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Microinstabilities in a Plasma Slab
with Steep Density Gradient

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## Abstract

An integral dispersiton relation in k-space was derived for the linear normal modes of electrostatic perturbations in an inhomogeneous plasma under the presence of a strong ambipolar electric field. This ambipolar field causes $\vec{E} \times \vec{B}$ drift with the velocity shear. Under a local method, $k\ell >> 1$, where k is the wavenumber and $\ell$ is the scale length, the effects of ambipolar field and magnetic curvature on drift wave were investigated in a cusped configuration system. For $\gamma_i < 0$, where $\gamma_i = \pm\omega_{Ei}^2/\omega_{ci}^2$; $|\omega_{Ei}|$ is the characteristic frequency in the ambipolar potential and $\omega_{ci}$ is the ion cyclotron frequency, the cusped magnetic curvature tends to have a stabilizing effect in addition to the ambipolar field on a drift instability. For $\gamma_i > 0$, however, the curvature effect seems not to be so efficient even in such a configuration system.

## I. Introduction

A systematic study of a rf plugging of plasma is succeeded with the aid of a plasma confinement and a heating in the open-ended magnetic cnfinement[1). A previous study showed that there exists a characteristic oscillation having the frequency close to $1.4\omega_{ci}$, when the width of a plasma becomes near to the ion Larmor radius $\rho_i$[2). In this case, the ion cyclotron damping loss of a rf field is small and the exterual rf field can penetrate resonantly into the plasma. When the width of plasma $\lambda$ becoms large for example, $\lambda \sim 5\rho_i$, the dispersion relation reduces similar to that of uniform plasma. Since, in this case, the characteristic frequency of the ion cyclotron wave becomes so near to $\omega_{ci}$ for $T_e = T_i$, the damping of the wave becomes very large and it becomes very difficult to excite such a wave in a plasma.

If a plasma is confined in a cusped magnetic field, the width of open end of the line of force becomes near to $\rho_i$, that is, plasma becomes a sheet plasma. The stability of this sheet plasma has essential relevance to a rf plugging of plasma. The sheet plasma with a width near to $\rho_i$ has a static electric field perpendicular to the sheet and the sheet plasma becomes very anisotropic in the temperature distribution.

Gradient-driven microinstabilities in an inhomogeneous plasma have attracted wide-spread interests in resent years in view of the heating and the anomalous transport in a magnetically confined plasma. In analyzing these instabilitis, the assumption is frequently made that the Larmor radius is very small compared with the scale length of plasma. Since the ion Larmor radius $\rho_i$ is comparable to the scale length $\lambda$ in a sheet plasma, however, the fluid description for ions cannot be used in this inhomogeneous thin plasma, requiring the kinetic description for ions. It is of considerable interests to perform a stability analysis of these instabilities.

In Sec.II, we derive an integral dispersion relation in k-space for electrostatic modes in an inhomogeneous collisionless plasma with an ambipolar field. In Sec.III, we develop a stability analysis by using a kind of local approximation. We discuss the effect of ambipolar field on a drift wave driven by a density gradient in Sec.IV. In Sec.V, we consider a plasma with a cusped configuration and investigate the effect of magnetic curvature on a drift wave. Section VI will be devoted to the discussions.

## II. Formulation

For a collisionless, low-$\beta$ plasma having a density inhomogeneity perpendicular to the uniform magnetic field $\vec{B}_0=B_0\hat{e}_z$, and with the ambipolar potential $\Phi_0=E_0/2\lambda_0\cdot x^2=M/2q\omega_E{}^2x^2$ ($\omega_E{}^2$ is defined as $qE_0/\lambda_0 M$ and $\lambda_0$ is a characteristic scale length of the field), the particle trajectories must satisfy a equation of motion and initial conditions

$$\frac{d\vec{r}'}{dt}=\vec{v}' \quad , \qquad \vec{r}'(t'=t) = \vec{r} \quad ,$$
$$\frac{d\vec{v}'}{dt}=\frac{q}{M}\left[E_0(x)+\frac{\vec{v}'}{c}\times\vec{B}_0\right], \quad \vec{v}'(t'=t) = \vec{v} \quad , \tag{1}$$

where primes are used to denote the trajectory variables, $\vec{r}=(x, y, z)$, and $E_0(x)= -\nabla\Phi_0$. The constants c, q, and M are the speed of light, charge and mass, and species subscripts are supressed. Equation (1) yields the conservation of energy and two components of canonical momentum:

$$|\vec{v}'|^2+\omega_E^2x'^2 = |\vec{v}|^2+\omega_E^2x^2 \quad ,$$
$$\vec{v}_y' + \omega_c x' = \vec{v}_y + \omega_c x \quad , \tag{2}$$
$$\vec{v}_z' = \vec{v}_z \quad ,$$

where $\omega_c=eB_0/Mc$ is the cyclotron frequency.

