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Nonlinear Heat and Particle Transport due to Collisional Drift Waves

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Synopsis

The previous nonlinear analysis[1)] of collisional drift waves in an inhomogeneous plasma is extended to include inhomogeneities in electron and ion temperatures. A slab plasma is adopted with the width ℓ in the direction of the density gradient. A systematic expansion in powers of $\varepsilon = |\kappa|\ell$ is used where κ is the degree of density (temperature) gradient in equilibrium state. The two-fluid equations are used where the thermal transport is considered besides the ion inertia and the effects of finite gyroradius and viscosity. A set of the model equations is proposed to describe the nonlinear evolution of a collisional drift wave under the presence of the temperature gradients. The nonlinear development of the drift wave is studied near marginal stability on the basis of the model equations. A new feature, hard excitation, has been found which is due to the effect of the nonlinear frequency shift and takes place easily when the ion temperature gradient is present.

§1. Introduction

In magnetically confined plasmas, nonlinear transport due to unstable waves is of primary concern. Although extensive studies have been made on this subject, we have only a few theories[1-4] in which attentions are paid to problems such as phenomena near marginal stability and transition to higher instabilities and consequent nonlinear transports. In order to treat these phenomena, we must have enough knowledges for nonlinear evolution of unstable modes which govern transport processes. For this aim, in the references[1,2] we have presented a nonlinear theory of unstable collisional drift wave and considered the consequent nonlinear transport. In these works, however, only the density gradient was taken into account. The thermal transport is generally larger and evidently more important in the confinement study of high temperature plasmas than the particle diffusion. In this paper we extend the previous theory to include the electron and ion temperature gradients and discuss the thermal transport besides the particle diffusion.

In §2, the basic equations are presented. In §3, nonlinear analysis is made by introducing an ordering scheme appropriate to an inhomogeneous plasma. In §4, the model equations in a simplified form are proposed for the nonlinear evolution of the mode of interest and the saturation level is determined. The transition of an unstable state to a state with more unstable modes is studied. In §5, the concluding remarks are presented.

§2. Basic Equations

We use the two-component fluid equations with the transport coefficients in the large $\Omega_j\tau_j$ limit ($j=e,i$) where Ω_j is cyclotron frequency and τ_j is collision time. The equations are valid for disturbances with the frequency $\omega<<\Omega_i$, and the wavelengths perpendicular $\lambda_\perp$ and parallel $\lambda_\parallel$ to the imposed magnetic field such that $\lambda_\perp>>a_i$ and $\lambda_\parallel>>\lambda_{mfp}$ where a_i is the mean ion gyroradius and λ_{mfp} is the collision mean free path. Further we assume the quasi-neutrality condition and consider electrostatic perturbations. The basic equations are

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}\cdot(n_e\vec{v}_e) = \mathcal{S}_e \quad , \tag{2.1}$$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}\cdot(n_i\vec{v}_i) = \mathcal{S}_i \quad , \tag{2.2}$$

$$0 = -\vec{\nabla}p_e - en_e[\vec{E} + \frac{1}{c}\vec{v}_e \times \vec{B}] + \vec{R} \quad , \tag{2.3}$$

$$m_in_i(\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla})\vec{v}_i = -\vec{\nabla}p_i + en_i[\vec{E} + \frac{1}{c}\vec{v}_i \times \vec{B}] - \vec{\nabla}\cdot\overleftrightarrow{\Pi}_i - \vec{R} \quad , \tag{2.4}$$

$$\frac{3}{2}n_e(\frac{\partial}{\partial t} + \vec{v}_e\cdot\vec{\nabla})T_e + p_e\vec{\nabla}\cdot\vec{v}_e = -\vec{\nabla}\cdot\vec{q}_e + Q_e \quad , \tag{2.5}$$

$$\frac{3}{2}n_i(\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla})T_i + p_i\vec{\nabla}\cdot\vec{v}_i = -\vec{\nabla}\cdot\vec{q}_i + Q_i \quad , \tag{2.6}$$

and the quasi-neutrality condition is

$$n_e \simeq n_i = n \quad , \tag{2.7}$$

where $n_{e,i}$, $\vec{v}_{e,i}$, and $T_{e,i}$ are electron and ion densities, fluid velocities and temperatures, respectively, and the source terms $\mathcal{S}_{e,i}$ are introduced to sustain the unperturbed steady state, and the electron inertia is neglected. We have used the Braginskii formulae[5] for the frictional force $\vec{R}$, the stress tensor $\overleftrightarrow{\Pi}_i$, the heat flux $\vec{q}$, and the heat generation Q. Their expressions are

$$\vec{R} = en\left(\frac{\vec{j}_\parallel}{\sigma_\parallel} + \frac{\vec{j}_\perp}{\sigma_\perp}\right) - \beta n\vec{\nabla}_\parallel T_e - \frac{3}{2}\frac{n\nu_{ei}}{\Omega_e}(\vec{b} \times \vec{\nabla}_\perp T_e) \quad , \tag{2.8}$$

$$\overleftrightarrow{\Pi}_i = \overleftrightarrow{\Pi}_{FLR} + \overleftrightarrow{\Pi}_{shear} \quad , \tag{2.9}$$

$$\vec{q}_e = -\beta\frac{T_e}{e}\vec{j}_\parallel - \alpha\frac{nT_e}{m_e\nu_{ei}}\vec{\nabla}_\parallel T_e - \frac{5}{2}\frac{cnT_e}{eB}(\vec{b} \times \vec{\nabla}_\perp T_e) \quad , \tag{2.10}$$

$$\vec{q}_i = \gamma\frac{nT_i}{m_i\nu_{ii}}\vec{\nabla}_\parallel T_i + \frac{5}{2}\frac{cnT_i}{eB}(\vec{b} \times \vec{\nabla}_\perp T_i) \quad , \tag{2.11}$$

$$Q_i = Q_\Delta \equiv 3\frac{m_e}{m_i}n\nu_{ei}(T_e - T_i) \quad , \tag{2.12}$$

$$Q_e = -\vec{R}\cdot\vec{u} - Q_\Delta = \frac{\vec{j}_\parallel^{\,2}}{\sigma_\parallel} + \frac{\vec{j}_\perp^{\,2}}{\sigma_\perp} - \frac{\beta}{e}\vec{j}_\parallel\cdot\vec{\nabla}_\parallel T_e - Q_\Delta \quad , \tag{2.13}$$

$$\vec{j}_\parallel = -en(\vec{v}_e - \vec{v}_i) = -en\vec{u} \quad , \tag{2.14}$$

where $\vec{b} = \vec{B}/|\vec{B}|$, and ν_{ei} and ν_{ii} are the electron-ion and ion-ion collision frequency. Here and hereafter we consider the case when Z = 1, then $\alpha = 3.16$, $\beta = 0.71$ and $\gamma = 3.9$. The perpendicular conductivity is given by $\sigma_\perp = ne^2/m_e\nu_{ei}$ and $\sigma_\parallel = 1.96\sigma_\perp$.

From eqs.(2.3),(2.4),(2.7),(2.8) and (2.9), we find $\vec{v}_e$, and $\vec{v}_i$ as

$$\vec{v}_{e\perp} = \vec{u}_E + \vec{u}_{de} + \vec{u}_D + \vec{u}_T \quad , \tag{2.15}$$

$$\vec{v}_{i\perp} = \vec{u}_E + \vec{u}_{di} + \vec{u}_D + \vec{u}_T$$

$$+ \frac{1}{nm_i\Omega_i}\vec{b} \times [\, nm_i\left(\frac{\partial}{\partial t} + \vec{v}^{(1)}_{i\perp}\cdot\vec{\nabla}_\perp\right)\vec{v}^{(1)}_{i\perp} + \vec{\nabla}_\perp\cdot\overleftrightarrow{\Pi}_i(\vec{v}^{(1)}_{i\perp})\,], \tag{2.16}$$

where $\vec{u}_E$ is the E × B drift velocity, $\vec{u}_{de,i}$ the electron (ion) diamagnetic velocity, $\vec{u}_D$ the diffusion velocity due to collisional friction, $\vec{u}_T$ the diffusion velocity due to thermal friction, respectively, as in the following;

$$\vec{u}_E = \frac{c}{B}(\vec{E} \times \vec{b}) \quad , \tag{2.17}$$

$$\vec{u}_{de} = -\frac{1}{m_e\Omega_e}\left(\vec{b} \times \frac{\vec{\nabla}p_e}{n}\right) \quad . \tag{2.18}$$

$$\vec{u}_{di} = \frac{1}{m_i\Omega_i}\left(\vec{b} \times \frac{\vec{\nabla}p_i}{n}\right) \quad , \tag{2.19}$$

$$\vec{u}_D = -\frac{\nu_{ei}}{nm_e\Omega_e^2}\vec{\nabla}_\perp(p_e + p_i) \quad , \tag{2.20}$$

$$\vec{u}_T = \frac{3}{2}\frac{\nu_{ei}}{m_e\Omega_e^2}\vec{\nabla}_\perp T_e \quad . \tag{2.21}$$

