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End Loss Reduction in Linear Devices by Means of Oscillating Multiple Mirrors

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ABSTRACT

We consider briefly the effects of adding oscillating multiple mirrors to a linear θ-pinch like plasma column. The plasma parameters chosen for the example shown correspond to those of a previously proposed linear reactor configuration. We show that it should be easy to dynamically stabilize the inherently unstable bumpy θ-pinch which results from adding static mirrors to a linear θ-pinch. We show further that the energy in the dynamic stabilization circuit can be used for plasma heating since the oscillating field components give rise to strongly Landau damped ion-acoustic waves.

1. INTRODUCTION

The purpose of this note is to consider a variaton of a recently proposed method for reducing particle end loss from linear θ-pinch like plasmas. Specifically we consider the application of multiple magnetic mirrors[1] applied to the uniform longitudinal magnetic field characteristic of a θ-pinch plasma. What follows can be equally applied to any linear solenoidal plasma configuration regardless of the initial phase of plasma production, i.e., implosion, electron beam or laser heated. There are a number of techniques which have been recently proposed for controlling particle end loss. Among these are material end plugs, cold gas end plugs, magnetic cusp end-fields and static multiple mirrors. In our opinion the multiple mirror concept is the most attractive for the reasons which we outline in the following.

We propose a version of the multiple mirror concept where an oscillating component of the mirror field is externally applied through an rf source. The total magnetic field consists of three parts; the expression for the z-component of the field is assumed to be $B_o + B_1 \sin kz + b \sin kz\, e^{i\omega t}$ where B_o is the main θ-pinch field, B_1 is the static mirror field and b is the time varying component of the mirror field. The frequency ω will be defined later and $k = 2\pi/\ell$ where ℓ is the period of the mirrors assumed to be equally spaced along the axis of the plasma column. The static mirror field is that magnetic field component responsible for the end loss reduction by means of the multiple mirror concept. It is this field component, however, that is also responsible for the "bumpy" θ-pinch equilibrium known to be $m = 1$ unstable according to linear stability theory.[2] It is primarily for this reason that the oscillating component of magnetic field is applied; we show that dynamic stabilization of the $m = 1$ mode may be possible at very reasonable power levels. An extra positive feature of the oscillating magnetic field is that the energy in the external rf source can be transferred directly to the ions by Landau damping of ion sound waves. In the following analysis it will be assumed that $b \ll B_1 < B_o$.

We do not discuss here in any detail the theory of particle trapping resulting from the application of multiple mirrors. We wish to use however the most recent result due to Lieberman[1] which shows the following scaling for the axial particle confinement time τ:

$$\tau \sim M^2L^2/\lambda v_{Ti} \ ,$$

where M is the maximum mirror ratio, L is the half length of plasma column, λ is the ion-ion mean free path and v_{Ti} is the mean ion thermal velocity. If the ordering scheme shown for the magnetic field component holds, then the ratio $M \cong (B_o + B_1)/[(B_o - B_1)(1 - \beta)^{1/2}]$. It is obvious that a high-β plasma column is desirable since the local mirror ratio in the plasma interior may be very large even though the externally applied mirror ratio is not large. This in turn works beneficially for the stability of the column since it is the externally applied mirror fields which control the growth rate of the $m = 1$ mode. Thus the conclusion at this point is that the multiple mirror configuration has a potential figure of merit of M^2L/λ over the classical value of θ-pinch confinement time. We admit that we have ignored possible effects on the trapping mechanism coming from the oscillating magnetic field. We have begun an investigation of the multiple mirror trapping under the influence of oscillating fields and will report on this in the future.

The remainder of this note is concerned with dynamic stabilization and rf heating considerations.

2. DYNAMIC STABILIZATION

The basic unstable mode which we are concerned with is the long wavelength, $m = 1$ mode. The stability of this mode in a bumpy θ-pinch was first considered by Haas and Wesson[2] using a thin skin approximation for the equilibrium. Use of the energy principle yields the following expression for the linear growth rate γ:

$$\gamma^2 = \frac{k^2 B_1^2}{\mu_o \rho_o} \frac{\beta(3-2\beta)}{4(1-\beta)^2} \left[\frac{1 - \beta(1+a^2/a_w^2)}{2 - \beta(1-a^2/a_w^2)} \right] \qquad (1)$$