As a function of the constants of the motion Eq.(2), the equilibrium distribution function $f_0(x, \vec{v})$ having the appropriate ambipolar field and the desired density inhomogeneity is assumed to be

$$f_0(x, \vec{v})=N_0\left(\frac{M}{2T}\right)^{3/2}(1-\delta+\gamma)^{-1/2}$$

$$\exp\{-\frac{M}{2T}[v_x^2+v_y^2+v_z^2+\omega_E^2x^2)+\frac{\delta-\gamma}{1-\delta+\gamma}(v_y+\omega_c x)^2]\} \ . \quad (3)$$

The z and x axes are chosen such that $B_0>0$ and perpendicular to it, respectively. In Eq.(3), $\gamma=\pm\omega_E^2/\omega_c^2$ and $\delta=\rho^2/\lambda^2$, where $\rho$ and $\lambda$ are the gyroradius and the scale lengh of the density inhomogenity, respectively. Also $N_0$ is the particle density at x=0.

The integration of $f_0(x, \vec{v})$ with respect to the velocity yields the desired unperturbed density as

$$n_0(x) = N_0\exp\left(-\frac{x^2}{2\lambda^2}\right) \ . \quad (4)$$

Now we should note that the equilibrium distribution funciton (3) has the effect of unequal temperatures such as $T_x=T_z\neq T_y$, which are obtained by averaging $v_\alpha{}^2$ with respect to the equilibrium distribution function (3).

In terms of the trajectory variables of Eq.(1), a solution of Vlasov equation for the perturbed distribuion function can be cast into the form

$$f(\vec{r}, \vec{v}, t) = -\frac{q}{M}\int_{-\infty}^{t} dt'\vec{E}(\vec{r}, t')\frac{\partial f_0'(\vec{v}')}{\partial\vec{v}'} \ , \quad (5)$$

where $f_0(x, \vec{v})$ as given by Eq.(3) may be employed. The solutions of Eq.(1) are given by

$$x'= x+\frac{(\omega_E^2x-\omega_c v_y)}{\Omega^2}((\cos\phi-1)+\frac{v_x}{\Omega}\sin\phi \ , \quad (6)$$

$$y'= y+\frac{\omega_E{}^2}{\Omega^3}(\omega_c x+v_y)\phi-\frac{\omega_c}{\Omega^3}(\omega_E^2x-\omega_c v_y)\sin\phi+\frac{\omega_c}{\Omega^2}v_x(\cos\phi-1) \ , \quad (7)$$

$$z' = z+\frac{v_z}{\Omega}\phi \ , \quad (8)$$

and

$$v'_x = -\frac{(\omega_E^2 x-\omega_c v_y)}{\Omega}\sin\phi + v_x\cos\phi \quad , \qquad (9)$$

$$v'_y = v_y - \frac{\omega_c}{\Omega^2}(\omega_E^2 x-\omega_c v_y)\cos\phi$$
$$-\frac{\omega_c}{\Omega} v_x\sin\psi + \frac{\omega_c}{\Omega^2}(\omega_E^2 x-\omega_c v_y) \quad , \qquad (10)$$

$$v'_z = v_z \quad , \qquad (11)$$

where $\Omega^2 = \omega_E^2 + \omega_c^2$ and $\phi = \Omega(t'+t)$. Equations (9)-(11) show that the equilibrium distribution function (3) includes the x dependent velocity shear effect due to $\vec{E} \times \vec{B}$ drift. It is considerable interest to investigate instabilities caused by the effects of anisotropy and the velocity shear effect.

Taking into account an inhomogenity of a system in the x direction, we now assume the potential as

$$\Psi(x, y, z, t) = \int_{-\infty}^{\infty} dk\ \Psi(k)\exp[i(kx+k_\perp y+ k_\parallel z-\omega t)]. \qquad (12)$$

This is equivalent to taking the y and the z Fourier transforms of the potential. Also, a functional dependence of the unperturbed distribution function is as follows

$$f_0(x, \vec{v}) = h_0[\frac{M}{2}(v_x^2+v_y^2+v_z^2+\omega_E^2x^2)]g(v_y+\omega_c x) \quad , \qquad (13)$$

equation (5) can be integrated over the particle orbit to obtain

$$f = \frac{q}{M}\int_{-\infty}^{\infty} dk'\ \Psi(k')\ \exp[i(k'x+k_\perp y+k_\parallel z-\omega t)]$$
$$\times[Mh'_0 g + i\int_{-\infty}^{0}\frac{d\phi}{\Omega}(\omega Mh'_0 g+k_\perp h_0 g')\exp\{i[k'(x'-x)$$
$$+ k_\perp(y'-y)+k_\parallel(z'-z) - \omega\frac{\phi}{\Omega}]\}] \quad , \qquad (14)$$

where the primes for h' and g' mean the derivations with respect to the variables, respectively.