Moreover, $\vec{v}^{(1)}_{i\perp}$ in (2.16) is defined as

$$\vec{v}^{(1)}_{i\perp} = \vec{u}_E + \vec{u}_{di} \quad . \tag{2.22}$$

After some manipulations, we deduce the following equations for $n_e \simeq n_i = n$ and $T_{e,i}$

$$\frac{\partial n}{\partial t} + \vec{u}_E\cdot\vec{\nabla}_\perp n + \vec{\nabla}_\parallel\cdot(n\vec{v}_e) + \vec{\nabla}_\perp\cdot(n\vec{u}_D) + \vec{\nabla}_\perp\cdot(n\vec{u}_T) = \mathcal{S}_e \, , \tag{2.23}$$

$$\frac{\partial n}{\partial t} + \vec{u}_E \cdot \vec{\nabla}_\perp n + \vec{\nabla}_\perp \cdot (n\vec{u}_D) + \vec{\nabla}_\perp \cdot (n\vec{u}_T)$$

$$+ \vec{\nabla}_\perp \cdot \{\frac{1}{m_i \Omega_i} \vec{b} \times [n m_i (\frac{\partial}{\partial t} + \vec{v}_{i\perp}^{(1)} \cdot \vec{\nabla}_\perp) \vec{v}_{i\perp}^{(1)} + (\vec{\nabla}_\perp \cdot \overleftrightarrow{\Pi}_i)] = \mathcal{S}_i \tag{2.24}$$

$$\frac{3}{2} n [\frac{\partial}{\partial t} + (\vec{u}_E + \vec{v}_{e\parallel}) \cdot \vec{\nabla}] T_e + p_e \vec{\nabla}_\parallel \cdot \vec{v}_{e\parallel}$$

$$= \vec{\nabla}_\parallel \cdot (\alpha \frac{n T_e}{m_e \nu_{ei}} \vec{\nabla}_\parallel T_e) + \frac{\beta}{e} T_e \vec{\nabla}_\parallel \cdot \vec{j}_\parallel + \frac{\vec{j}_\parallel^{\,2}}{\sigma_\parallel} + \frac{\vec{j}_\perp^{\,2}}{\sigma_\perp} - Q_\Delta \ , \tag{2.25}$$

$$\frac{3}{2} n [\frac{\partial}{\partial t} + (\vec{u}_E \cdot \vec{\nabla}_\perp)] T_i = \vec{\nabla}_\parallel \cdot (\gamma \frac{n T_i}{m_i \nu_{ii}} \vec{\nabla}_\parallel T_i) + Q_\Delta \ , \tag{2.26}$$

where

$$\vec{j}_\parallel = \sigma_\parallel [\vec{E}_\parallel + \frac{T_e}{en} \vec{\nabla}_\parallel n + \frac{(1+\beta)}{e} \vec{\nabla}_\parallel T_e] \ , \tag{2.27}$$

$$\vec{j}_\perp = \frac{c}{B} \vec{b} \times [\vec{\nabla}_\perp (p_e + p_i)] \ . \tag{2.28}$$

In calculating eqs.(2.25) and (2.26), the following relations are used,

$$\frac{3}{2} n \vec{u}_{de,i} \cdot \vec{\nabla}_\perp T_{e,i} + p_{e,i} \vec{\nabla}_\perp \cdot \vec{u}_{de,i} + \vec{\nabla}_\perp \cdot [\pm \frac{5}{2} \frac{c n T_{e,i}}{eB} (\vec{b} \times \vec{\nabla}_\perp T_{e,i})] = 0 . \tag{2.29}$$

Moreover, the perpendicular components of $\vec{\nabla}_\perp \cdot \overleftrightarrow{\Pi}_{FLR}$ and $\vec{\nabla}_\perp \cdot \overleftrightarrow{\Pi}_{shear}$ are[5]

$$[\vec{\nabla}_\perp \cdot \overleftrightarrow{\Pi}_{FLR} (\vec{v}_{i\perp}^{(1)})]_\perp = - \frac{1}{2} \frac{1}{\Omega_i} [p_i \Delta_\perp + (\vec{\nabla}_\perp p_i) \cdot \vec{\nabla}_\perp] (\vec{v}_{i\perp}^{(1)} \times \vec{b})$$

$$+ (\vec{b} \times \vec{\nabla}_\perp p_i) \cdot \vec{\nabla}_\perp \vec{v}_{i\perp}^{(1)} \} \ , \tag{2.30}$$

$$[\vec{\nabla}_\perp\cdot\overleftrightarrow{\Pi}_{shear}(\vec{v}_{i\perp}^{(1)})] = -\frac{3}{10}\frac{1}{\Omega_i{}^2}[p_i\nu_{ii}\Delta_\perp\vec{v}_{i\perp}^{(1)} + (\vec{\nabla}_\perp p_i\nu_{ii})\cdot\vec{\nabla}_\perp\vec{v}_{i\perp}^{(1)} - (\vec{b}\times\vec{\nabla}_\perp p_i\nu_{ii})\cdot\vec{\nabla}_\perp(\vec{v}_{i\perp}^{(1)}\times\vec{b})] \qquad (2.31)$$

where $\Delta_\perp = \vec{\nabla}_\perp\cdot\vec{\nabla}_\perp$.

§3. Nonlinear Analysis

We adopt a slab model with the following profiles of density and temperatures in the equilibrium state

$$N(x) = N_0(1+\kappa x) \quad , \qquad (3.1)$$

$$T_e(x) = T_0^e(1+\kappa_e x) \quad , \qquad (3.2)$$

$$T_i(x) = T_0^i(1+\kappa_i x) \quad , \qquad (3.3)$$

where κ, κ_e and κ_i are supposed to be almost of the same order. We assume that in equilibrium the magnetic field $\vec{B} = \vec{b}|\vec{B}|$ is uniform and static, and the equilibrium current $\vec{j}_{\parallel 0}$ is sustained by the external electrostatic field $E_{\parallel 0}$, that is, $\vec{j}_{\parallel 0} = \sigma_\parallel \vec{E}_{\parallel 0}$.

We assume a smallness parameter $\varepsilon \equiv |\kappa|\ell \ll 1$ and introduce the following ordering scheme[1);]

$$\tilde{n}/N \sim e\tilde{\Psi}/T_e \sim O(\varepsilon), \quad \omega/\Omega_i \sim \omega_n^*/\Omega_i \sim O(\varepsilon k_\perp^2 a_i^2), \quad k_y\ell \sim O(\varepsilon^0),$$

$$\nu_{ii}/\Omega_i \sim O(\varepsilon^2), \quad k_\parallel^2 D_{c\parallel}/\omega_n^* \sim O(\varepsilon^{-1}), \quad k_y^2 D_{c\perp}/\omega_n^* \sim O(\varepsilon), \qquad (3.4)$$

where $\tilde{n}$ and $\tilde{\Psi}$ are the perturbed density, and electrostatic potential, ω and k_y, $k_\parallel = k_z$ are their frequency and wave numbers. Moreover, ω_n^*, $\omega_{T_{e,i}}^*$, the electron diffusion coefficient along the magnetic field $D_{c\parallel}$, the plasma diffusion coefficient perpendicular to the magnetic field $D_{c\perp}$ and the electron parallel heat conductivity $\chi_{c\parallel}^e$ are defined as

$$\omega_n^* = -k_y T_e \kappa / m_e \Omega_e \quad , \tag{3.5}$$

$$\omega_{T_e}^* = -k_y T_e \kappa_e \,/ m_e \Omega_e \quad , \tag{3.6}$$

$$\omega_{T_i}^* = k_y T_i \kappa_i / m_i \Omega_i \quad , \tag{3.7}$$

$$D_{c\parallel} = 1.96\ T_e / m_e \nu_{ei} \quad , \tag{3.8}$$

$$D_{c\perp} = (T_e + T_i) \nu_{ei} / m_e \Omega_e^2 \quad , \tag{3.9}$$

$$\chi_{c\parallel}^e = 3.16\ T_e / m_e \nu_{ei} \quad . \tag{3.10}$$

The density n and the temperatures $T_{e,i}$ are divided into the unperturbed and the perturbed quantities as

$$n = N(x)(1+\rho) \quad , \tag{3.11}$$

$$T_e = T_e(x)(1+\zeta_e) \quad , \tag{3.12}$$

$$T_i = T_i(x)(1+\zeta_i) \quad . \tag{3.13}$$

That is, we define the dimensionless perturbations

$$\rho = \frac{\tilde{n}}{N(x)} \ , \ \psi = \frac{e\tilde{\phi}}{T_e(x)} \ , \ \zeta_e = \frac{\tilde{T}_e}{T_e(x)} \ , \quad \zeta_i = \frac{\tilde{T}_i}{T_i(x)} \quad . \tag{3.14}$$

As in Ref.1, by separating the linear terms and the nonlinear ones, eqs.(2.23)~(2.26) can be expressed in a matrix form as

$$LU = S \quad , \qquad U = \begin{bmatrix} \rho \\ \psi \\ \zeta_e \\ \zeta_i \end{bmatrix} \quad , \tag{3.15}$$

where L is the linear operator and S is the nonlinear terms. The expressions of L and S are given in the Appendix. By the orderings given in (3.4), we may expand

$$\begin{aligned} L &= L^{(0)} + \varepsilon L^{(1)} + \varepsilon^2 L^{(2)} + \cdots \quad , \\ U &= \varepsilon U^{(1)} + \varepsilon^2 U^{(2)} + \cdots \quad , \\ S &= \varepsilon^2 S^{(2)} + \varepsilon^3 S^{(3)} + \cdots \quad , \end{aligned} \tag{3.16}$$

also,

$$\begin{aligned} \frac{\partial}{\partial t} &= \varepsilon \frac{\partial}{\partial t_1} + \varepsilon^2 \left(\frac{\partial}{\partial t_2} - \vec{v}_g \frac{\partial}{\partial y_1} \right) + \cdots \quad , \\ \frac{\partial}{\partial y} &= \frac{\partial}{\partial y_0} + \varepsilon \frac{\partial}{\partial y_1} + \cdots \quad , \end{aligned} \tag{3.17}$$

where $\vec{v}_g$ is the group velocity to be determined in the course of calculation. As ω is thought to be the first order in ε as shown in (3.4), the time derivative begins in the first order in ε.

We will proceed into the third order in ε, while we keep the quantities of the second order in $b = k_y^2 a_i^2/2$ (we have used the iteration in the calculation of eq.(2.16)). Therefore we assume $\varepsilon \gg b$.

[1] 1st order calculation

From eq.(3.15), we get

$$L^{(0)}U^{(1)}$$

$$= \begin{bmatrix} -D_{C''}\frac{\partial^2}{\partial z^2}, & D_{C''}\frac{\partial^2}{\partial z^2}, & -(1+\beta)D_{C''}\frac{\partial^2}{\partial z^2}, & 0 \\ 0, & 0, & 0, & 0 \\ -(1+\beta)D_{C''}\frac{\partial^2}{\partial z^2}, & (1+\beta)D_{C''}\frac{\partial^2}{\partial z^2}, & -[(1+\beta)D_{C''}+\chi^e_{C''}]\frac{\partial^2}{\partial z^2}, & 0 \\ 0, & 0, & 0, & -\chi^i_{C''}\frac{\partial^2}{\partial z^2} \end{bmatrix} U^{(1)}=0 \tag{3.18}$$

where $\chi^i_{C''} = 3.9\ T_i/m_i\nu_{ii}$.

