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RESEARCH REPORT

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Possibility of Stable Implosion

of Structured Pellet I

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Abstract

The scheme for stable implosion is proposed and discussed. The asymptotic theory which is one branch of homogeneous isentropic compression scheme is derived and ascertained by numerical simulations.

§1. Introduction

So far many authors[1-10] have proposed models for laser-driven implosion and have derived valuable results to laser fusion. In the previous paper[11], we have implied that the time dependences are different between the mechanical power and the absorbed laser power especially for an optimal implosion of the structured target.

According to these implosion models, the spherical target has the Rayleigh-Taylor instability[12] because of their continuous inward acceleration on the pellet surface. In this paper, we discuss the possibility of stable implosion scheme of the structured pellet. In section 2, the basic idea is schematically shown and an asymptotic theory which is one branch of Kidder's homogeneous isentropic scheme[3] is derived. In section 3, we show that numerical results agree well with our theory whose solution has a quite different form from those given by Kidder. In section 4, the possibility of stable implosion is discussed.

§2. Basic Idea

In Fig. 1, the scheme of stable implosion is schematically illustrated. In the figure, r is the radial coordinate, t the time, A_1-A_2- ... -A_5 is the path of the 'piston' whose speed remains constant, the solid lines A_1-O_1, O_1-A_2, ... are loci of shock waves whose Mach numbers are M_1, M_2, ... and a dotted line shows the path of a fluid particle. From the Rankine-Hugoniot relations, the Mach number M_n of the reflected shock wave is given by

$$M_n = \frac{\alpha + \sqrt{\alpha^2 + 4}}{2}, \quad \alpha \equiv \frac{(\gamma+1)(M_{n-1}^2 - 1)}{\sqrt{[2\gamma M_{n-1}^2 + 1 - \gamma][(\gamma-1)M_{n-1}^2 + 2]}} \quad (n = 2, 3, 4 \cdots), \quad (1)$$

where γ is the ratio of the specific heats.

We assume now that the strength of the initial shock wave is infinite, i.e. $M_1 = \infty$, then from eq.(1) we have a series of the Mach numbers $M_1 = \infty$, $M_2 = \sqrt{5}$, $M_3 = \sqrt{3}$, $M_4 = \sqrt{21}/3$, ------- which implies that M_n can be considered as unity for $n \geq 5$. The pathes of fluid particles are approximated by straight lines after they cross the fifth shock wave. In this scheme, fluid particles have no acceleration. Stable implosion can be expected.

Taking this into account, we now quote Kidder's formalism[3] which is read as follows;

$$r(r_0, t) = r_0 h(t), \quad u(r_0, t) = \dot{r} = r_0 \dot{h}(t), \tag{2}$$

where r_0 and r are respectively the Lagrangian and the Eulerian coördinates of fluid particles, u is the fluid velocity and $h(t)$ is the scale factor. Dots on variables mean the differentiation with respect to the time. The equation of motion reduces to

$$h^{3\gamma-2} \ddot{h} = -\frac{1}{\rho_0 r_0} \frac{d p_0}{d r_0} = \text{const.}, \tag{3}$$

where ρ and p are the density and the pressure respectively, and the subscript o denotes initial values. Kidder has derived the time dependence of the scale factor $h(t)$ for specific profiles of the initial density and pressure as

$$h(t) = \sqrt{(1+b\tau)(1-\tau)} \qquad b = 0 \text{ or } 1, \tag{4}$$

where

$$\tau = t/t_c \qquad (t_c \text{ is a constant}).$$

On the other hand, eq.(3) has another special solution

$$h(t) = A + Bt, \tag{5}$$

when the density and pressure profiles are homogeneous at the initial time. This solution has an interesting feature that trajectories of fluid particles are straight lines $r = r_0(A+Bt)$ in the r-t space. If we specify the 'piston' speed as U_p and the initial target radius as R_0, we can determine the integral constants A and B respectively as