Equation (14) is combined with the Poisson equation

$$\nabla^2\Psi(\vec{r}, t) = 4\pi \sum_j q \int d^3\vec{v}\ f\ (\vec{r}, t, \vec{v}) \quad , \tag{15}$$

to yield

$$(k^2+k_\perp^2+k_\parallel^2)\Psi(k) = 4\pi \sum_j q^2 \int_{-\infty}^{\infty} \frac{dx}{2\pi} \int_{-\infty}^{\infty} dk'\ \Psi(k')\exp[i(k'-k)x]$$

$$\times\int_{-\infty}^{\infty} d^3\vec{v}\ [\ h_0'g + i \int_{-\infty}^{0} \frac{d\phi}{\Omega}\ (\omega_0 h_0'g + \frac{k_\perp h_0}{M}\ g')$$

$$\exp\{i[k'(x'-x)+ k_\perp(y'-y)+ k_\parallel(z'-z)- \omega\frac{\phi}{\Omega}\ ]\}] \quad . \tag{16}$$

Substitution of Eqs.(3), (6)-(11) into Eq.(16) gives

$$(k^2+k_\perp^2+k_\parallel^2)\Psi(k)$$

$$= 4\pi\sum_j q^2\int_{-\infty}^{\infty} \frac{dk'}{2\pi}\Psi(k')\,[-\frac{N_0}{T}\sqrt{2\pi}\lambda\exp\{-\ \frac{\lambda^2(k'-k)^2}{2}\}$$

$$+ i \int_{-\infty}^{\infty} d^3\vec{v} \int_{-\infty}^{\infty} \frac{d\phi}{\Omega} \int_{-\infty}^{\infty} \frac{dx}{2\pi}\ [-\ \frac{\omega}{T} - \frac{k_\perp}{T}\ \frac{\delta-\gamma}{1-\delta+\gamma}\ (v_y+\omega_c x)]\exp[i(k'-k)x]$$

$$\times\ N_0(\frac{M}{2\pi T})^{3/2}(1-\delta+\gamma)^{-1/2}\times\ \exp\{-\ \frac{M}{2T}[(v_x^2+v_y^2+v_z^2+\omega_E^2x^2)$$

$$+ \frac{\delta-\gamma}{1-\delta+\gamma}\ (v_y+\omega_c x)^2\}]\ \times\ \exp\{i[k'\frac{(\omega_E^2x-\omega_c v_y)}{\Omega^2}\ (\cos\phi-1)$$

$$+ \frac{k'v_x}{\Omega}\ \sin\phi\ -\ \frac{k_\perp\omega_c}{\Omega^3}\ (\omega_E^2x-\omega_c v_y)\ \sin\phi\ +\ \frac{k_\perp\omega_c v_x}{\Omega^2}(\cos\phi-1)$$

$$+ \frac{k_\perp\omega_E^{\ 2}}{\Omega^3}\ (\omega_c x+v_y)\phi\ +\ k_\parallel v_z\ \frac{\phi}{\Omega}\ -\ \frac{\omega\phi}{\Omega}]\} \quad . \tag{17}$$

Introducing the new variable $\tilde{v}_y= v_y+ \omega_c x$, equation (17) can be integrated with respect to coordinate x, velocities ($v_x$, $\tilde{v}_y$, $v_z$), and $\phi$. After the lengthy calculations, we have

$$F(k)\Psi(k) = \int_{-\infty}^{\infty} \frac{\lambda dk'}{\sqrt{2\pi}} \Psi(k') K(k, k') \exp[- \frac{\lambda^2 (k'-k)^2}{2}] \quad (18)$$

with

$$F(k) = (k^2 + k_\perp{}^2 + k_\parallel{}^2) \quad , \quad (19)$$

$$K(k, k') = -4\pi \sum_j \frac{N_0 q_j^2}{T_j} \{1 + \sum_{n=-\infty}^{\infty} I_n(b_j) \exp[-b_j - in\phi_{0j}(k,k')]$$

$$\times \exp[\frac{\lambda^2 \delta_j k_\perp{}^2}{(1+\gamma_j)^2} \frac{(k' - k)^2}{kk' + \frac{k_\perp^2}{1+\gamma_j} + \sqrt{(\frac{k_\perp{}^2}{1+\gamma_j} + k'^2)(\frac{k_\perp{}^2}{1+\gamma_j} + k^2)}}]$$