From eq.(3.18), we find $\rho^{(1)} = \psi^{(1)}$ and $\zeta_e^{(1)} = \zeta_i^{(1)} = 0$.

We may write

$$\rho^{(1)} = \psi^{(1)} = f = h(x,y_1,t_2) + \sum_{k_{''}}[g(x,y_1,t_2)e^{2ik_{''}z}+c.c.]$$

$$+\sum_{k_{''},k_y}\{[f_+(x,y_1,t_2)e^{ik_{''}z}+f_-(x,y_1,t_2)e^{-ik_{''}z}]e^{-i\omega t_1+ik_y y_0}+c.c. \tag{3.19}$$

where h and g denote the modifications of the background density, and f_+ and f_- denote the amplitudes of the drift wave. This expression of $\rho^{(1)}$ and $\psi^{(1)}$ is based upon the consideration that the modifications of the background density by the zero-frequency harmonic is the primary mechanism for saturation of the instability.[1)] It should be noted that ω is taken to be real in the present representation.

[2] 2nd order calculation

We have

$$L^{(0)}U^{(2)} + L^{(1)}U^{(1)} = S^{(2)} \quad .$$

The components of this equation are

$$-D_{c\parallel}\frac{\partial^2}{\partial z^2}\left(\rho^{(2)}-\psi^{(2)}\right)-(1+\beta)D_{c\parallel}\frac{\partial^2}{\partial z^2}\zeta_e^{(2)}+\left(\frac{\partial}{\partial t_1}+v_n^*\frac{\partial}{\partial y_0}\right)f$$

$$=-D_{c\parallel}\frac{\partial^2}{\partial z^2}\left(\frac{1}{2}f^2\right), \qquad (3.20)$$

$$\left\{\frac{\partial}{\partial t_1}[1+(1+\lambda)\hat{b}]+[v_n^*-\frac{1}{2}v_{T_i}^*\hat{b}]\frac{\partial}{\partial y_0}\right\}f$$

$$=\frac{1}{4}\lambda(\lambda+1)\Omega_i a_i^4\left(-\frac{\partial f}{\partial y_0}\frac{\partial}{\partial x}+\frac{\partial f}{\partial x}\frac{\partial}{\partial y_0}\right)\Delta_\perp f, \qquad (3.21)$$

$$-(1+\beta)D_{c\parallel}\frac{\partial^2}{\partial z^2}\left(\rho^{(2)}-\psi^{(2)}\right)-[(1+\beta)^2D_{c\parallel}+\chi^e_{c\parallel}]\frac{\partial^2}{\partial z^2}\zeta_e^{(2)}$$

$$+[-u_0\frac{\partial}{\partial z}+\frac{3}{2}v_{T_e}^*\frac{\partial}{\partial y_0}]f=-(1+\beta)D_{c\parallel}\frac{\partial^2}{\partial z^2}\left(\frac{1}{2}f^2\right), \qquad (3.22)$$

$$-\chi^i_{c\parallel}\frac{\partial^2}{\partial z^2}\zeta_i^{(2)}-\frac{3}{2}v_{T_i}^*\frac{\partial}{\partial y_0}f=0, \qquad (3.23)$$

where

$$\Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y_0^2}, \qquad \hat{b}=-\frac{1}{2}a_i^2\Delta_\perp, \qquad \lambda=T_e/T_i,$$

$$v_n^*=-T_e\kappa/m_e\Omega_e, \qquad v_{T_e}^*=-T_e\kappa_e/m_e\Omega_e, \qquad v_{T_i}^*=T_i\kappa_i/m_i\Omega_i$$

From (3.20), (3.22) and (3.23), we get

$$\rho^{(2)}-\psi^{(2)}=\frac{1}{2}f^2+\sum_{k_\parallel k_y}\{[(-i)(\delta_k+\delta_k')(f_+e^{ik_\parallel z}+f_-e^{-ik_\parallel z})e^{-i\omega t_1+ik_y y_0}+c.c.]$$

$$+[(-i)\delta_k''(f_+e^{ik_\parallel z}-f_-e^{-ik_\parallel z})e^{-i\omega t_1+ik_y y_0}+c.c.], \qquad (3.24)$$

$$\zeta_e^{(2)}=\sum_{k_\parallel,k_y}[i(\frac{1+\beta}{\mu}\frac{D_{c\parallel}}{\chi^e_{c\parallel}}\delta_k+\frac{1}{1+\beta}\delta_k')(f_+e^{ik_\parallel z}+f_-e^{-ik_\parallel z})e^{-i\omega t_1+ik_y y_0}+c.c.]$$

$$+[\frac{i}{1+\beta}\delta_k''(f_+e^{ik_\parallel z}-f_-e^{-ik_\parallel z})e^{-i\omega t_1+ik_y y_0}+c.c.], \qquad (3.25)$$

$$\zeta_i^{(2)}=\sum_{k_\parallel,k_y}[i\delta_{T_i}(f_+e^{ik_\parallel z}+f_-e^{-ik_\parallel z})e^{-i\omega t_1+ik_y y_0}+c.c.], \qquad (3.26)$$

where

$$\delta_k = \mu(\omega_n^* - \omega)/k_\parallel^2 D_{c\parallel} , \qquad \mu = 1+(1+\beta)^2 D_{c\parallel}/\chi_{c\parallel}^e ,$$

$$\delta_k' = -(3/2)(1+\beta)\omega_{T_e}^* /k_\parallel^2\chi_{c\parallel}^e , \qquad \delta_k'' = (1+\beta)k_\parallel u_0/k_\parallel^2\chi_{c\parallel}^e ,$$

$$\delta_{T_i} = (3/2)\omega_{T_i}^* /k_\parallel^2\chi_{c\parallel}^i .$$

The quantities δ_k, δ_k', and δ_k'' are the phase shifts between $\tilde{n}$ and $\tilde{\psi}$, which are due to the density gradient, the electron temperature gradient and the unperturbed fluid velocity, respectively.

We use the expression of f given by eq.(3.19) for f in eq.(3.21), then we obtain

$$\begin{aligned} &\{(-i\omega)[1+(1+\lambda)\hat{b}] + ik_y v_n^* - \tfrac{1}{2}ik_y v_{T_i}^* \hat{b} \\ &+ \tfrac{1}{4}\lambda(\lambda+1)\Omega_i a_i^4 (\frac{\partial^3 h}{\partial x^3} - \frac{\partial h}{\partial x}\Delta_\perp)(ik_y)\}\begin{bmatrix} f_+ \\ f_- \end{bmatrix} \\ &+ \tfrac{1}{4}\lambda(\lambda+1)\Omega_i a_i^4 [\frac{\partial^3}{\partial x^3}\begin{bmatrix} g \\ g^* \end{bmatrix} - \frac{\partial}{\partial x}\begin{bmatrix} g \\ g^* \end{bmatrix}\Delta_\perp](ik_y)\begin{bmatrix} f_- \\ f_+ \end{bmatrix} = 0. \qquad (3.27) \end{aligned}$$

where $\Delta_\perp = \partial^2/\partial x^2 - k_y^2$, and g^* is the complex conjugate of g. This equation gives the dispersion relation for ω(real) which includes the nonlinear frequency shift. If one neglects the nonlinear terms in (3.27), the linear dispersion relation will be recovered as

$$\omega = (\omega_n^* - \tfrac{1}{2}b\omega_{T_i}^*)/[1 + (1+\lambda)b] , \qquad (3.28)$$

where $b = k_y^2 a_i^2/2$.

In the above, we have assumed that f in eq.(3.19) contains the terms up to the first harmonic. Strictly speaking, the right-hand side of eq.(3.21) generates the second harmonic with respect to the frequency ω. However, the second harmonic can be neglected for the following reason. From the coupling strength of the harmonic generation in Eq.(3.21), we see that the generated second harmonic is smaller than the original first harmonic by about one eighth. Furthermore, if we assume $f \propto \sin(k_x x) e^{ik_y y_0}$ which is one of our dominant solution as will be shown below, the second harmonic is not generated by this mode.

The particle and the heat flux are obtained from the second order calculation. The particle fluxes along the density for the electrons and the ions are

$$\Gamma_{ex} \equiv \langle n\, v_x \rangle_e = \langle n\, v_x \rangle_i \equiv \Gamma_{ix}$$

$$= \sum_{k_\parallel, k_y} \frac{2cN_0T_e}{eB} \frac{k_y}{k_\parallel^2 \chi^e_{c\parallel}} [\{ \frac{(1+\beta)^2 + \chi^e_{c\parallel}/D_{c\parallel}}{1+(1+\lambda)b} [(1+\lambda)b\omega^*_n + \frac{1}{2}b\omega^*_{T_i}]$$

$$- \frac{3}{2}(1+\beta)\omega^*_{T_e}\}(|f_+|^2+|f_-|^2) + (1+\beta)k_\parallel u_0(|f_+|^2-|f_-|^2)] \quad . \tag{3.29}$$

The heat fluxes are

$$Q_{ex} \equiv \langle nT_e v_x \rangle_e$$

$$= \sum_{k_\parallel, k_y} \frac{2cN_0T_e}{eB} \frac{k_y}{k_\parallel^2 \chi^e_{c\parallel}} [\{ \frac{\beta(1+\beta) + \chi^e_{c\parallel}/D_{c\parallel}}{1+(1+\lambda)b} [(1+\lambda)b\omega^*_n + \frac{1}{2}b\omega^*_{T_i}]$$

$$- \frac{3}{2}\beta\omega^*_{T_e}\}(|f_+|^2+|f_-|^2) + \beta k_\parallel u_0(|f_+|^2-|f_-|^2)] \quad , \tag{3.30}$$

$$Q_{ix} \equiv \langle nT_i v_x \rangle_i$$

$$= \sum_{k_\parallel, k_y} \frac{2cN_0T_eT_i}{eB} \frac{k_y}{k_\parallel^2 \chi^e_{c\parallel}} [\{ \frac{(1+\beta)^2 + \chi^e_{c\parallel}/D_{c\parallel}}{1+(1+\lambda)b} [(1+\lambda)b\omega^*_n + \frac{1}{2}b\omega^*_{T_i}]$$

$$- \frac{3}{2}(1+\beta)\omega^*_{T_e} - \frac{3}{2}\frac{\chi^e_{c\parallel}}{\chi^i_{c\parallel}}\omega^*_{T_i}\}(|f_+|^2+|f_-|^2) + (1+\beta)k_\parallel u_0(|f_+|^2-|f_-|^2)] .$$

