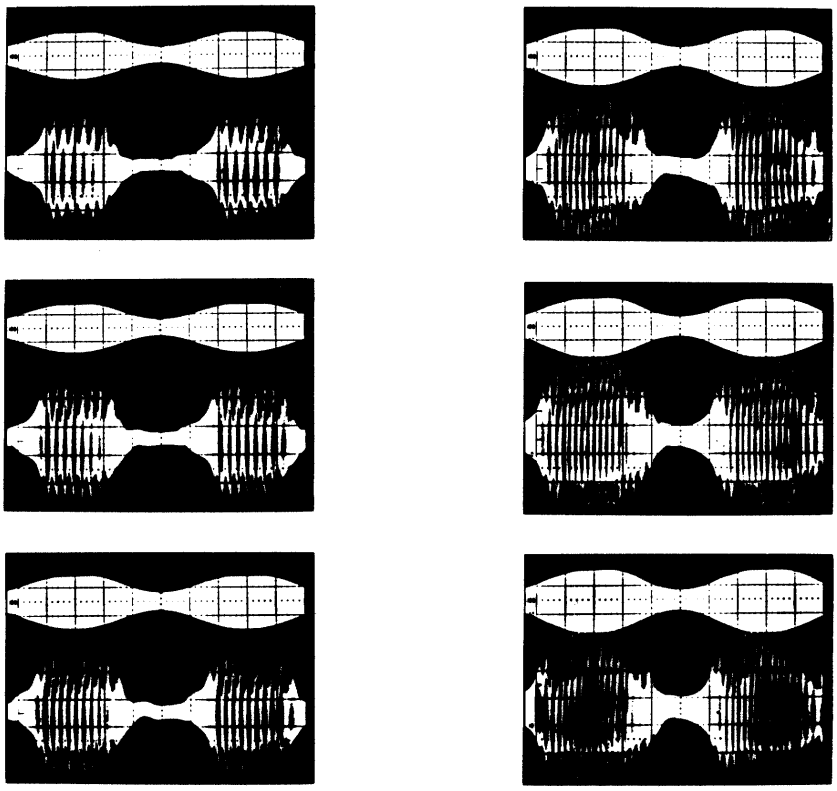
(Fig.5)


1/(interval between spikes) (μs)^-1
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
750
800
850
900
RF frequency (kHz)
modulation period
= 96.7 μs (fixed)

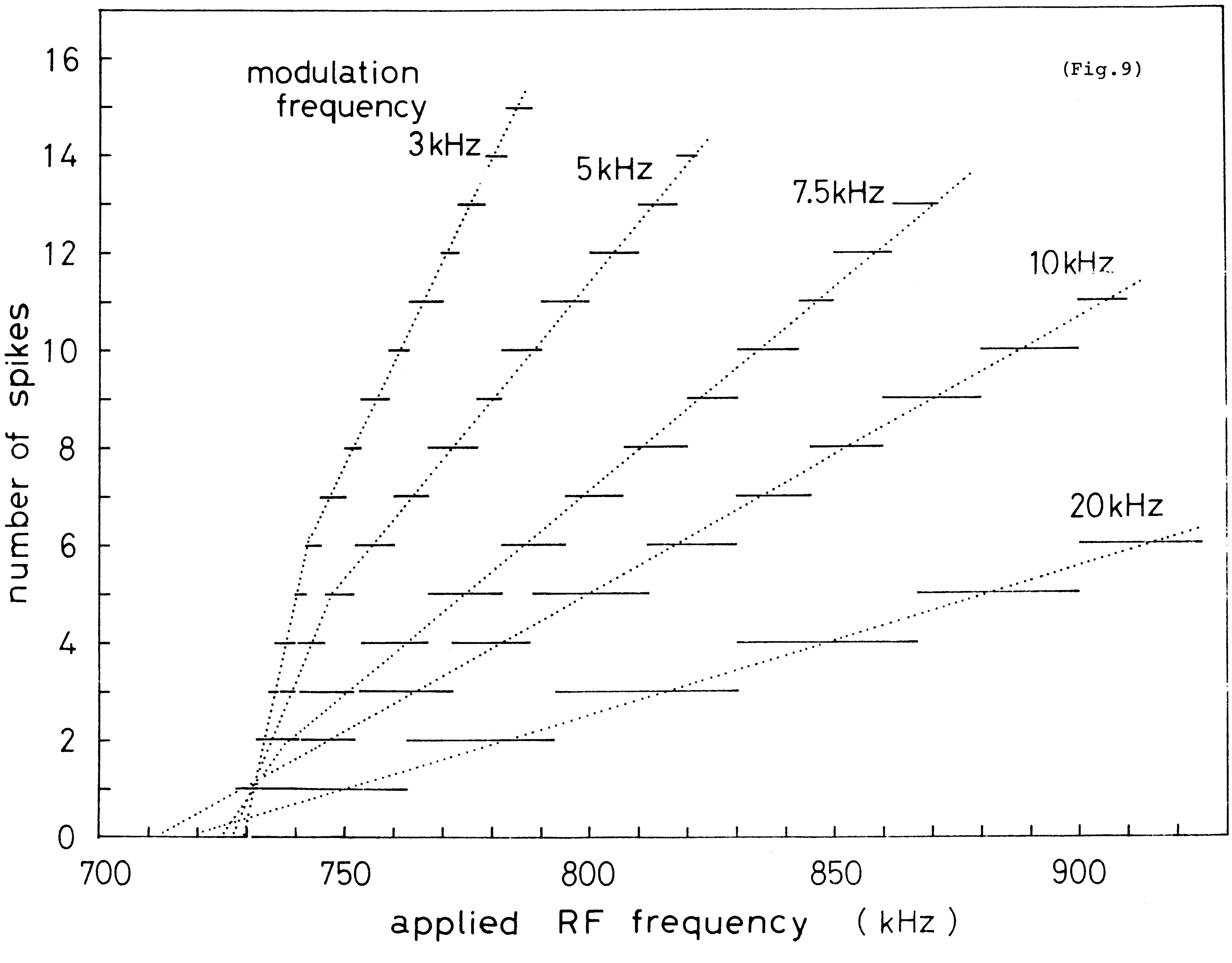
(Fig.6)