$$\times [\frac{\omega + i\lambda^2(\delta_j - \gamma_j) k_\perp \omega_{cj}(k'-k)}{\sqrt{2v_j^2 (k_\parallel^2 + \frac{(1-\delta_j+\gamma_j)}{\delta_j(1+\gamma_j)} \gamma_j^2 k_\perp^2)}} Z(\zeta_n^j) - \frac{\omega_{cj}(\delta_j - \gamma_j)\gamma_j \lambda^2 k_\perp^2}{2v_j^2 (k_\parallel^2 + \frac{(1-\delta_j+\gamma_j)}{\delta_j(1+\gamma_j)} \gamma_j^2 k_\perp^2)} Z'(\zeta_n^j)]\} , \quad (20)$$

where $I_n(b_j)$, $Z(\zeta_n^j)$ and $Z'(\zeta_n^j)$ are the modified Bessel function, the plasma dispersion function and the differential of them, and $v_j$ mean the thermal velocity for j-th spicies of plasma particles. In Eq.(21), $b_j$ and $\zeta_n^j$ are defined by

$$b_j(k,k') = \rho_j^2 [(\frac{k_\perp^2}{1+\gamma_j} + k'^2)(\frac{k_\perp^2}{1+\gamma_j} + k^2)]^{1/2} \quad , \quad (21)$$

$$\zeta_n^j = \frac{\omega - n\Omega_j - i\lambda^2 (k'-k) \frac{(1-\delta_j+\gamma_j)}{(1+\gamma_j)} \gamma_j \omega_{cj}}{\sqrt{2v_j{}^2 (k_\parallel{}^2 + \frac{(1-\delta_j+\gamma_j)\gamma_j{}^2}{\delta_j(1+\gamma_j)} k_\perp{}^2)}} \quad , \quad (22)$$

and the initial phase $\phi_0(k, k')$ is also given by

$$\cos\phi_{0j} = \frac{\left(kk' + \frac{k_\perp^2}{1+\gamma_j}\right)}{\sqrt{\left(kk' + \frac{k_\perp^2}{1+\gamma_j}\right) + \frac{k_\perp^2}{1+\gamma_j}(k'-k)^2}} , \tag{23a}$$

$$\sin\phi_{0j} = \frac{(k'-k)k_\perp/(1+\gamma_j)^{1/2}}{\sqrt{\left(kk' + \frac{k_\perp^2}{1+\gamma_j}\right) + \frac{k_\perp^2}{1+\gamma_j}(k'-k)^2}} , \tag{23b}$$

Now we note that in the second term of the numerator of Eq.(22), $\omega_{cj}$ is replaced by $\Omega_j = (\omega_{cj}^2 + \omega_{Ej}^2)^{1/2}$ due to the particle gyration in the ambipolar potential. The third term in the numerator and also the second term in the braces of the denominator of Eq.(22) are corrections induced by a velocity shear due to the ambipolar field. Equation (19) is an integral dispersion relation in k-space which is applicable to any electrostatic modes in an inhomogenious collisionless plasma. While it is difficult to find analytic solution of Eq.(19), one may discuss wide range of instabilities by using a numerical analysis. But the detail of such a numerical analysis awaits future investigation.

## III. Stability Analysis

Since Eq.(18) with Eqs.(19)-(23) is quite general, so we may discuss several instabilities for electrostatic modes on the basis of this equation. One should find analytic solutions by solving a nonlocal equation in k-space such as Eq.(18) provided $\ell \sim \lambda$ ($\ell$ is a characteristic wavelength), however, one may discuss properties for several instabilies even under the local approximation. In this paper we restrict ourselves to a local solution of Eq.(18).

Let us now introduce a transformation as

$$\Psi(k) = \psi(k)\ \exp\,(i\beta k) \quad , \tag{24}$$

where $\beta$ is an arbitrary constant. Equation (24) means that $\Psi(x+\beta)$ can be represented as Eq.(12), namely

$$\Psi(x+\beta) = \int_{-\infty}^{\infty} dk\ \psi(k)\ \exp\,(ikx) \quad . \tag{25}$$