where ρ_o is the equilibrium density, k the wavenumber of the applied magnetic field, a and a_w the plasma and wall radius respectively. This dispersion relation is valid only for the "gross" mode which has $n = 0$ where n is the number of radial nodes. According to (1), the gross mode is stabilized for $\beta > \beta_w$ where

$$\beta_w = 1/(1 + (a^2/a_w^2)) \qquad (2)$$

Recently, diffuse profile theory[3] has shown that for $\beta > \beta_w$ "localized" modes with radial structure appear and depending on the value of a/a_w can have very fast growth rates. The eigenfunctions for the higher n modes indeed show a localized structure which may be suppressed by finite Larmor radius stabilization.[4] Eigenfunctions with small but not zero radial node numbers have been found at $\beta > \beta_w$ which have rather broad radial extent. Finite Larmor radius effects should not be too effective in suppressing these modes; however, it has been shown[3] that by increasing the steepness of the profiles the growth rates of these modes can be made small. At high β and high temperatures we expect the plasma profiles to be flat over the center of the column with reasonably steep gradients. Thus there is reason to believe that the growth rates of modes for which $\beta > \beta_w$ will be smaller than the maximum for $\beta < \beta_w$ as long as β does not approach too close to unity.

For the present, let us choose $\beta = 0.8$ and a value of $a_w^2/a^2 = 4.5$ (facilitating a comparison with numerical results[3] for diffuse profile stability) and compute from (1) the growth rate of the m = 1 gross mode as

$$\gamma^2 \simeq 0.1\ k^2 B_1^2/\rho_o \mu_o \quad . \tag{3}$$

The following discussion of dynamic stabilization of the mode whose growth rate is given by (3) is due to Berge's[5] early work which considers a thin skin model at the bumpy θ-pinch. A requirement for dynamic stabilization is that the externally applied oscillation frequency ω be much larger than the growth rate γ. Let us require $\gamma = 0.1\ \omega$ and choose a maximum mirror ratio $M = 2.7$ with a background θ-pinch field of 250 kilogauss. The choice of θ-pinch parameters is based on reactor considerations due to Logan[6] et. al. The choice of M yields a value of $B_1 = 25$ kilogauss. Borrowing reactor parameters again, we choose $\ell = 5\text{m}$ and $n_o \sim 8 \times 10^{16}\ \text{cm}^{-3}$. These choices yield a value of $\omega/2\pi \sim 10^5$ Hz. We shall see that this value of ω is very attractive from the standpoint of heating as well as dynamic stabilization. We now invoke the sufficient stability condition due to Berge[5];

$$\frac{Y^2 + 1}{G(\beta)} > \frac{4\delta^2}{\varepsilon^2} \quad , \tag{4}$$

where

$$Y = \frac{x_o J_o(x_o)}{J_1(x_o)} \left(1 - \beta + \frac{\Gamma\beta}{2} A\right) \quad ,$$

$$x_o = ka \frac{\omega}{kC_s} \left\{ \frac{1 - \frac{2}{\Gamma} \frac{1-\beta}{\beta} \left(\frac{kC_s}{\omega}\right)^2}{1 + \frac{2}{\Gamma} \frac{1-\beta}{\beta} - \left(\frac{kC_s}{\omega}\right)^2} \right\} \quad ,$$

$$G(\beta) = \frac{\beta(1-\beta)(3-2\beta)}{2-\beta} \quad ,$$

$$C_s^2 = \frac{\Gamma P}{\rho} \quad ,$$

$$A = \frac{1}{1 - \left(\frac{kC_s}{\omega}\right)^2} \quad .$$

The stability calculation has assumed that a cylindrically symmetric, "m = 0", oscillating magnetic field is applied to the column. Here the small parameter δ can be considered to be equivalent to a/ℓ and ε is equivalent to b/B_o. If we use the following values: $\Gamma = 5/3$, $\beta = 0.8$, $T_i \simeq 4.5$ keV, $ka = 6\pi/500$, $\delta = 3/500$ and the value for ω obtained by using $n = 8 \times 10^{16}$ cm^{-3}, where n is the deuterium particle density we find that condition (4) is satisfied for $\varepsilon \geq 7 \times 10^{-4}$ or $b \sim 7 \times 10^{-4}\ B_o = 155$ gauss. Thus, if b > 155 gauss, dynamic stabilization of the gross m = 1 mode should be expected. The frequency ω turns out to be around 100 kHz which is comfortably low. We shall see that if b is made much larger than the value needed for stabilization then considerable ion heating is obtained by the external rf source.