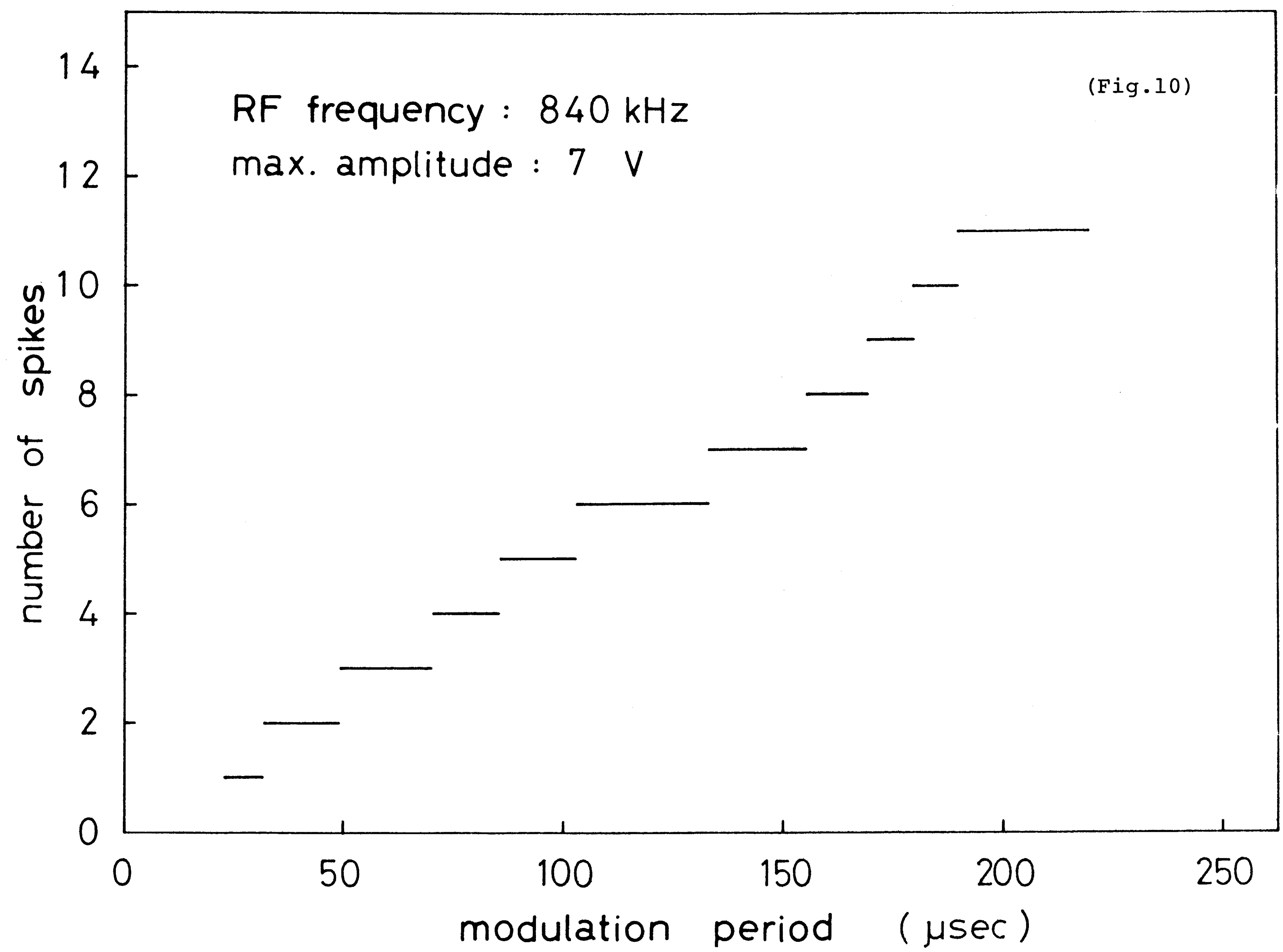

fmod = 3.3 kHz
fRF = 600 kHz
63 % modulation
0.10
0.09
0.08
0.07
0.06
0.05
1 / (interval between spikes) (μs)-1
10
15
20
25
max. amplitude of RF voltage (V)