Substitution of Eq.(24) into Eq.(18) gives

$$F(k)\psi(k)\exp(i\beta k) = \sum_j \int_{-\infty}^{\infty} \frac{dk'}{\sqrt{2\pi}}\psi(k')\{K_j^{(0)}+K_j^{(1)}\exp[S_j(k,k')]\}$$

$$\times\ \exp\,[i\beta k' - \frac{\lambda^2(k'-k)^2}{2}] \quad , \tag{26}$$

with

$$K_j^{(0)} = -k_{Dj}^2 \quad , \tag{27}$$

$$K_j^{(1)} = -k_{Dj}^2 \sum_{n=-\infty}^{\infty} I_n(b_j)\ \exp[-b_j - in\phi_{0j}(k,\ k')]$$

$$[\frac{\omega+i\lambda^2(\delta_j-\gamma_j)k_\perp\omega_{cj}(k'-k)}{\sqrt{2v_j^2(k_\parallel^2+\frac{(1-\delta_j+\gamma_j)}{\delta_j(1+\gamma_j)}\gamma_j^2k_\perp^2)}}Z(\zeta_n^j) - \frac{\omega_{cj}^2(\delta_j-\gamma_j)\gamma_j\lambda^2k_\perp^2}{2v_j^2(k_\parallel^2+\frac{(1-\delta_j+\gamma_j)}{\delta_j(1+\gamma_j)}\gamma_j^2k_\perp^2)}Z'(\zeta_n^j) \quad , \tag{28}$$

$$S_j(k,k') = \frac{\lambda^2\delta_j k_\perp^2}{(1+\gamma_j)^2}\ \frac{(k'-k)^2}{kk'+\frac{k_\perp^2}{1+\gamma_j}+\sqrt{(\frac{k_\perp^2}{1+\gamma_j}+k'^2)(\frac{k_\perp^2}{1+\gamma_j}+k^2)}} \quad . \tag{29}$$

Let us carry out the integration with respect to k' in Eq.(26) by the method of steepest decent. After the lengthy calculation, we have

$$F(k)\psi(k)\exp(i\beta k)$$

$$=\sum_j \exp(i\beta k-\frac{\beta^2}{2\lambda^2})\int_{-\infty}^{\infty}\frac{d\eta}{\sqrt{2\pi}}\psi(k+i\frac{\beta}{\lambda^2}+\frac{\eta}{\lambda})[K_j^{(0)}+K_j^{(1)}(k,k'=k+i\frac{\beta}{\lambda^2}+\frac{\eta}{\lambda})]$$

$$\times\exp(-\frac{\eta^2}{2})\quad, \tag{30}$$

where the saddle point is $(k'-k) = i\beta/\lambda^2$ and we also put $(k'-k) = i\beta/\lambda^2+\eta/\lambda$. Now we consider a case in which $k>>\beta/\lambda^2$, $\eta/\lambda$. Expanding all quatities in Eq.(30) into power sireis with respect to $\eta/\lambda$, the lowest order equation reduces to

$$F(k)\psi(k) = \sum_j \exp(-\frac{\beta^2}{\lambda^2})\psi(k+i\frac{\beta}{\lambda^2})[K_j^{(0)}+K_j^{(1)}(k,k'=k+i\frac{\beta}{\lambda^2})]\quad, \tag{31}$$

where

$$K_j^{(1)}(k,k'=k+i\frac{\beta}{\lambda^2})\simeq -k_{Dj}^2\sum_{n=-\infty}^{\infty}I_n(b_j)\exp(-b_j)$$

$$\times\{\frac{\omega-(\delta_j-\gamma_j)k_\perp\omega_{cj}\beta}{\sqrt{2v_j^2(k_\parallel^2+\frac{(1-\delta_j+\gamma_j)}{\delta_j(1+\gamma_j)}\gamma_j^2k_\perp^2)}}Z(\zeta_{n0}^j)-\frac{\omega_{cj}^2(\delta_j-\gamma_j)\gamma_j\lambda^2k_\perp^2}{2v_j^2(k_\parallel^2+\frac{(1-\delta_j+\gamma_j)}{\delta_j(1+\gamma_j)}\gamma_j^2k_\perp^2)}Z'(\zeta_{n0}^j)\}\,, \tag{32}$$

$$\zeta_{n0}^j=\frac{\omega-n\Omega_j+\beta k_\perp\frac{(1-\delta_j+\gamma_j)}{1+\gamma_j}\gamma_j\omega_{cj}}{\sqrt{2v_j^2(k_\parallel^2+\frac{(1-\delta_j+\gamma_j)}{\delta_j(1+\gamma_j)}\gamma_j^2k_\perp^2)}}\quad. \tag{33}$$

In Eq.(31) with Eqs.(32) and (33), we should notice that the initial phase $\phi_{0j}(k,k')$ is order of $\eta/\lambda$, and $b_j$ can be approximated as

$$b_j = \rho_j^{\,2}\left(\frac{k_\perp^{\,2}}{1+\gamma_j} + k^2\right) \quad , \tag{34}$$

provided $k \gg \beta/\lambda^2$.