3. rf ABSORPTION

In this section we consider the effect of the oscillating component of the magnetic field on the plasma column. Since $b/B_o < b/B_1 \ll 1$ we will use linearized equations of motion and in particular we choose the guiding center plasma (GCP) model due to Grad.[7] The plasma column is considered to be cylindrically symmetric and infinitely long; there is only a magnetic field in the longitudinal direction. All quantities are perturbed according to $g(r,\theta,z,t) = g(r)\exp i(-\omega t + m\theta + kz)$ and consistent with the last section we set $m = 0$. The governing equations of motion in the form[8] convenient for our analysis are:

$$\frac{dp^*}{dr} = b_1\, r\xi_r \quad , \tag{5}$$

$$\frac{dr\xi_r}{dr} = b_2\, p^* \tag{6}$$

where $p^* = \underline{p}_\perp + \underline{b}\cdot\underline{B_o}/\mu_o$ is the total perturbed perpendicular pressure and ξ_r is the radial plasma displacement. The coefficients appearing in (5) and (6) are:

$$b_1 = -\frac{A}{r}$$

$$\frac{1}{b_2} = -\frac{B_o^2}{r\mu_o}\, a_1$$

$$A = -\rho_o\omega^2 + \left(1 - \frac{P_{\parallel o} - P_{\perp o}}{B_o^2/\mu_o}\right)\frac{k^2 B_o^2}{\mu_o}$$

$$a_1 = 1 + \frac{P_{\perp o}}{B_o^2} 2\mu_o + \mu_o \Theta_1 \quad .$$

In the above the subscript $(\ )_o$ denotes equilibrium quantity. For clarity we suppress the subscript in the following. Kinetic effects show up in the coefficient a_1 through the function Θ_1. Assuming a bi-Maxwellian distribution function for both ions and electrons Θ_1 can be simplified to:[8]

$$\Theta_1 = I(2,0) - I^2(1,1)/I(0,2) \quad ,$$

$$I(\ell,n) = I^+(\ell,n) + (-1)^n I^-(\ell,n) \quad ,$$

$$I^\pm(2,0) = \frac{2}{P_{\parallel}^\pm} \left(\frac{P_\perp^\pm}{B_o}\right)^2 (1 + \xi^\pm Z(\xi^\pm)) \quad ,$$

$$I^\pm(1,1) = -\frac{1}{P_{\parallel}^\pm} \frac{P_\perp^\pm}{B_o} (1 + \xi^\pm Z(\xi^\pm)) \quad ,$$

$$I^\pm(0,2) = -\frac{1}{P_{\parallel}^\pm} (1 + \xi^\pm Z(\xi^\pm)) \quad ,$$

where $\pm$ refer to ion, electron, respectively and $Z(\xi)$ is the plasma dispersion function with argument ξ defined as

$$\xi^\pm = \frac{\omega}{k} \left(\frac{m^\pm}{2T_{\parallel}^\pm}\right)^{1/2} \quad .$$

Equations (5) and (6) can be combined to give the following equation:

$$\frac{d}{dr}\left(\frac{r}{A}\frac{dp^*}{dr}\right) - \frac{r\mu_o}{B_o^2} \frac{1-\beta}{(1 + (1-\beta)\mu_o\Theta_1)} p^* = 0 \quad . \qquad (7)$$

The θ-pinch equilibrium relations, $P_\perp = (B_o^2 - B^2(r=0))/2\mu_o$ and $\beta = 1 - B^2(r=0)/B_o^2$, have been used in arriving at (7). For the plasma parameters described earlier the following relations hold.

$$\frac{\omega}{k} \sim \left(\frac{2T_{||}^+}{m_+}\right)^{1/2} << \left(\frac{2T_{||}^-}{m_-}\right)^{1/2} .$$