[3] 3rd order calculation

The third order equation of eq.(3.15) is

$$L^{(0)}U^{(3)}+L^{(1)}U^{(2)}+L^{(2)}U^{(1)}=S^{(3)} \qquad (3.32)$$

which has four components. Their expressions are

$$-D_{c\parallel}\frac{\partial^2}{\partial z^2}[\rho^{(3)}-\psi^{(3)}]-(1+\beta)D_{c\parallel}\frac{\partial^2}{\partial z^2}\zeta_e^{(3)}+[\frac{3}{2}u_0\frac{\partial}{\partial z}+(1+\beta)\kappa x\, D_{c\parallel}\frac{\partial^2}{\partial z^2}]\zeta_e$$

$$+[\frac{\partial}{\partial t_2}+(v_n^*-v_g)\frac{\partial}{\partial y_1}-\kappa x v_n^*\frac{\partial}{\partial y_0}-D_{c\perp}\Delta_\perp]f$$

$$=\bar{\gamma}(v_n^*\frac{\partial}{\partial y_0}+\kappa x\, D_{c\parallel}\frac{\partial^2}{\partial z^2})[\frac{1}{2}f^2+\psi^{(2)}-\rho^{(2)}]$$

$$-(\frac{\partial}{\partial t_1}+v_n^*\frac{\partial}{\partial y_0})\rho^{(2)}+S_e^{(3)}, \qquad (3.33)$$

$$[1+(1+\lambda)\hat{b}]\frac{\partial f}{\partial t_2}+\{-v_g[1+(1+\lambda)\hat{b}]+v_n^*-\frac{1}{2}v_{T_i}^*\hat{b}-(1+\lambda)a_i^2\frac{\partial^2}{\partial t_1\partial y_0}\}\frac{\partial f}{\partial y_1}$$

$$-\kappa x v_n^*\frac{\partial f}{\partial y_0}+\frac{1}{2}a_i^2[(\kappa-\kappa_i)v_n^*+(\kappa+\kappa_i)v_{T_i}^*]\frac{\partial^2 f}{\partial x\partial y_0}$$

$$-\frac{1}{2}a_i^2[(\kappa+\kappa_i)+\lambda(\kappa+2\kappa_e)]\frac{\partial^2 f}{\partial t_1\partial x}+[-D_{c\perp}\Delta_\perp+\frac{3}{10}(1+\lambda)\nu_{ii}\hat{b}^2]f$$

$$=(v_n^*\frac{\partial}{\partial y_0}+\lambda\frac{\partial}{\partial t_1}\hat{b})[\rho^{(2)}-\psi^{(2)}]-\{\frac{\partial}{\partial t_1}[1+(1+\lambda)\hat{b}]$$

$$+v_n^*\frac{\partial}{\partial y_0}-\frac{1}{2}v_{T_i}^*\frac{\partial}{\partial y_0}\hat{b}\}\rho^{(2)}-[\frac{\partial}{\partial t_1}\hat{b}-(2\lambda)^{-1}v_n^*\frac{\partial}{\partial y_0}\hat{b}]\zeta_i^{(2)}+S_i' \qquad (3.34)$$

$$-(1+\beta)D_{c\parallel}\frac{\partial^2}{\partial z^2}[\rho^{(3)}-\psi^{(3)}]-[(1+\beta)D_{c\parallel}+\chi_{c\parallel}^e]\frac{\partial^2}{\partial z^2}\zeta_e^{(3)}+u_0\frac{\partial}{\partial z}\rho^{(2)}$$

$$+(\frac{3}{2}v_{T_e}^*\frac{\partial}{\partial y_0}-2u_0\frac{\partial}{\partial z})\psi^{(2)}+(1+\beta)\kappa x D_{c\parallel}\frac{\partial^2}{\partial z^2}[\rho^{(2)}-\psi^{(2)}]+\frac{3}{2}v_{T_e}^*\frac{\partial}{\partial y_1}f$$

$$+\{\frac{3}{2}\frac{\partial}{\partial t_1}+(5+\frac{7}{2}\beta)u_0\frac{\partial}{\partial z}+[(1+\beta)D_{c\parallel}+\chi_{c\parallel}^e]\kappa x\frac{\partial^2}{\partial z^2}\}\zeta_e^{(2)}=S_{T_e}^{(3)}, \qquad (3.35)$$

$$-\chi_{c\parallel}^i\frac{\partial^2}{\partial z^2}\zeta_i^{(3)}-\frac{3}{2}v_{T_i}^*\frac{\partial}{\partial y_0}\psi^{(2)}+\frac{3}{2}\frac{\partial}{\partial t_1}\zeta_i^{(2)}-\frac{3}{2}v_{T_i}^*\frac{\partial}{\partial y_1}f=S_{T_i}^{(3)}, \qquad (3.36)$$

where $S_e^{(3)}$, $S_i^{(3)}$, $S_{T_e}^{(3)}$ and $S_{T_i}^{(3)}$ are the third order nonlinear terms which are given in the Appendix.

In the calculations of the second and third order in ε, we have retained the quantities of the order of $\varepsilon^2 b^2$ and $\varepsilon^3 b^2$, while neglecting the quantities of the order of εb^3 and $\varepsilon^2 b^3$.

As was argued in Ref.1, we plausibly require for the first harmonic component of $\rho^{(2)}$ the following relation

$$
\begin{aligned}
& i\{\omega[\,1+(1+\lambda)\hat{b}]\ -\omega_n^* + \tfrac{1}{2}b\omega_{T_i}^*\}\ \rho^{(2)} \\
&= -\tfrac{1}{4}\lambda(\lambda+1)\Omega_i a_i^4[(\vec{b}\times\vec{\nabla}_\perp f\cdot\vec{\nabla}_\perp)\Delta_\perp\rho^{(2)}+(\vec{b}\times\vec{\nabla}_\perp\rho^{(2)}\cdot\vec{\nabla}_\perp)\Delta_\perp f]
\end{aligned}
$$

which is reduced from the first harmonic part of eq.(3.34). This relation exactly holds when we replace $\rho^{(2)}$ by the first harmonic component of f, as seen in eq.(3.21).

In order to obtain the evolution equations for h and g, we average eq.(3.33) over t_1, y_0 and z, and over t_1 and y_0, respectively. Also we obtain two evolution equations for f_+ and f_- from the first harmonic part of eq.(3.34). The results are

$$
\begin{aligned}
\frac{\partial h}{\partial t_2} &= D_{c\perp}\frac{\partial^2 h}{\partial x^2} + \Sigma[2\frac{\omega_n^*(\delta_k+\delta_k')}{\kappa}\frac{\partial}{\partial x}(|f_+|^2+|f_-|^2)] \\
&\quad + \Sigma 2\frac{\omega_n^*\delta_k''}{\kappa}\frac{\partial}{\partial x}(|f_+|^2-|f_-|^2)\ , \qquad (3.37)
\end{aligned}
$$

$$
\frac{\partial g}{\partial t_2} = D_{c\perp}\frac{\partial^2 g}{\partial x^2} + \Sigma[2\frac{\omega_n^*(\delta_k+\delta_k')}{\kappa}\frac{\partial}{\partial x}(f_+f_-^*)] \qquad (3.38)
$$

$$(\frac{\partial}{\partial t_2} - \Delta v_g \frac{\partial}{\partial y_1}) f_+ = [\omega_n^*(\delta_k+\delta_k'+\delta_k'')(1+\frac{1}{\kappa}\frac{\partial h}{\partial x}) - \frac{1}{2}\frac{a_i^2}{\kappa}\omega_n^*\delta_{T_i}\frac{\partial^3 h}{\partial x^3}$$

$$+ \hat{0} + i\delta\omega] f_+ + [\frac{\omega_n^*(\delta_k+\delta_k'+\delta_k'')}{\kappa}\frac{\partial g}{\partial x} + i\delta\omega'] f_- \tag{3.39}$$

$$(\frac{\partial}{\partial t_2} - \Delta v_g \frac{\partial}{\partial y_1}) f_- = [\omega_n^*(\delta_k+\delta_k'+\delta_k'')(1+\frac{1}{\kappa}\frac{\partial h}{\partial x}) - \frac{1}{2}\frac{a_i^2}{\kappa}\omega_n^*\delta_{T_i}\frac{\partial^3 h}{\partial x^3}$$

$$+ \hat{0} + i\delta\omega] f_- + [\frac{\omega_n^*(\delta_k+\delta_k'+\delta_k'')}{\kappa}\frac{\partial g^*}{\partial x} + i\delta\omega'] f_+ , \tag{3.40}$$

where the summation Σ is taken for k_y and $k_{\shortparallel}$, and

$$\hat{0} = D_{c\perp}\Delta_\perp - \frac{3}{10}(1+\lambda)\nu_{ii}(\frac{1}{2}a_i^2\Delta_\perp)^2 ,$$

$$\Delta_\perp = \frac{\partial^2}{\partial x^2} - k_y^2 , \quad \Delta v_g = (1+\lambda)(v_n^*\hat{b} + \omega a_i^2 k_y) - \frac{1}{2}v_{T_i}^*\hat{b} \tag{3.41}$$

In eqs.(3.37)-(3.40), we have chosen $v_g = v_n^*$ to drop the derivative with respect to y_1. On the right-hand side of eqs.(3.39) and (3.40), $\delta\omega$ and $\delta\omega'$ denote the frequency shifts

$$\delta\omega = \kappa x\omega_n^*[\,1+0(b)\,] \quad , \quad \delta\omega' \sim \delta\omega b \quad .$$