If we neglect the charge separation term in Eq.(31) and consider the case of $k \gg \beta/\lambda^2$, the dispersion relation is given as

$$\sum_{j=e,i} [K_j^{(0)} + K_j^{(1)}(k,k')]\psi(k) = 0 \quad , \tag{35}$$

where $K_j^{(0)}$ and $K_j^{(1)}(k,k')$ are defined by Eqs.(27) and (28), respectively.

## IV. Effect of Ambipolar Field on Drift Wave

It is physically plausible that the important waves in an inhomogeneous plasma are drift waves. Let us consider the effects of an ambioplar field on a drift wave. In the drift limit, $v_i \ll \omega/k_\parallel \ll v_e$, only the leading zeroth term in $I_n(b_j)$ is retained, and only the finite Lamor effect from ions $b_i$ is retained. In this limit, the dispersion relation can be written as

$$1 + \frac{T_i}{T_e} + \frac{T_i}{T_e}\left\{\frac{\omega-\tilde{\omega}_{*e}}{\sqrt{2}v_e\tilde{k}_{\parallel e}} Z(\zeta_{00}^e) - \frac{\omega_{ce}^2(\delta_e-\gamma_e)\gamma_e\lambda^2k_\perp^2}{2v_e^2\tilde{k}_{\parallel e}^{\,2}} Z'(\zeta_{00}^e)\right\}$$

$$+ I_0(b_i)\exp(-b_i)\left\{\frac{\omega-\tilde{\omega}_{*i}}{\sqrt{2}v_i\tilde{k}_{\parallel i}} Z(\zeta_{00}^j) - \frac{\omega_{ci}^2(\delta_i-\gamma_i)\gamma_i\lambda^2k_\perp^2}{2v_i^2\tilde{k}_{\parallel i}^{\,2}} Z'(\zeta_{00}^j)\right\} = 0 \quad ,$$

(36)

where

$$\tilde{\omega}_{*j} = (\delta_j - \gamma_j) k_\perp \omega_{cj} \beta \quad , \tag{37}$$

$$\zeta_{00}^j = \frac{\omega + \beta k_\perp \frac{(1-\delta_j+\gamma_j)}{1+\gamma_j} \gamma_j \omega_{cj}}{\sqrt{2} v_j \tilde{k}_{\parallel j}} \quad , \tag{38}$$

$$\tilde{k}_{\parallel j}^2 = k_\parallel^2 + \frac{(1+\gamma_j-\delta_j)\gamma_j^2}{\delta_j(1+\gamma_j)} k_\perp{}^2 \quad . \tag{39}$$

Since in the drift limit, $v_i << \omega/k_\parallel << v_e$, and the lowest significant order contribution from the electron Z-function that appears in Eq.(36) is its residue, we have

$$\begin{aligned} &Z(\zeta_{00}^e) \simeq i\sqrt{\pi}\, \exp[-(\zeta_{00}^e)^2], \qquad Z'(\zeta_{00}^e) \simeq -2 \quad , \\ &Z(\zeta_{00}^i) \simeq -1/\zeta_{00}^i \quad , \qquad Z'(\zeta_{00}^i) \simeq 1/(\zeta_{00}^i)^2 \quad , \end{aligned} \tag{40}$$

where the contribution arising from resonant ions is neglected compared with that from electrons. Consequently the dispersion relation for a drift wave including the effect of the velocity shear, reduces to

$$\begin{aligned} 1 + \frac{T_i}{T_e} + \frac{T_i}{T_e} \frac{\omega - \tilde{\omega}_{*e}}{\sqrt{2} v_e \tilde{k}_{\parallel e}} \left[ i\sqrt{\pi} - \sqrt{2}\, \frac{\omega + \beta k_\perp \frac{(1+\gamma_e-\delta_e)}{1+\gamma_e} \gamma_e \omega_{ce}}{v_e \tilde{k}_{\parallel e}} \right] \\ + \frac{T_i}{T_e} \frac{\omega_{ce}^2 (\delta_e - \gamma_e) \gamma_e \lambda^2 k_\perp{}^2}{v_e^2 \tilde{k}_{\parallel e}^2} - \frac{\omega - \tilde{\omega}_{*i}}{\omega + \beta k_\perp \frac{(1+\gamma_j-\delta_j)}{1+\gamma_j} \gamma_j \omega_{cj}} I_0(b_i) \exp(-b_i) = 0. \end{aligned} \tag{41}$$