$$A = 1, \qquad B = -\frac{R_0}{U_p} \equiv -\frac{1}{t_s}, \tag{6}$$

from the condition that all the fluid particles must converge simultaneously at the center. Then eq.(5) reduces to

$$h(t) = 1 - t/t_s. \tag{5'}$$

From the conservation of mass, we have

$$\rho r^2 dr = \rho_0 r_0^2 dr_0. \tag{7}$$

The isentropic relation (which is approximately valid after the fifth shock wave passes)

$$p\rho^{-\gamma} = p_0 \rho_0^{-\gamma}, \tag{8}$$

together with eq.(7) leads the time dependence of the density ρ, the pressure p and mechanical power E_M on the boundary surface to

$$\rho = \rho_0 (1 - t/t_s)^{-3}, \tag{9-a}$$

$$p = p_0 (1 - t/t_s)^{-3\gamma} = p_0 (1 - t/t_s)^{-5}, \tag{9-b}$$

$$E_M \equiv \rho u r^2 \big|_{\text{at surface}} = p_0 U_p R_0^2 (1 - t/t_s)^{-3\gamma+2}, \qquad = p_0 U_p R_0^2 (1 - t/t_s)^{-3}, \tag{9-c}$$

for $\gamma = 5/3$.

§3. Numerical simulation

In the preceding section, we derive the time dependence of the various physical quantities when the pellet implodes stably. In the theory, the target structure is not taken explicitly into consideration. But the real pellet has no piston at the outer boundary. The reflections of shock waves at the boundary necessarily imply the existence of the high density 'tamper' there. In this section, we show the results of the numerical simulation carried out for the structured pellet.

The details of our one-dimensional spherical hydrodynamic code are reported in reference 13 . For simplicity, fusion yield, bremsstrahlung and absorption of the laser light are neglected. The parameters used here are as follows: the initial temperature is $1\,eV$, the initial number density is $5 \times 10^{22}\,cm^{-3}$. A D-T solid sphere whose initial radius is $400\,\mu m$ is surrounded by a heavier material (50 times heavier than D-T solid) with the thickness of $10\,\mu m$.

Let us suppose that, the outer surface of the 'tamper' moves with a constant speed $U_p = 1.12 \times 10^6\,cm/sec$. Then the pressure and the mechanical power at the outer surface are numerically obtained. Figure 2 indicates sampled trajectories of fluid particles. As clearly seen in the figure, the trajectories are nearly straight for $t \gtrsim 22$nsec, i.e. after the fifth shock wave passes. In the figure, the shaded region represents the high density 'tamper'.

The mechanical power E_M is plotted in Fig.3 versus the time. It is very interesting to see that peak times of E_M in Fig.3 correspond to the arrival times in Fig.2 of outwardly propagating shock waves at the outer boundary. The mechanical power E_M is drawn in logarithmic scale in Fig.4. In the final stage of collapse, i.e. $t \sim ts$, we observe $E_M \propto (1-t/t_s)^{-3}$ which coincides with our theoretical result (9-c) for $\gamma=5/3$. In the earlier stage ($t \sim 0$), a in $E_M \propto (1-t/t_s)^{-a}$ is smaller than 3, because shock waves have finite strengthes. Figure 5 shows the pressure at the outer surface versus the time. The time dependence is in good accordance with our theoretical result (9-b). It should be noted that the pressure given by eq.(9-b) significantly differs from Kidder's result[3] $p \propto (1-t/t_s)^{-5/2}$.

§4. Discussions

In the preceding sections, we derive the time dependence of the pressure and the mechanical power at the outer surface when a structured pellet implodes stably. As pointed out in the previous paper[11], the absorbed laser power does not necessarily have the time dependence which is equal to that of the mechanical power, especially in the case of the structured pellet. The conversion from the laser energy to the mechanical power will be discussed in details in the subsequent paper.