The quantity $\gamma_k = \omega_n^*(\delta_k+\delta_k'+\delta_k'')$ corsesponds to the linear growth rate and the factor of $\kappa^{-1}\frac{\partial h}{\partial x}$ means the reduction of the growth rate due to the change in gradient of the background density. The phase difference between ρ and ψ, δ_k, must be determined by solving eq.(3.27) under appropriate boundary conditions. We use the relation derived from eq.(3.27)

$$\frac{\delta_k}{\mu}\begin{bmatrix} f_+ \\ f_- \end{bmatrix} = \{ \frac{(1+\lambda)\omega_n^*}{k_{\shortparallel}^2 D_{c\shortparallel}}[(1+\frac{1}{\kappa}\frac{\partial h}{\partial x})\hat{b} + \frac{1}{2}a_i^2\frac{1}{i\kappa}\frac{\partial^3 h}{\partial x^3}] + \frac{\omega_{T_i}^*}{2k_{\shortparallel}^2 D_{c\shortparallel}}\hat{b}\}\begin{bmatrix} f_+ \\ f_- \end{bmatrix}$$

$$+ \frac{(1+\lambda)\omega_n^*}{k_{\shortparallel}^2 D_{c\shortparallel}}[\frac{1}{\kappa}\frac{\partial}{\partial x}\begin{bmatrix} g \\ g^* \end{bmatrix}\hat{b} + \frac{1}{2}a_i^2\frac{1}{i\kappa}\frac{\partial^3}{\partial x^3}\begin{bmatrix} g \\ g^* \end{bmatrix}]\begin{bmatrix} f_- \\ f_+ \end{bmatrix} . \tag{3.42}$$

§4. Model Equations and Transition from Laminar to Turbulent State

Nonlinear behaviours of the system under consideration are to be described by the set of these four equations for h, g, f_+ and f_- given in the preceding section. However, the essential properties of quasi-linear coupling can be treated by the following simplified set of the model equations for h, the amplitude of the modification of the background density and $f \equiv f_+$, the amplitude of the unstable drift wave. Namely

$$\frac{\partial h}{\partial t_2} = D_{c\perp}\frac{\partial^2 h}{\partial x^2} + 2\sum_{k_\parallel, k_y} \frac{\mu\gamma_L}{|\kappa| k_y^2}\frac{\partial}{\partial x}\left[\left(1+\nu - \frac{1}{|\kappa|}\frac{\partial h}{\partial x}\right) f\Delta_\perp f + \frac{1}{|\kappa|}\frac{\partial^3 h}{\partial x^3} f^2\right]$$
$$-4\,\frac{(1+\beta)D_{c\parallel}}{(1+\lambda)\chi^e_{c\parallel}}\,\frac{\gamma_L}{a_i^2\, k_y^2|\kappa|}\left(-\frac{3}{2}\frac{\omega^*_{T_e}}{\omega^*_n} + \frac{k_\parallel u_0}{\omega^*_n}\right)\frac{\partial}{\partial x} f^2 \;, \qquad (4.1)$$

$$\frac{\partial f}{\partial t_2} = \frac{\mu\gamma_L}{k_y^2}\left(1+\nu - \frac{1}{|\kappa|}\frac{\partial h}{\partial x}\right)\left(1 - \frac{1}{|\kappa|}\frac{\partial h}{\partial x}\right)(-\Delta_\perp f)$$
$$-\frac{3}{40}(1+\lambda)\nu_{ii} a_i^4 \Delta_\perp^2 f + \frac{3}{2}\,\frac{D_{c\parallel}\gamma_L}{(1+\lambda)\chi^i_{c\parallel}}\,\frac{\omega^*_{T_i}}{k_y^2\;|\kappa|\omega^*_n}\,\frac{\partial^3 h}{\partial x^3} f$$
$$-\left[\frac{\mu\gamma_L}{k_y^2\;|\kappa|}\frac{\partial^3 h}{\partial x^3} - 2\frac{(1+\beta)D_{c\parallel}\gamma_L}{(1+\lambda)\chi^e_{c\parallel} a_i^2 k_y^2}\left(-\frac{3}{2}\frac{\omega^*_{T_e}}{\omega^*_n} + \frac{k_\parallel u_0}{\omega^*_n}\right)\right]\left(1 - \frac{1}{|\kappa|}\frac{\partial h}{\partial x}\right) f \;, \qquad (4.2)$$

where

$$\gamma_L = \omega_n^{*2}(1+\lambda)(1/2)(k_y^2\; a_i^2)/k_\parallel^2 D_{c\parallel} \;,$$

$$\nu = \omega^*_{T_i}/[2(1+\lambda)\omega^*_n] \;.$$

These equations are obtained from eqs.(3.37), (3.39) and (3.42).

We rewrite the eqs.(4.1) and (4.2) in dimensionless form as

$$\frac{\partial H}{\partial \tau} = \alpha\frac{\partial^2 H}{\partial \xi^2} + 2\frac{\partial}{\partial \xi}[\mu(1+\nu- \frac{\partial H}{\partial \xi})F\Delta F+\mu\frac{\partial^3 H}{\partial \xi^3} F^2]- 4\nu''\frac{\partial}{\partial \xi} F^2 \quad , \tag{4.3}$$

$$\frac{\partial F}{\partial \tau} = -\mu(1+\nu- \frac{\partial H}{\partial \xi})(1- \frac{\partial H}{\partial \xi})\Delta F - \eta\, \Delta^2 F-(\mu\frac{\partial^3 H}{\partial \xi^3} - 2\nu'')(1- \frac{\partial H}{\partial \xi})F$$

$$+ \nu'\frac{\partial^3 H}{\partial \xi^3} F \quad . \tag{4.4}$$

where

$$H = (\pi/|\kappa|\ell)h \quad , \quad F = (\pi/|\kappa|\ell)f \quad , \quad k = k_y\ell/\pi = k_y/k_x \quad ,$$

$$\tau = \gamma_L t_2 (\pi/\ell k_y)^2 \quad , \quad \xi = (\pi/\ell)x = k_x x \quad , \quad \alpha = k_y^2 D_{c\perp}/\gamma_L \quad ,$$

$$\nu' = \frac{3\ D_{c\parallel}}{2(1+\lambda)\chi^i_{c\parallel}} \frac{\omega^*_{Ti}}{\omega^*_n} \quad , \quad \nu'' = \frac{(1+\beta)D_{c\parallel}}{k_x^2(1+\lambda)\chi^e_{c\parallel}a_i{}^2}(-\frac{3}{2}\frac{\omega^*_{Te}}{\omega^*_n} + \frac{k_\parallel u_0}{\omega^*_n}) \quad ,$$

$$\Delta \equiv \frac{\partial^2}{\partial \xi^2} - k^2 \ .$$

Moreover η is an important parameter which is proportional to the ratio of the viscosity damping to the linear growth rate and is defined as

$$\eta = \frac{3}{40}(1+\lambda)(k_x a_i)^4 k^2 \frac{\nu_{ii}}{\gamma_L} = \frac{3}{20}(k_x a_i)^2 \frac{\nu_{ii} k_\parallel{}^2 D_{c\parallel}}{\omega_n^{*2}} \quad . \tag{4.5}$$

Without loss of generality, we hereafter may fix $k_\parallel$ and k_y values. In experiments, $k_\parallel$ is usually specified by the length of device or by the configuration of the magnetic field and k_y takes a discrete value since the y direction corresponds to the azimuthal direction of plasma column.

When we require the fixed boundary conditions $F = H = 0$, at both ends of the slab, the equations (4.3) and (4.4) can be solved by decomposing F and H into the Fourier components

$$F = \sum_p F_p \sin(p\xi) \quad , \qquad H = \sum_p H_p \sin(p\xi) \quad . \tag{4.6}$$

Hence we have

$$\frac{dH_p}{d\tau} = -\Gamma_p H_p + \sum_{p'p''} [\mu(1+\nu) K_{pp'p''} + 2\nu'' M_{pp'p''}] F_{p'} F_{p''}$$

$$- \frac{\mu}{2} \sum_{p'p''p'''} (N_{pp'p''p'''} - S_{pp'p''p'''}) H_{p'} F_{p''} F_{p'''} \quad , \qquad (4.7)$$

$$\frac{dF_p}{d\tau} = \gamma_p F_p - \sum_{p'p''} [\frac{\mu(2+\nu)}{2} K_{p'pp''} + \nu'' M_{p'pp''} + \frac{1}{2}(\mu-\nu') U_{pp'p''}] H_{p'} F_{p''}$$

$$+ \frac{\mu}{4} \sum_{p'p''p'''} (N_{p''p'pp'''} - S_{p''p'pp'''}) H_{p'} H_{p''} F_{p'''} \quad . \qquad (4.8)$$

and $$\gamma_p = [\mu(1+\nu) - \eta(p^2+k^2)](p^2+k^2) + 2\nu'' \quad , \qquad (4.9)$$

where

$$\Gamma_p = \alpha p^2 \quad .$$

$$K_{pp'p''} = p(p''^2 + k^2)(-\delta_{p,p'+p''} + \delta_{p,p''-p'} + \delta_{p,p'-p''}) \quad ,$$

$$M_{pp'p''} = p(-\delta_{p,p'+p''} + \delta_{p,p''-p'} + \delta_{p,p'-p''}) \quad .$$

$$N_{p\,p'p''p'''} = pp'(p'''^2 + k^2)\ (-\delta_{p,p'+p''+p'''} + \delta_{p,p'+p''-p'''}$$
$$+ \delta_{p,p'-p''+p'''} - \delta_{p,p'-p''-p'''} + \delta_{p,-p'-p''+p'''}$$
$$+ \delta_{p,-p'+p''-p'''} - \delta_{p,-p'+p''+p'''}) \quad ,$$

$$S_{pp'p''p'''} = pp'^3 (-\delta_{p,p'+p''+p'''} + \delta_{p,p'+p''-p'''}$$
$$+ \delta_{p,p'-p''+p'''} - \delta_{p,p'-p''-p'''} + \delta_{p,-p'-p''+p'''}$$
$$+ \delta_{p,-p'+p''-p'''} - \delta_{p,-p''+p''+p'''}) \quad ,$$

$$U_{pp'p''} = p'^3 (-\delta_{p,p'+p''} + \delta_{p,p'-p''} - \delta_{p,p''-p'}) \quad ,$$

and $\delta_{p,p'}$ is the Kronecker's delta.