Writing $\omega = \omega_r + i\omega_i$ with $\omega_r >> \omega_i$ and $b_i << 1$, for convinience, we have

$$\omega_r \simeq \frac{\omega_{*e}}{F}\left[(1-b_i)+\frac{\gamma_i}{\delta_i}\right] \quad , \tag{42}$$

$$\omega_i \simeq \frac{\sqrt{\pi}}{F^3}\frac{1}{\sqrt{2}v_e\tilde{k}_{\parallel e}}\left(1+\frac{T_e}{T_i}\right)\frac{(\omega_{*e})^2}{1+\gamma_j}(1-b_i)\left(b_i+\frac{T_i}{T_e}\frac{\gamma_i}{1+\gamma_j}\right) \quad , \tag{43}$$

with

$$F = 1+\frac{T_e}{T_i}b_i+\frac{\omega_{ce}^2}{v_e^2\tilde{k}_{\parallel e}^2}\frac{(\delta_e-\gamma_e)}{1+\gamma_e}\gamma_e[k_\perp\beta(1+\gamma_e-\delta_e)+\lambda^2k_\perp^2(1+\gamma_e)] \quad , \tag{44}$$

where $\omega_{*e} = -(T_e/eB)(\beta/\lambda^2)k_\perp$.

For no ambipolar field, $T_e/T_i b_i \ll 1$, and $\beta=\lambda$, equations (42) and (43) can be approximated to be

$$\omega_r \simeq \omega_{*e}\left[1-\left(1+\frac{T_e}{T_i}\right)b_i\right] \quad , \tag{45}$$

$$\omega_i \simeq \frac{\sqrt{\pi}}{\sqrt{2}v_e k_\parallel}\left(1+\frac{T_e}{T_i}\right)(\omega_{*e})^2 b_i\left[1-\left(1+\frac{T_e}{T_i}\right)b_i\right] \quad , \tag{46}$$

which correspond to the usual dispersion relation for the drift wave. The effect of the $(1+T_e/T_i)b_i$ term of Eqs.(45) and (46) on stability has been considered by Liu et al.[3] The interesting feature of Eqs.(42) and (43) is that the term in $\gamma_i$ acts to higher $\omega$ from $\omega_{*e}$ and so might be expected to result in an instability in a manner similar to that of finite Larmor effect in a case of $\gamma_i>0$. On the contrary, for $\gamma_i<0$, the term in $\gamma_i$ acts to lower $\omega$ from $\omega_{*e}$ and results in a supprësion of instability provided

$$|\gamma_i| = \frac{|\omega_{Ei}^2|}{\omega_{ci}^2} > \frac{\frac{T_e}{T_i}b_i}{\left(1+\frac{T_e}{T_i}b_i\right)} \quad . \tag{47}$$

## V. Effect of Magnetic Curvature on Density Gradient Instabilities

Let us extend the present formalism into a system including a effect of magnetic curvature such as a cusped configuration. To do so, we consider a cusped system with an ambipolar field $E_0(x)$ in which the density gradient is in the x direction and the magnetic field $\vec{B}_0 = B_0[-x/L,\ 0,\ z/L]$, L is the distance from the plasma center to the part of line cusp[2]. Under this configuration, the gravitational field equivalent to the effect of magnetic curvature is almost parallel to the x direction. In this case, the particle trajectories must satisfy the equation of motion and initial conditions instead of Eq.(1),

$$\frac{d\vec{r}'}{dt} = \vec{v} \quad , \qquad \vec{r}'(t'=t) = \vec{r} \quad ,$$

$$\frac{d\vec{v}'}{dt} = \frac{q}{M}\left[E_0(x) + \frac{\vec{v}'}{c} \times B_0\right] + \vec{F}_g, \qquad \vec{v}'(t'=t) = \vec{v} \quad , \tag{48}$$

where the gravitational force may be approximated to be

$$\vec{F}_g = -\frac{\vec{n}}{R} v_j^2 \simeq -g\frac{x}{L}\hat{e}_x \quad , \qquad g = \frac{v_j^2}{L} \quad . \tag{49}$$

The unperturbed distribution function which is a function of constant of the motion, can be taken to be

$$f_0(x,\ \vec{v}) = N_0\left(\frac{M}{2\pi T}\right)^{3/2}(1-\delta+\gamma)^{-1/2}$$

$$\times \exp\left\{-\frac{M}{2T}\left[(v_x^2+v_y^2+v_z^2+\hat{\omega}_E^2 x^2) + \frac{\delta-\gamma}{1-\delta+\gamma}(v_y+\omega_c x)^2\right]\right\} , \tag{50}$$

where $\hat{\omega}_E^2=\omega_E^2+g/L$. We note that the formalism disscussed in Sec.II-Sec.IV is also applicable to a cusped system by replacing $\omega_E^2$ by $\hat{\omega}_E^2$. Finally, the dispersion relation for a drift wave is given as