If we assume isotropic and equal temperatures, $T_e = T_i$, equation (7) reduces to

$$\frac{d}{dr}\left(\frac{r}{A}\frac{dp^*}{dr}\right) - \frac{r\mu_o}{B_o^2}\,\frac{(1-\beta)\ (\alpha+1)}{(\alpha+1) - \frac{\beta}{2}\,(\alpha^2+6\alpha+1)}\, p^* = 0 , \qquad (8)$$

where

$$\alpha = 1 + \xi^+\ Z(\xi^+) .$$

Assuming a thin skin θ-pinch equilibrium (8) can be put into the following form:

$$\frac{d^2p^*}{dr^2} + \frac{1}{r}\frac{dp^*}{dr} - k^2\, Q\, p^* = 0 , \qquad (9)$$

where

$$Q = \left(1 - \frac{\omega^2}{k^2V_A^2}\right)\frac{(1-\beta)\ (\alpha+1)}{(\alpha+1) - \frac{\beta}{2}\,(\alpha^2+6\alpha+1)}$$

and $V_A^2 = B_o^2(1-\beta)/\rho_o\mu_o$. The solution of (9) which is regular at the origin is given by $I_o(\sqrt{Q}\ kr)$ where I_o is a modified Bessel function. Use of the well known vacuum solutions for the perturbed magnetic field in cylindrical geometry plus jump conditions for b_r, the radial perturbed magnetic field, and p^* across the plasma-vacuum interface yields the following dispersion relation:

$$\frac{\sqrt{Q}}{1 - \dfrac{\omega^2}{k^2 v_A^2}} \; \frac{I_o'(\sqrt{Q}\, ka)}{I_o(\sqrt{Q}\, ka)} -$$

$$- (1-\beta)^{1/2} \, \frac{I_1(ka)K_1(ka_w) - I_1(ka_w)K_1(ka)}{I_o(ka)K_1(ka_w) - I_1(ka_w)K_o(ka)} = 0 \qquad (10)$$

The boundary condition $b_r = 0$ at $r = a_w$ has been used in deriving (10). For the parameters which we have considered $ka \ll 1$ and $ka_w \ll 1$ hold so that (10) may be reduced to a simpler expression as follows:

$$\frac{\beta}{2} \, (\alpha^2 + 6\alpha + 1) - (\alpha + 1) \left\{ 1 + \frac{(1-\beta)^{1/2}}{\left(\frac{aw}{a}\right)^2 - 1} \right\} = 0. \qquad (11)$$

Results from a numerical solution of (11) for $(a_w/a)^2 = 4.5$ are shown in Fig. 1 where both real and imaginary parts of the frequency are plotted as a function of β. For $\beta = 0.8$, the damping rate of the $m = 0$, $k \neq 0$ oscillation is large; $\gamma \sim kv_{Ti}$ where v_{Ti} is the ion thermal velocity. The damping is due to ion Landau damping. The rf energy goes directly into the ions and only a few collisions are necessary to thermalize the ion distribution. Similar estimates for damping at these frequencies have been given by Stepanov.[9] The actual mode which is damped is the kinetic counterpart of the MHD slow magnetoacoustic mode.[10] An estimate of the ion heating rate can be obtained from the following equation where it has been assumed that uniform heating occurs over a cross-section of the plasma column with constant profiles;

$$\frac{dn_i T_i}{dt} \sim \gamma \frac{b^2}{2\mu_o} \quad . \qquad (12)$$

If we use $n_i \sim 8 \times 10^{16}$ cm^{-3}, $b = b_o \times 10^3$ gauss and the value of γ at $\beta = 0.8$ then $dT_i/dt \sim 2\, b_o^2 \times 10^5$ eV/sec. The equivalent rf power necessary can be computed from $\omega b^2 V/2\mu_o$ where V is the volume per unit length between the plasma and the wall. For the parameters chosen earlier this number is 25 b_o^2 megawatts per meter. This power level is high and is a result of choosing b such that significant heating is obtained. The minimum power level necessary for dynamic stabilization is .6 megawatts per meter, a very reasonable level.

4. CONCLUSIONS

We have shown that it should be possible to dynamically stabilize a bumpy θ-pinch in an interesting reactor like regime. The bumpy θ-pinch results from applying equally spaced static multiple mirrors to a longitudinal θ-pinch magnetic field. The addition of multiple mirrors implies a drastic reduction in particle end loss. A part of the power in the rf stabilization circuit can be transferred directly to the ions by rf heating.

A major assumption in the above has been that the theory of multiple mirror end loss reduction applies to a high density, high β plasma with a fluctuating background magnetic field. It is believed that since the $\omega \sim v_{Ti}/\lambda_{ii}$ no serious effects result but this is not proved. We intend to study this problem.