From the anlysis for (4.6) and (4.7), we obtain the following results for each stage which has more unstable modes with the decreasing "order parameter" η included in (4.9).

(1) Equilibrium

There is no unstable mode, if

$$\eta > \eta_c \equiv [\mu(1+\nu)(1+k^2)+2\nu'']/(1+k^2)^2 \ .$$

We can find the stable steady state with $F_p = H_p = 0$.

(2) First stage

If $\quad \eta_c > \eta > \eta_1 \equiv [\mu(1+\nu)(4+k^2)+2\nu'']/(4+k^2)^2,$

then one mode F_1 becomes unstable associated with the modification of background density H_2. In this stage, we obtain a simple set of the equations for F_1 and H_2 as

$$\frac{d}{d\tau} H_2 = -\Gamma_2 H_2 - [2\mu(1+\nu)(1+k^2)+4\nu'']F_1^2 + 4\mu(3-k^2)H_2 F_1{}^2, \quad (4.10)$$

$$\frac{d}{d\tau} F_1 = \gamma_1 F_1 + [\mu(2+\nu)(1+k^2)+2\nu''+4(\nu'-\mu)]H_2 F_1$$

$$-2\mu(3-k^2)H_2{}^2 F_1 \quad , \quad (4.11)$$

where

$$\Gamma_2 = 4\alpha \quad , \quad \gamma_1 = [\mu(1+\nu) - \eta(1+k^2)](1+k^2)+2\nu'' = (1+k^2)^2(\eta_c - \eta).$$

We then find the steady state solutions, (which is indicated by the notation S[1]), with the saturation levels

$$H_{2s} = \frac{c+(1+k^2)^2\eta_c \pm\sqrt{d}}{4\mu(3-k^2)} \quad , \quad (4.12)$$

$$F_{1s} = \pm\left\{\frac{\alpha\ [c+(1+k^2)^2\eta_c \pm\sqrt{d}]}{\mu(3-k^2)\,[c-(1+k^2)^2\eta_c \pm\sqrt{d}]}\right\}^{1/2} \quad , \quad (4.13)$$

where

$$c = \mu(k^2 - 3) + 4\nu' \quad ,$$

$$d = [c+(1+k^2)^2\eta_c]^2 + 8\mu(3-k^2)(1+k^2)^2(\eta_c-\eta) \quad . \qquad (4.14)$$

From the stability analysis of S[1] state, only the solution with the choice of the negative sign for $\pm\sqrt{d}$ in eqs.(4.12) and (4.13) is acceptable. The dependence of H_{2s} on the "order parameter" η is shown schematically, in Fig.1. The excitation as shown in Fig.1 (a) is a soft type in which the amplitude varies continuously with η under the condition $c + (1+k^2)^2\eta_c \gtrless 0$. On the contrary, for $c + (1+k^2)^2\eta_c < 0$ and $3 > k^2$, as shown in Fig.1 (b), the amplitude of H_{2s} increases abruptly to some finite value as soon as the value η becomes less than the value η_c, and drops abruptly to zero for $\eta=\eta_0>\eta_c$ where

$$\eta_0 = \eta_c + [c+(1+k^2)^2\eta_c]^2/[8\mu(3-k^2)(1+k^2)^2] . \qquad (4.15)$$

This is the hard excitation.[6)]

In this stage, the particle flux and the heat fluxes are given by

$$\Gamma_x$$

$$= D_{c\perp}|\kappa|N_0[1+ \frac{\sqrt{d}+2\mu(1-k^2)-\mu(1+k^2)\nu-4\nu'-2\nu''}{2\mu(3-k^2)}] \ . \qquad (4.16)$$

$$Q_{ex}(x=\ell/2)$$

$$= N_0\chi^e_{c\perp}|\kappa_e|T_e + N_0D_{c\perp}|\kappa|T_e\frac{\sqrt{d}+2\mu(1-k^2)-\mu(1+k^2)\nu-4\nu'-2\nu''}{2\mu(3-k^2)}$$

$$\times[1+ \frac{(4\ell|\kappa|/\pi)(1+\beta)(1+k^2)D_{c\parallel}/\chi^e_{c\parallel}+8\nu''/(1+\beta)}{\sqrt{d}+4\mu k^2+\mu(1+k^2)\nu-4\nu'+2\nu''}] \ , \qquad (4.17)$$

$Q_{ix}(x=\ell/2)$

$$= N_0\chi^e_{c\perp}|\kappa_i|T_i + N_0 D_{c\perp}|\kappa|T_i \frac{\sqrt{d}+2\mu(1+k^2)-\mu(1+k^2)\nu-4\nu'-2\nu''}{2\mu(3-k^2)}$$

$$\times[1+\frac{8\ell^2\nu'/(\pi^2 a_i^2)}{\sqrt{d}+4\mu k^2+\mu(1+k^2)\nu-4\nu'+2\nu''}] \quad , \qquad (4.18)$$

where

$$\chi^e_{c\perp} = 4.66\nu_{ei}T_e/m_e\Omega_e^2 \quad , \quad \chi^i_{c\perp} = 2\nu_{ii}T_i/m_i\Omega_i^2 \quad .$$

(3) Second stage

If $\eta_1 > \eta > \eta_2 \equiv [\mu(1+\nu)(9+k^2)+2\nu'']/(9+k^2)^2$, then F_1 and F_2 become unstable associated with H_1 and H_2. Consequently we must consider four coupled equations for F_1, F_2, H_1 and H_2. There is another branch of steady state S[2], with all $F_1, F_2, H_1, H_2 \neq 0$ in addition to S[1]. The stability of these solutions depend on the value the parameters such as k, ν, ν' and ν'' .

(4) Subsequent stages

As η decreases further, the number of unstable modes increases, and the state becomes turbulent.

§5 The Concluting Remarks

The nonlinear evolution of the collisional drift wave and the consequent particle and heat transport have been studied by using a systematic expansion.

The most important interaction is shown to be quasi-linear one in real space. The elctron and ion temperature perturbations besides the density one are considered. In the first order, we find that the amplitude of the density perturbation is equal to that of the potential perturbation. The temperature perturbations are found to be zero, which comes from that in the present ordering scheme the electron and ion heat conductivities along the magnetic field are large enough. In the second order, the characteristic frequency is determined by taking the modification of the background density into consideration and the phase shift between the densiy perturbation and the potential one are obtained. The temperature perturbations are given in this order. In the third order, the nonlinear evolution equations for the modification of the background density and the drift wave are derived. The fact that the modification of the background density flattens the initial density gradient and reduces the growth rate of the drift wave is consistently incorporated into these equations. The particle and heat fluxes are calculated. It should be noted that in the particle and electron heat fluxes the electron and ion temperature gradients contribute to carry particle and heat from the lower to the higher temperature region, contrary to the ordinary effect of the density gradient. We may understand that this effect is originated from a tendency of nT_e= const. through the parallel motions of the electrons. In the ion heat flux, the contribution of the ion temperature gradient behaves in the same way as that of the density gradient.

Interesting properties of nonlinear behaviours of the system under consideration have been revealed by the proposed model equations given in eq.(4.3) and (4.4). Nonlinear states are characterized by the order parameter η defined in eq.(4.5) from marginal stability to successive states with increasing number of unstable modes. In the first stage where only one mode is unstable, we find the steady state solution which is nonlinearly stable. A remarkable feature is the occurence of the hard excitation which arises likely when the ion temperature gradient is in the same direction as the density gradient. We also have the case of the soft excitation according to the values of parameters.

For the subsequent stages the analysis is not easy. We have worked out a numerical computation for the isothermal case[7)] and found that in the second stage where two modes are unstable, there exists no steady solution, the whole problem becomes time-dependent and the amplitudes of the interacting modes are bounded. We may expect similar results for the non-isothermal case. Numerical computation for this case is in progress and will be reported elsewhere.

References

1) T. Hatori and Y. Terashima: J. Phys. Soc. Japan 42 (1977) No.3 (to be published).

2) T. Hatori, K.I. Nishi-Kawa and Y. Terashima: Presented at 6th Int. Conf. Plasma Physics and Controlled Nuclear Fusion Research (1976), CN-35/D9.

3) W. M. Tang et al: Presented at 6th Int. Conf. Plasma Physics and Controlled Nuclear Fusion Research (1976), CN-35/B15-2.

4) R. Saison and H. K. Wimmel: Presented at 6th Int. Conf. Plasma Physics and Controlled Nuclear Fussion Research (1976), CN-35/B15-1.

5) S. J. Braginskii: Reviews of Plasma Physics ed. M. A. Leontovich (Consultant Bureau, New York, 1965) Vol.1. p.205.

6) B. B. Kadomtsev: Plasma Turbulence (Academic Press, New York, 1965) p.5.

7) T. Hatori and Y. Terashima: Research Report of Institute of Plasma Physics, Nagoya University, IPPJ-250 (1976).

Figure Captions

Fig.1. (a) Soft exitation for $c+(1+k^2)^2\eta_c > 0$.

The amplitude of H_{2s} varies continuously with η.

(b) Hard excitation for $c+(1+k^2)^2\eta_c < 0$ and $k^2 < 3$.

The amplitude of H_{2s} increases abruptly to some finite value as soon as the value η becomes less than η_c. When η increases, it drops abruptly to zero for $\eta = \eta_0 > \eta_c$. The stable and unstable states are indicated by the solid and dushed lines, respectively.