$$\omega_r \simeq \frac{\omega_{*e}}{\hat{F}}[(1-b_i)+\frac{\hat{\gamma}_i}{\delta_i}] \quad , \tag{51}$$

$$\omega_i \simeq \frac{\sqrt{\pi}}{\hat{F}^3}\frac{1}{\sqrt{2}v_e\hat{k}_{\parallel e}}(1+\frac{T_e}{T_i})\frac{(\omega_{*e})^2}{1+\hat{\gamma}_i}(1-b_i)(b_i+\frac{T_i}{T_e}\frac{\hat{\gamma}_i}{1+\hat{\gamma}_i}) \quad , \tag{52}$$

where $\hat{\gamma}_i=\hat{\omega}^2_{Ej}/\omega^2_{cj}$, $\hat{k}^2_{\parallel j}=\tilde{k}^2_{\parallel j}(\gamma_j\to\hat{\gamma}_j)$, and $\hat{F}=F(\gamma_j\to\hat{\gamma}_j)$. Krall and Rosenbluth have considered the curvature effect by introducing an equivalent gravitational field, and have found that the favarable curvature can stabilize a electrostatic drift instability[4]. Also, Kitao investigated the effect of magnetic curvature on stability of a Alfvèn wave[5]. For the cusped configuration considered in this section, equations(51) and (52) showed that the stabilization condition is satisfied provided

$$|\hat{\gamma}_i| > \frac{b_i\frac{T_e}{T_i}}{1+b_i\frac{T_e}{T_i}} \simeq b_i\frac{T_e}{T_i} \quad . \tag{53}$$

For $\gamma_i<0$, the cusped magnetic curvature tends to have a stabilizing effect in addition to the ambipolar field on a drift instability. For $\gamma_i>0$, however, the curvature effect seems not to be so efficient in a cusped configuration system because the inequality (53) is a quite severe condition in such a system.

## VI. Discussions

We here summarize briefly the results obtained in the present paper. First, we derived an integral dispersion relation in k-space, which describes the linear normal modes of electrostatic perturbations in an inhomogeneous plasma under the action of an ambipolar field. In analyzing instabilities in an inhomogeneous plasma, there exist two methods, namely, the local method and the nonlocal one. The local method is valid only when the wavelength is much smaller than the scale length, while the nonlocal method is most effecient when the wavelength is comparable to the scale length. Eq.(18) does not depend on the Larmor radius being small compared with the scale lenght. Both methods for Eq.(18) may be employed to find the normal modes and related instabilities in the plasma over wide range of densities and density gradients. Recently, Gerver et al. investigated the normal modes in a loss cone plasma with density varing sinusoidally in space[6]. They showed that it is possible to use a local method to find the normal modes and their instabilities, even when the Larmor radius is comparable to the scale length, provided the wavelength is much less than the scale length.

In the present paper, we developed a stability analysis for Eq.(18) by using a local method. Particularly, we discussed the effect of ambipolar field and also the effect of magnetic curvature on a drift wave in a cusped configuration system. For the negative ambipolar field ($E_0<0$), the cusped magnetic curvature tends to have a stabilizing effect in addition to the ambipolar field on a drift instability. For the positive ambipolor field ($E_0>0$), however, the curvature effect seems not to be so efficient, even in a cusped configuration system. Analysis of normal modes of experimental plasmas with Lamor radius comparable to scale length awaits futuer investigation.

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References


1) S. Hiroe, T. Sato, T. Watari, S. Hidekuma, R. Kumazawa, K. Adati, T. Shoji, S. Kawasaki, T. Hatori, H. Obayashi, T. Watanabe, N. Watanabe, S. Miyake, A. Miyahara and K. Takayama; Proceeding of the 6th International Conference of Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgarden, IAEA-CN-35/H5 Vol. 2 (1977)

2) T. Hatori, T. Watanabe, H. Obayashi, T. Sato, S. Hiroe, T. Watari, T. Shoji, K. Adati, J. Jacquinot and A. Miyahara, Proc. 5th International Conf. on Plasma Physics and Controlled Nuchear Fusion Research, Tokyo (1974), Vol.1, p.663

3) C. S. Liu, M. N. Rosenbluth, and C. W. Horton Jr., Phys. Rev. Lett. 29. 1489 (1972)

4) N. A. Krall and M. N. Rosenbluth, Phys. Fluids 8, 1488 (1965)

5) K. Kitao, J. Phys. Soc. Japan 26, 802 (1969)

6) M. J. Gerver, C. K. Birdsall, A. B. Langdon, and D. Fuss, Phys. Fluids 20, 291 (1977)