Our theory does not fit well for $t \lesssim 22$ nsec, which can be seen from Fig.2. Concerning the pulse shape of the mechanical power for $t \lesssim 22$ nsec (see Fig.3), there may be no difficulty for us to produce it experimentally, because of its long period. The most serious fact, we must point out, is that the inner surface of the high density 'tamper' experiences the oscillatory acceleration. We model this phenomenon using the following equation for the amplitude a of surface perturbation as

$$\ddot{a}(t) = gk\,a(t) \cdot \cos(\omega_0 t), \tag{10}$$

where g is the amplitude of acceleration, k the wave number of surface wave and ω_0 the frequency of the oscillatory acceleration. Mathieu equation of this type leads to the stable oscillation if the condition $\sqrt{gk}/\omega_0 \lesssim 1$ is satisfied . That is to say, if the frequency of oscillation is greater than the growth rate of the Rayleigh-Taylor instability, there is no trouble. Thus the instability must be examined in our example for $15 \lesssim t \lesssim 20$ nsec, i.e. for the purely decelerating period of the inner surface of the high density 'tamper'. We may have some hints to solve this problem. Figure 6 shows sampled trajectories of fluids particles calculated for the same conditions as those used in Fig.2, except for the fact that the 'piston' is constantly accelerated for $0 \leq t \leq 5$ nsec as seen in the figure. There is no serious deceleration on the inner surface. These will be also discussed in the future.

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Figure Captions

Fig.1 : The scheme of stable implosion.

Fig.2 : Trajectories of fluid particles in r-t space.

Fig.3 : The mechanical power E_M versus the time.

Fig.4 : The mechanical power versus the time. The chain line denotes the asymptotic theory.

Fig.5 : The pressure on the outer surface versus the time. The chain line shows our theory.

Fig.6 : Trajectories of fluid particles in r-t space.