Appendix

Expressions of $L^{(0)}$, $L^{(1)}$ and $L^{(2)}$ are

$$L^{(0)}=\begin{bmatrix} -D_{C''}\frac{\partial^2}{\partial z^2}, & D_{C''}\frac{\partial^2}{\partial z^2}, & -(1+\beta)D_{C''}\frac{\partial^2}{\partial z^2}, & 0 \\ 0, & 0, & 0, & 0 \\ -(1+\beta)D_{C''}\frac{\partial^2}{\partial z^2}, & (1+\beta)D_{C''}\frac{\partial^2}{\partial z^2}, & -[(1+\beta)D_{C''}+\chi^e_{C''}]\frac{\partial^2}{\partial z^2}, & 0, \\ 0, & 0, & 0, & -\chi^i_{C''}\frac{\partial^2}{\partial z^2} \end{bmatrix}$$

$$L^{(1)}=|L^{(1)}_{ij}| \quad (i,j = 1,2,3,4,)$$

where

$$L^{(1)}_{11} = \frac{\partial}{\partial t_1} + \kappa \times D_{C''}\frac{\partial^2}{\partial z^2}, \qquad L^{(1)}_{12} = v^*_n \frac{\partial}{\partial y_0} - \kappa \times D_{C''}\frac{\partial^2}{\partial z^2},$$

$$L^{(1)}_{13} = \frac{3}{2}u_0\frac{\partial}{\partial z} + (1+\beta)\kappa \times D_{C''}\frac{\partial^2}{\partial z^2}, \qquad L^{(1)}_{14} = 0,$$

$$L^{(1)}_{21} = \frac{\partial}{\partial t_1}(1+\hat{b}) - \frac{1}{2}v^*_{T_i}\hat{b}\frac{\partial}{\partial y_0}, \qquad L^{(1)}_{22} = v^*_n\frac{\partial}{\partial y_0} + \lambda\frac{\partial}{\partial t_1}\hat{b},$$

$$L^{(1)}_{23} = 0, \qquad L^{(1)}_{24} = \frac{\partial}{\partial t_1}\hat{b} - (2\lambda)^{-1}v^*_n\hat{b}\frac{\partial}{\partial y_0},$$

$$L^{(1)}_{31} = u_0\frac{\partial}{\partial z} + (1+\beta)\kappa \times D_{C''}\frac{\partial^2}{\partial z^2},$$

$$L^{(1)}_{32} = \frac{3}{2}v^*_{T_e}\frac{\partial}{\partial y_0} - 2u_0\frac{\partial}{\partial z} - (1+\beta)\kappa \times D_{C''}\frac{\partial^2}{\partial z^2},$$

$$L^{(1)}_{33} = \frac{3}{2}\frac{\partial}{\partial t_1} + (5+\frac{7}{2}\beta)u_0\frac{\partial}{\partial z} + (1+\beta)\kappa \times D_{C''}\frac{\partial^2}{\partial z^2} + \kappa \times \chi^e_{C''}\frac{\partial^2}{\partial z^2},$$

$$L^{(1)}_{34} = 0,$$

$$L_{41}^{(1)} = 0 \quad , \qquad L_{42}^{(1)} = -\frac{3}{2} v_{Ti}^{*}\frac{\partial}{\partial y_0} \quad ,$$

$$L_{43}^{(1)} = 0 \quad , \qquad L_{44}^{(1)} = \frac{3}{2}\frac{\partial}{\partial t_1} \quad ,$$

$$L^{(2)} = |L_{ij}^{(2)}| \quad (i,j = 1,2,3,4)$$

where

$$L_{11}^{(2)} = \frac{\partial}{\partial t_2} - v_g\frac{\partial}{\partial y_1} - D_{c\perp}\Delta_\perp - (\kappa x)^2 D_{c\parallel}\frac{\partial^2}{\partial z^2} \, ,$$

$$L_{12}^{(2)} = v_n^{*}\frac{\partial}{\partial y_1} - \kappa x v_n^{*}\frac{\partial}{\partial y_0} + (\kappa x)^2 D_{c\parallel}\frac{\partial^2}{\partial z^2} \, ,$$

$$L_{13}^{(2)} = \frac{1}{4}\nu_{ei}a_e^2\Delta_\perp \quad , \qquad L_{14}^{(2)} = -(2\lambda)^{-1}\nu_{ei}a_e^2\Delta_\perp \quad ,$$

$$L_{21}^{(2)} = \left(\frac{\partial}{\partial t_2} - v_g\frac{\partial}{\partial y_1}\right)(1+\hat{b}) - D_{c\perp}\Delta_\perp + \frac{3}{10}\nu_{ii}\hat{b}^2 - \frac{1}{2}v_{Ti}\hat{b}\frac{\partial}{\partial y_1}$$

$$+ \Omega_i^{-1}(\lambda^{-1}v_n^{*} - v_{Ti}^{*})\frac{\partial^2}{\partial x\partial t_1} + \Omega_i^{-1}v_{T_i}^{*}(v_{T_i}^{*} - \lambda^{-1}v_n^{*})\frac{\partial^2}{\partial x\partial y_0} \, ,$$

$$L_{22}^{(2)} = \lambda\left(\frac{\partial}{\partial t_2} - v_g\frac{\partial}{\partial y_1}\right)\hat{b} + \frac{3}{10}\lambda\nu_{ii}\hat{b}^2 + v_n^{*}\frac{\partial}{\partial y_1} - \kappa x v_n^{*}\frac{\partial}{\partial y_0}$$

$$+ \Omega_i^{-1}(v_n^{*} + 2v_{Te}^{*})\frac{\partial^2}{\partial x\partial t_1} - \Omega_i^{-1}v_n^{*}(v_{Ti}^{*} + \lambda^{-1}v_n^{*})\frac{\partial^2}{\partial x\partial y_0} \, ,$$

$$L_{23}^{(2)} = \frac{1}{4}\nu_{ei}a_e^2\Delta_\perp \quad ,$$

$$L_{24}^{(2)} = \left(\frac{\partial}{\partial t_2} - v_g\frac{\partial}{\partial y_1}\right)\hat{b} - (-2\lambda)^{-1}\nu_{ei}a_e^2\Delta_\perp + \frac{3}{4}\nu_{ei}\hat{b}^2$$

$$+ 2(\lambda\Omega_i)^{-1}v_n^{*}\frac{\partial^2}{\partial x\partial t_1} + (\lambda\Omega_i)^{-1}v_n^{*}v_{Ti}^{*}\frac{\partial^2}{\partial x\partial y_0} \quad ,$$

$$L_{31}^{(2)} = -(1+\beta)(\kappa x)^2 D_{c\parallel}\frac{\partial^2}{\partial z^2} \, ,$$

$$L_{32}^{(2)} = \frac{3}{2}v_{Te}^{*}\frac{\partial}{\partial y_1} + (1+\beta)(\kappa x)^2 D_{c\parallel}\frac{\partial^2}{\partial z^2} \, ,$$

$$L_{33}^{(2)} = \frac{3}{2}\left(\frac{\partial}{\partial t_2} - v_g\frac{\partial}{\partial y_1}\right) , \qquad L_{34}^{(2)} = 0 ,$$

$$L_{41}^{(2)} = 0 , \qquad L_{42}^{(2)} = -\frac{3}{2}v_{T_i}^{*}\frac{\partial}{\partial y_1} ,$$

$$L_{43}^{(2)} = 0 , \qquad L_{44}^{(2)} = \frac{3}{2}\left(\frac{\partial}{\partial t_2} - v_g\frac{\partial}{\partial y_1}\right) ,$$

where a_e is the electron gyroradius.

Nonlinear terms S are expressed in terms of dimensionless quantities $\rho = \frac{\tilde{n}}{N(x)}$, $\psi = \frac{e\tilde{\phi}}{Te(x)}$, $\zeta_e = \frac{\tilde{T}_e}{Te(x)}$ and $\zeta_i = \frac{\tilde{T}_i}{Ti(x)}$

$$S = \begin{bmatrix} S_e \\ S_i \\ S_{Te} \\ S_{Ti} \end{bmatrix}$$

$$S_e = -\rho v_n^{*}\frac{\partial\psi}{\partial y} - \frac{T_e}{m_e\Omega_e}(\vec{b}\times\vec{\nabla}_\perp\psi)\cdot\vec{\nabla}_\perp\rho + D_{c\parallel}\frac{\partial^2}{\partial z^2}[\log(1+\rho)-\rho]$$

$$+ D_{c\perp}\frac{1}{2N^2}\Delta_\perp(N^2\rho^2) + \psi v_{T_e}^{*}\frac{\partial\rho}{\partial y} - D_{c\parallel}\zeta_e\frac{\partial^2}{\partial z^2}\rho - D_{c\parallel}\frac{\partial}{\partial z}\zeta_e\frac{\partial}{\partial z}\rho ,$$

$$S_i = -\rho v_n^{*}\frac{\partial\psi}{\partial y} - \frac{T_e}{m_e\Omega_e}(\vec{b}\times\vec{\nabla}_\perp\psi)\cdot\nabla_\perp\rho + D_{c\perp}\frac{1}{2N^2}\Delta_\perp(N^2\rho^2) + \psi v_{Te}^{*}\frac{\partial\rho}{\partial y}$$

$$+ \frac{1}{2}\lambda a_i^2(\vec{\nabla}_\perp\rho\cdot\vec{\nabla}_\perp + \rho\Delta_\perp)\frac{\partial\psi}{\partial t_1} + \frac{1}{2}a_i^2(\Delta_\perp\rho + \vec{\nabla}_\perp\rho\cdot\vec{\nabla}_\perp)\frac{\partial\rho}{\partial t_1}$$

$$+ \frac{1}{8}\Omega_i a_i^4\{(\vec{\nabla}_\perp\zeta_i\cdot\vec{b}\times\vec{\nabla}_\perp)\Delta_\perp\rho - (\vec{\nabla}_\perp\rho\cdot\vec{b}\times\vec{\nabla}_\perp)\Delta_\perp\zeta_i$$

$$+2\left[\frac{\partial^2}{\partial x\partial y}\zeta_i\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\rho - \frac{\partial^2}{\partial x\partial y}\rho\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\zeta_i\right]\}$$

$$- \frac{1}{4}\lambda\Omega_i a_i^4\{\Delta_\perp\rho(\vec{b}\times\vec{\nabla}_\perp\psi\cdot\vec{\nabla}_\perp\rho) + \vec{\nabla}_\perp\rho\cdot\vec{\nabla}_\perp(\vec{b}\times\vec{\nabla}_\perp\psi\cdot\vec{\nabla}_\perp\rho)$$

$$-(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp})\Delta_{\perp}\rho+(\vec{b}\times\vec{\nabla}_{\perp}\rho\cdot\vec{\nabla}_{\perp})\Delta_{\perp}\psi$$