In conclusion, it would appear that a high β, high density, multiple mirror experiment should be performed. The most likely candidate for such an experiment is a long implosion heated θ-pinch with $T_i \sim 1$ KeV, $n \sim 10^{16}$, $\lambda_{ii} \sim 5$ m. About five to six λ_{ii}'s are necessary resulting in a device 25$\sim$30 meters long. Such an experiment would be, in our opinion, the logical extension of the results of Logan[6] et. al to the more interesting high density, high β regime.

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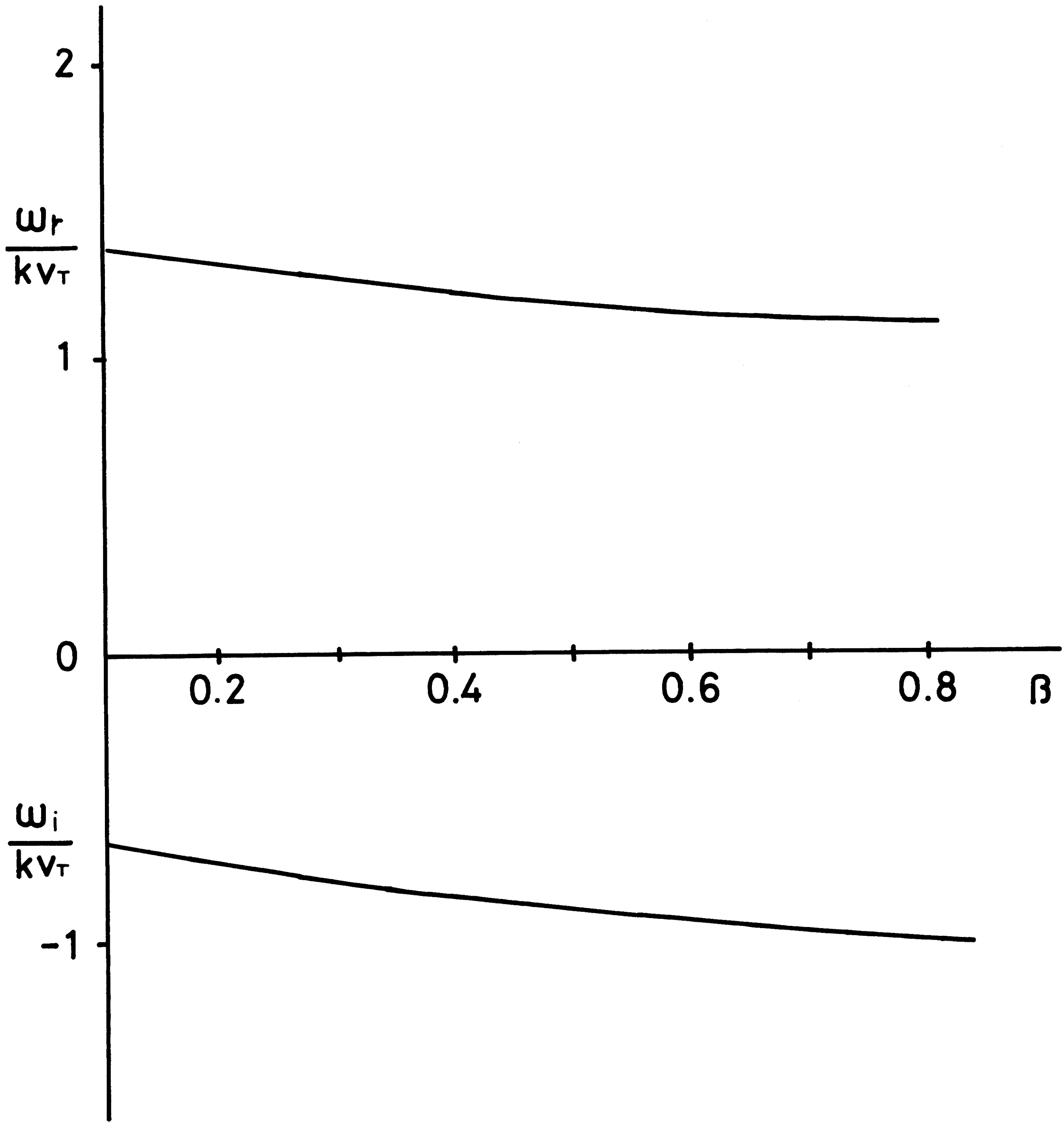

Fig. 1: Solution of dispersion relation; real (ω_r) and imaginary (ω_i) frequencies normalized to thermal frequency as a function of β.