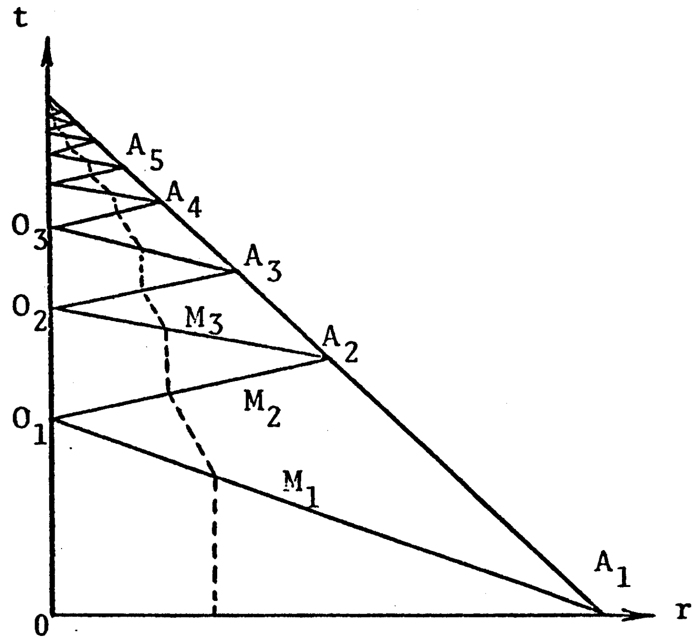
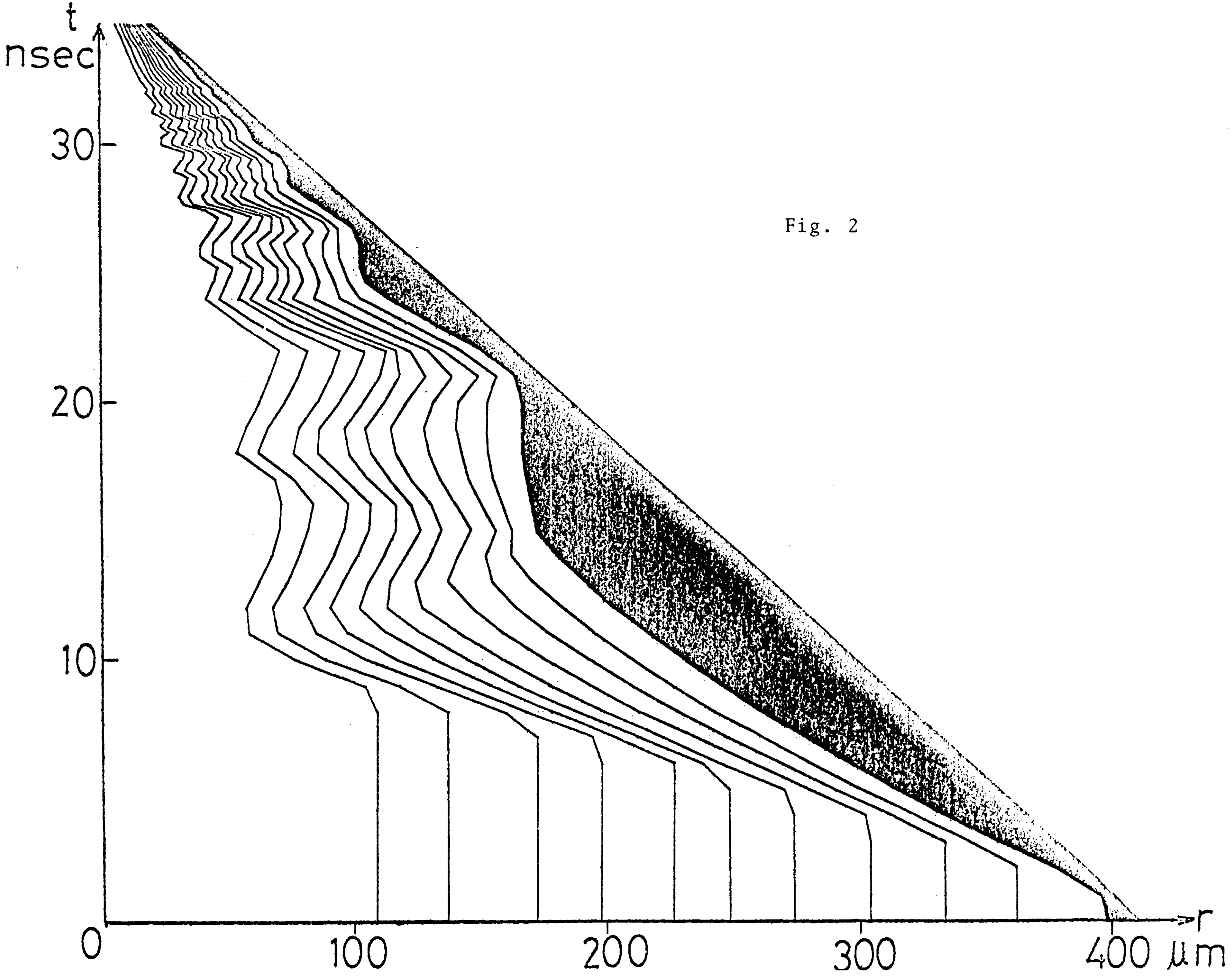