$$-2[((\vec{\nabla}_{\perp}\frac{\partial\psi}{\partial x})\frac{\partial}{\partial y}-(\vec{\nabla}_{\perp}\frac{\partial\psi}{\partial y})\frac{\partial}{\partial x})\cdot\vec{\nabla}_{\perp}\rho]$$

$$+\kappa[-\frac{\partial\rho}{\partial y}\Delta_{\perp}\psi-(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp})\frac{\partial\rho}{\partial x}$$

$$+\rho\frac{\partial}{\partial y}(\Delta_{\perp}\psi)+2\vec{\nabla}_{\perp}\rho\cdot\vec{\nabla}_{\perp}\frac{\partial\psi}{\partial y}]\}$$

$$+\frac{1}{4}\lambda^{2}\Omega_{i}a_{i}^{4}[(1+\rho)(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp})\Delta_{\perp}\psi+\kappa(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp})\frac{\partial\psi}{\partial x}$$

$$+\vec{\nabla}_{\perp}\rho\cdot(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp})\vec{\nabla}_{\perp}\psi]$$

$$-\frac{1}{4}\lambda\Omega_{i}a_{i}^{4}\{\kappa_{i}[-(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp})\frac{\partial\rho}{\partial x}+(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp})\frac{\partial\psi}{\partial x}+\Delta_{\perp}(\rho\frac{\partial\psi}{\partial y})]$$

$$-\Delta_{\perp}(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp}\zeta_{i})\}$$

$$+\frac{1}{4}\lambda\Omega_{i}a_{i}^{4}\{(\kappa+\kappa_{i})\rho\frac{\partial}{\partial y}\Delta_{\perp}\psi+(\vec{b}\times\vec{\nabla}_{\perp}\rho\cdot\vec{\nabla}_{\perp})\Delta_{\perp}\psi+(\vec{b}\times\vec{\nabla}_{\perp}\zeta_{i}\cdot\vec{\nabla}_{\perp})\Delta_{\perp}\psi$$

$$+\frac{\partial^{2}}{\partial x\partial y}\psi[(\frac{\partial^{2}}{\partial x^{2}}-\frac{\partial^{2}}{\partial y^{2}})(\rho+\zeta_{i})+2(\kappa+\kappa_{i})\frac{\partial\rho}{\partial x}]$$

$$-(\frac{\partial^{2}}{\partial x^{2}}-\frac{\partial^{2}}{\partial y^{2}})\psi[(\kappa+\kappa_{i})\frac{\partial\rho}{\partial y}+\frac{\partial^{2}\rho}{\partial x\partial y}+\frac{\partial^{2}\zeta_{i}}{\partial x\partial y}]\}$$

$$S_{T_e}=-\frac{3}{2}\frac{T_e}{m_e\Omega_e}(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp}\zeta_e)-\chi^{e}_{c\parallel}\rho\frac{\partial^{2}}{\partial z^{2}}\zeta_e$$

$$-D_{c\parallel}[(1+\beta)(1-\rho)\frac{\partial^{2}}{\partial z^{2}}(\frac{\rho^{2}}{2}-\frac{\rho^{3}}{3})+(1+\beta)^{2}\rho\frac{\partial^{2}}{\partial z^{2}}\zeta_e$$

$$-(1+\beta)\zeta_e\frac{\partial^{2}}{\partial z^{2}}\rho+(1+\beta)\rho\frac{\partial^{2}}{\partial z^{2}}(\rho^{(2)}-\psi^{(2)})$$

$$+\frac{\partial}{\partial z}\log(1+\rho)\frac{\partial}{\partial z}(\rho^{(2)}-\psi^{(2)}-\frac{1}{2}\rho^{2})]$$

$$-u_0\rho\frac{\partial}{\partial z}\rho\quad,$$

$$S_{T_i}=\frac{3}{2}\frac{T_e}{m_e\Omega_e}(\vec{b}\times\vec{\nabla}_{\perp}\psi\cdot\vec{\nabla}_{\perp}\zeta_i)-\chi^{i}_{c\parallel}\rho\frac{\partial^{2}}{\partial z^{2}}\zeta_i$$

The expressions of $S_e^{(3)}$, $S_i^{(3)}$, $S_{T_e}^{(3)}$ and $S_{T_i}^{(3)}$ which appear in eqs.(3.33) - (3.36) are

$$S_e^{(3)} = \frac{T_e}{m_e\Omega_e}\,(\vec{b}\times\vec{\nabla}_\perp f)\cdot\vec{\nabla}_\perp[\,\psi^{(2)} - \rho^{(2)}] + D_{c''}\frac{\partial^2}{\partial z^2}[\,\tfrac{1}{3}f^3 - \tfrac{1}{2}f\rho^{(2)}] \quad ,$$

$$S_i^{(3)} = \frac{T_e}{m_e\Omega_e}\,(\vec{b}\times\vec{\nabla}_\perp f)\cdot\vec{\nabla}_\perp[\,\psi^{(2)} - \rho^{(2)}] + \tfrac{1}{4}\lambda\Omega_i a_i^4(\vec{b}\times\vec{\nabla}_\perp\zeta_i^{(2)}\cdot\vec{\nabla}_\perp)\Delta_\perp f$$

$$+\ \tfrac{1}{4}\lambda(\lambda+1)\Omega_i a_i^4[(\vec{b}\times\vec{\nabla}_\perp f\cdot\vec{\nabla}_\perp)\Delta_\perp\rho^{(2)} + (\vec{b}\times\vec{\nabla}_\perp\rho^{(2)}\cdot\vec{\nabla}_\perp)\Delta_\perp f] \quad ,$$

$$S_{T_e}^{(3)} = -\frac{3}{2}\frac{T_e}{m_e\Omega_e}\,(\vec{b}\times\vec{\nabla}_\perp f)\cdot\vec{\nabla}_\perp\zeta_e^{(2)} + (1+\beta)D_{c''}\frac{\partial^2}{\partial z^2}[\,\tfrac{1}{3}f^3 - \tfrac{1}{2}f\rho^{(2)}] \quad ,$$

$$S_{T_i}^{(3)} = -\frac{3}{2}\frac{T_e}{m_e\Omega_e}\,(\vec{b}\times\vec{\nabla}_\perp f)\cdot\vec{\nabla}_\perp\zeta_i^{(2)} \quad .$$

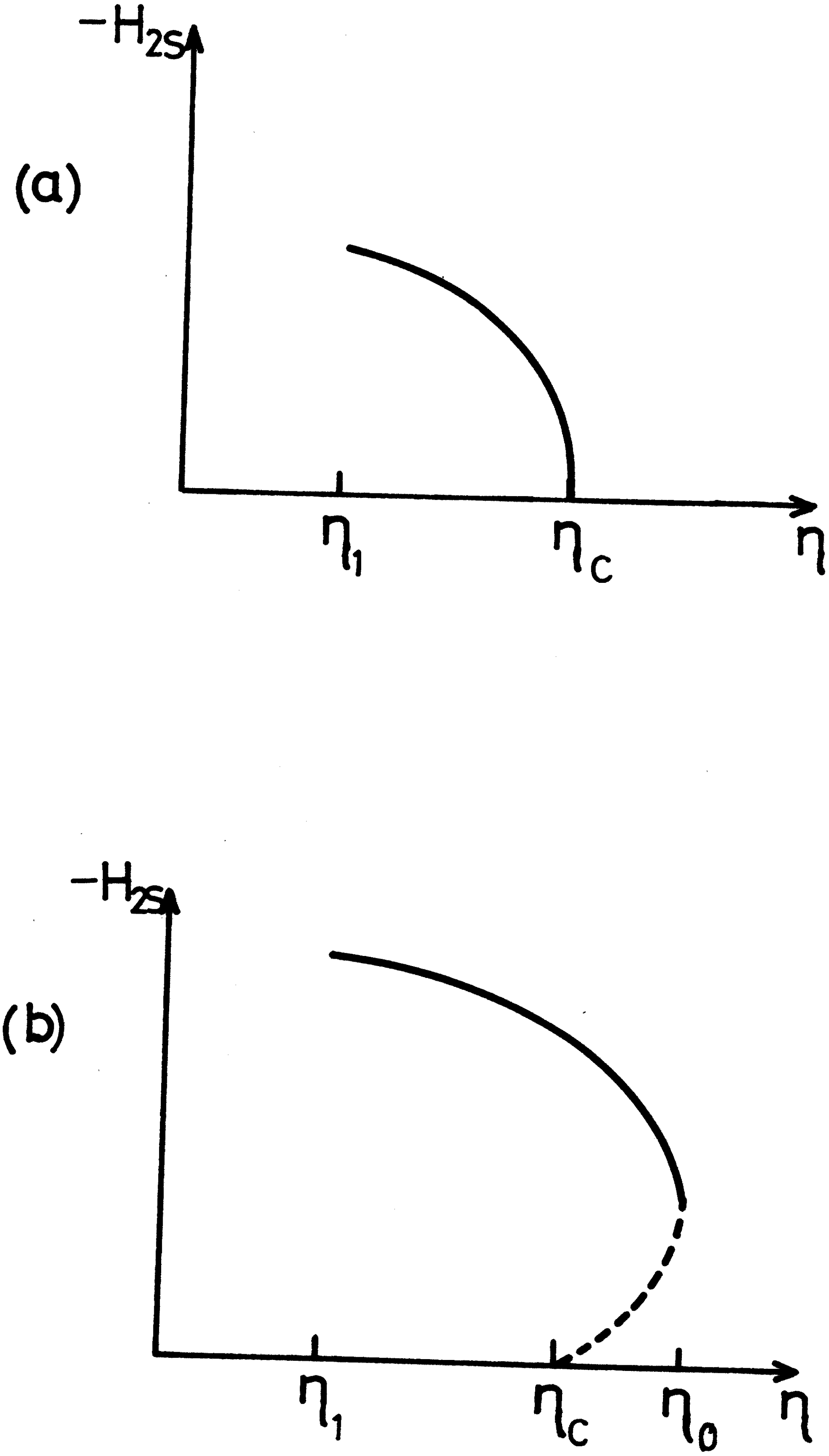

FIG. 1