(Fig.7)

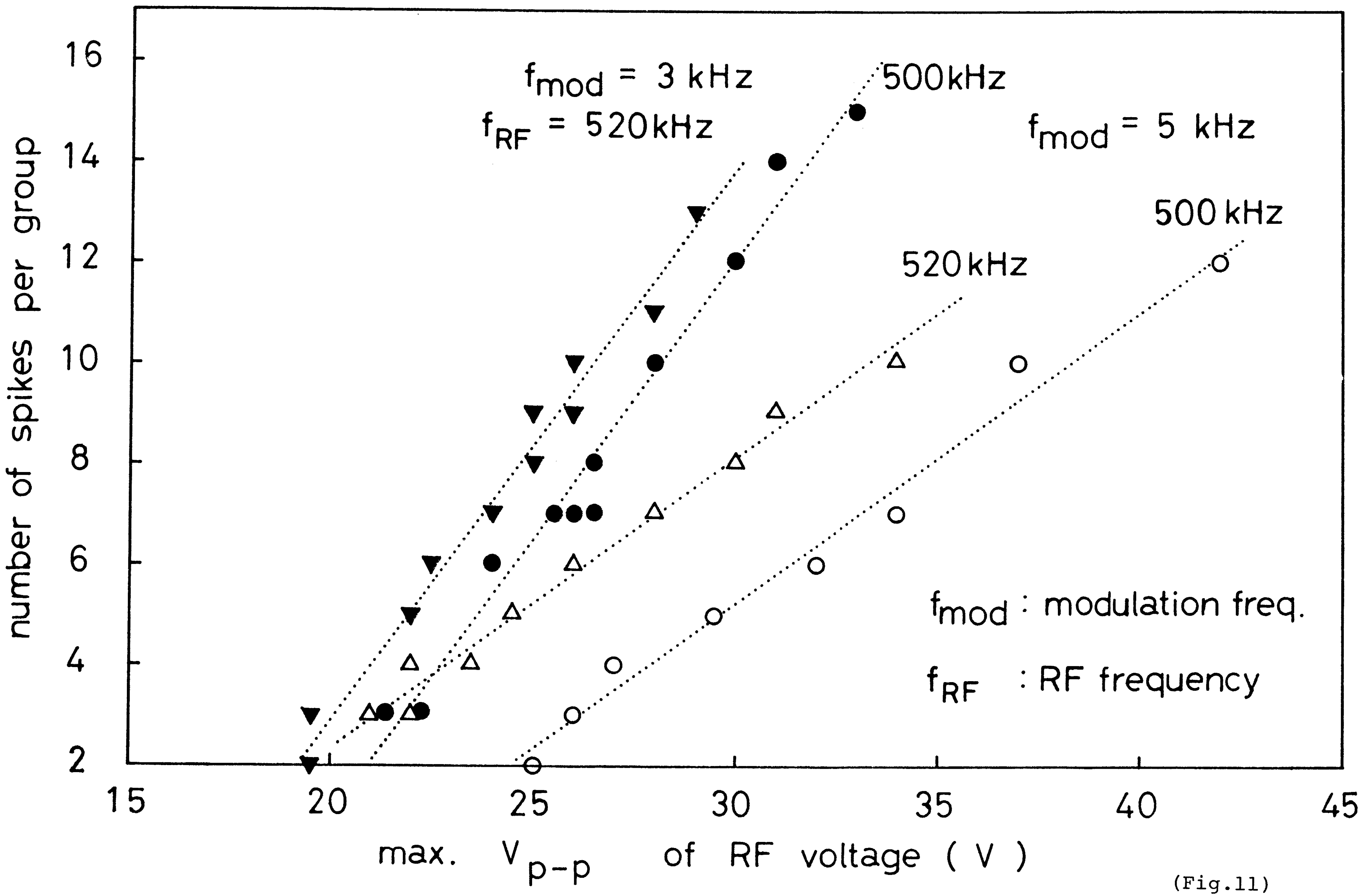


(Fig.8)


(Fig.9)
modulation frequency
3kHz
5kHz
7.5kHz
10kHz
20kHz
number of spikes
0
2
4
6
8
10
12
14
16
700
750
800
850
900
applied RF frequency (kHz)



(Fig.10)
RF frequency : 840 kHz
max. amplitude : 7 V
number of spikes
0
2
4
6
8
10
12
14
0
50
100
150
200
250
modulation period (µsec)



number of spikes per group
max. V_{p-p} of RF voltage (V)
f_{mod} = 3 kHz
f_{RF} = 520kHz
500kHz
f_{mod} = 5 kHz
500 kHz
520kHz
f_{mod} : modulation freq.
f_{RF} : RF frequency

(Fig.11)