Fig. 1

Fig. 2



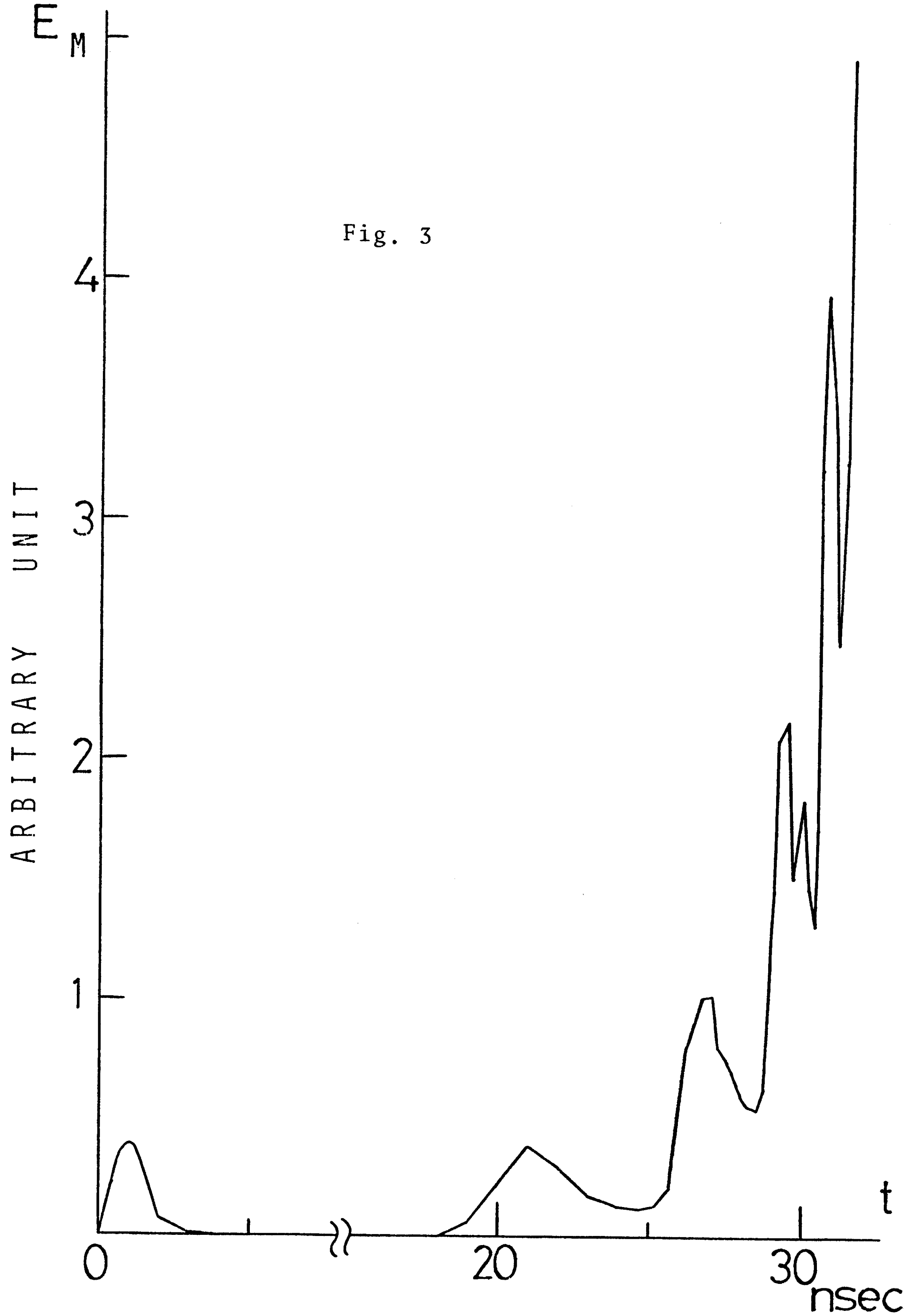


Fig. 3

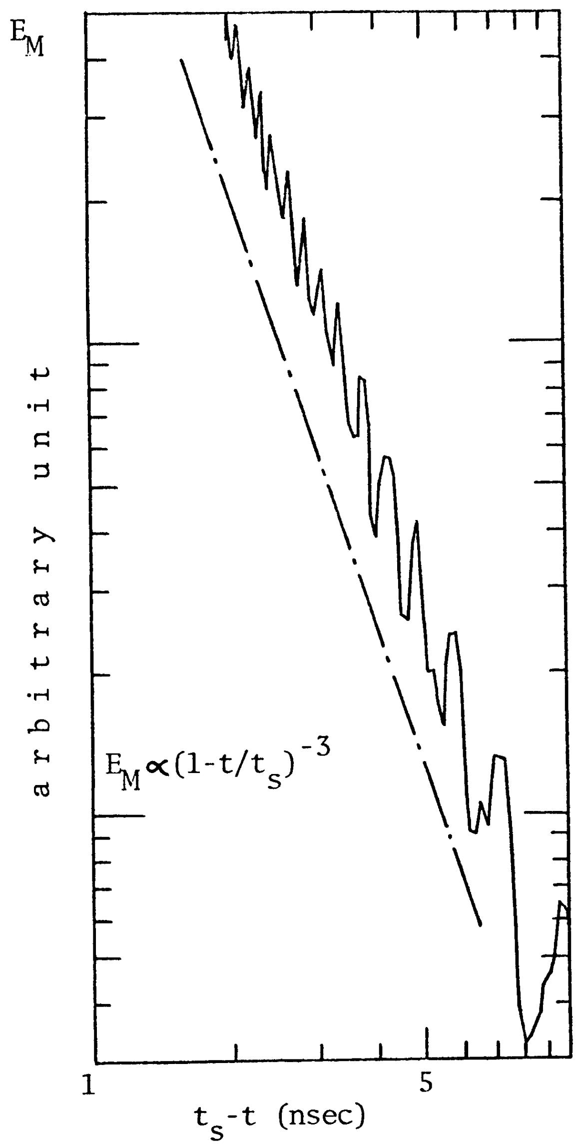


Fig. 4

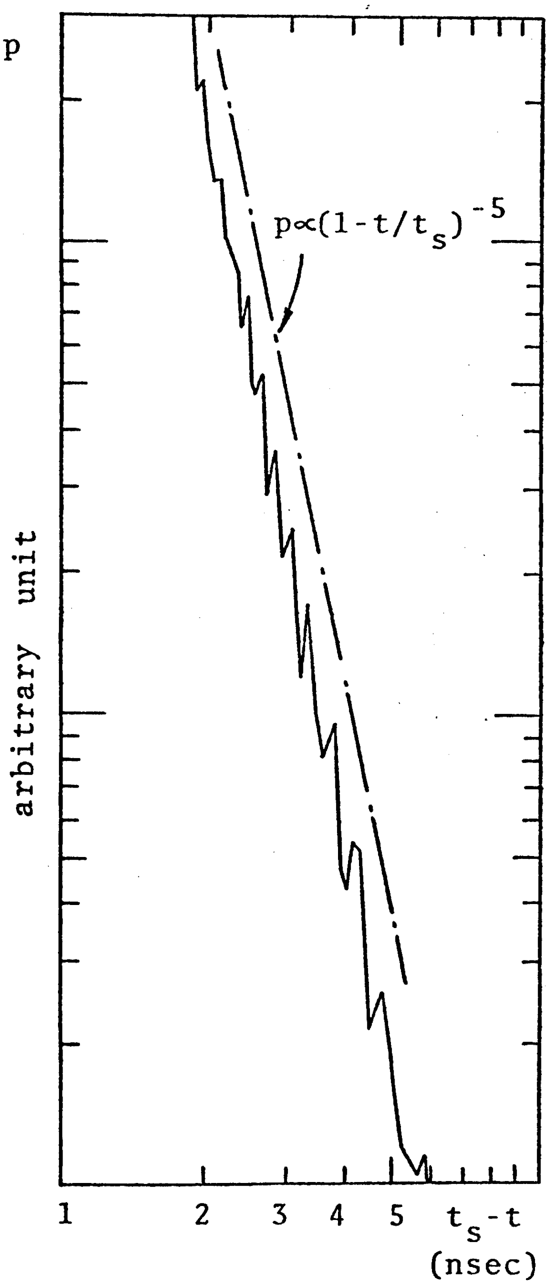


Fig. 5

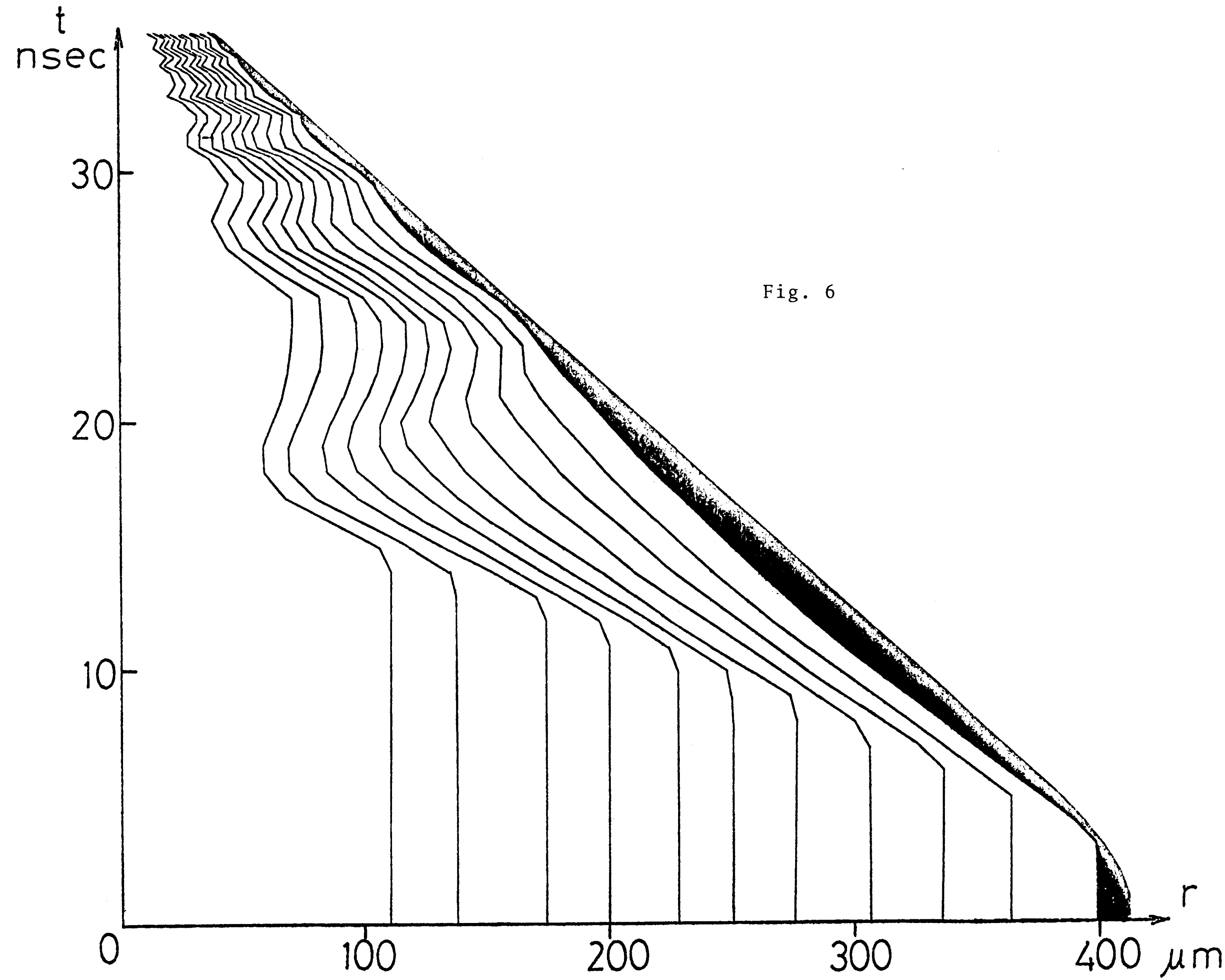


Fig. 6