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RESEARCH REPORT

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Anomalous Effect of Small Collisions on the Nonlinear Interaction in a Small Cold Beam and a Warm Plasma System

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Abstract

It is found that a nonlinear wave-particle interaction based on the single wave model can be changed strongly by introducing small collisions into plasma electrons. These collisions are too small to change the linear stage. However, they alter the phase relation between the single wave and the trapped electrons in the nonlinear stage. As the result, the wave amplitude of the first minimum decreases anomalously and the persistent trapped-particle oscillations in collisionless case are destroyed.

A nonlinear wave-particle interaction in a small cold beam-plasma system has been investigated both theoretically[1] and experimentally[2]. Theoretical predictions[1] based on the single wave model agree with experimental observations[2] through the initial trapping of beam electrons and up to the first amplitude oscillation. Beyond this point, however, experiments[2] exhibit a rapid decay of the saturated wave rather than the persistent trapped-particle oscillations predicted by the theory[1]. Recently, Dimonte and Malmberg[6] observed destruction of trapped-particle oscillations simulating background plasma by a traveling wave tube.

A spatial evolution of a beam-plasma instability during steady injection of a small cold beam into a warm plasma has been studied by a particle simulation[5]. A typical example of results is shown in Fig.1. Behavior of an unstable monochromatic wave is in agreement with the theory up to the point where it saturates due to beam trapping. A discrepancy between simulation results and the theory[1], however, appears after this point. The most different features are as follows: The wave damps strongly to the first minimum and fails to regrow to the amplitude of the first maximum. The ratio of the amplitude of the first maximum to that of the first minimum is about three times as large as the expected value from the theory[1]. In addition to this, motion of beam electrons, i.e. the phase space loci of the electrons shown in Fig.1(B), do not become coincident with the theoretical results after this point.

In order to examine some discrepancies from the theory which can occur in both the laboratory experiments and the particle simulation, we have extended the theory to the more rigorous one including the collisional effect of the plasma electrons. In Ref.1, the collisional effects and the higher order of the temperature effect have been neglected. In the warm background plasma with collisions, however, the behavior of the wave amplitude and beam electrons in a nonlinear stage can be changed strongly from that expected by the collisionless trapping model, even if collisions are too small to change the linear stage. As an effective collision frequency, we may adopt the reciprocal of the slowing down time associated with the Coulomb collisions in a dense plasma. Even when collisions are negligible, however, some weakly nonlinear processes(like parametric instabilities) may result in an effective damping of the saturated wave.[3,6] Since the beam velocity may be much greater than the plasma thermal velocity, collisional effects of the beam electrons are neglected. In beam-plasma experiments where plasma is generated by the injected beam, however, electron-neutral collisions of the beam electrons may be important for the development of the nonlinear stage. These collisional effects may also have another effect which is different from that considered here.

We consider the spatial evolution of a single wave of frequency ω. Following Ref.1, we can treat the plasma as a linear dielectric medium. The plasma dielectric function $\varepsilon(\omega,k)$ is given in the form

$$\varepsilon(\omega,k) = 1 - \frac{\omega_p^2}{\omega(\omega+i\nu)}\left(1 + 3\frac{k^2 v_t^2}{\omega^2}\right), \qquad (1)$$

where ω_p is the plasma frequency, v_t is the thermal velocity of the plasma electrons, k is the wave number and ν is the effective collision frequency of the plasma electrons. The system of equations is obtained in terms of a natural extension of the model by O'Neil et al.[1] and Jungwirth and Krlin[3]:

$$\left(A + iB + i\frac{d}{d\eta} + C\frac{d^2}{d\eta^2}\right)E(\eta) = \frac{i}{N}\sum_{j=1}^{N} e^{i\xi_j}, \qquad (2)$$

$$\frac{d^2\xi_j}{d\eta^2} = \left(1 + \varkappa\frac{d\xi_j}{d\eta}\right)^3 E(\eta)\, e^{-i\xi_j} + \text{c.c.}\ , \qquad (3)$$

where

$$A = \varkappa^2 \cdot \frac{\omega_{pt}^2}{\omega_b^2} \cdot \left[2\frac{\delta\omega}{\omega_{pt}} + \left(\frac{\delta\omega}{\omega_{pt}}\right)^2\right],$$

$$B = \varkappa^2 \cdot \frac{\nu}{\omega_{pt}} \cdot \frac{\omega_{pt}^2}{\omega_b^2} \cdot \left(1 + \frac{\delta\omega}{\omega_{pt}}\right)^{-1},$$

$$C = \frac{\varkappa}{Z}\ , \qquad \varkappa = \left(\frac{1}{6} \cdot \frac{n_b}{n_p} \cdot \frac{v_b^2}{v_t^2}\right)^{\frac{1}{3}},$$

and

$$\omega_{pt}^2 = \omega_p^2\left(1 + 3\frac{v_t^2}{v_b^2}\right).$$

Equation (2) is obtained by usual manipulation after the Taylor expansion of the dielectric function up to the second order of $k-k_0$ where $k_0 = \omega / v_b$. The real part of the dielectric function $\varepsilon_r(\omega, k_0)$ need not be necessarily equal to zero. In these equations, n_p and n_b are the plasma and beam densities, respectively, ω_b is the plasma frequency of the beam electrons, v_b is the initial beam velocity, $\delta\omega$ is the difference between the frequency of the most unstable mode ω_{pt} and ω, and $\varkappa$ is the spatial scaling factor, where $(3^{1/2}/2)\varkappa k_0$ is the spatial growth rate in the collisionless warm plasma. The normalized electric field $E(\eta)$ of the wave and the spatial coordinate η are defined in terms of x and $\varkappa$ by $E(\eta) = eE(x)/(mv_b \omega \varkappa^2)$ and $\eta = \varkappa x(\omega/v_b)$, respectively. The phase space coordinate ξ_j of the j-th beam electron is defined as $\xi_j = \omega [t_j(x) - x/v_b]$. The function $t_j(x)$ is the time when the j-th beam electron passes the point x. The velocity $\dot{x}_j$ in the laboratory frame is obtained from $\dot{\xi}_j = d\xi_j/d\eta$ by using the relation $\dot{x}_j = v_b/(1+\varkappa \dot{\xi}_j)$. In Eq.(2), the first term of the left hand side is derived from the detuning,[3] the second is the collisional effect, and the third and the fourth are the temperature effects.

The typical results of the effect of collisions on the wave is shown in Fig.2. The dashed curve shows the amplitude squared vs η in collisionless case. The solid curve is the amplitude squared vs η when $\nu/\omega_{pt} = 2 \times 10^{-3}$. Some other parameters are chosen to fit the simulation, i.e. $n_b/n_p = 5 \times 10^{-4}$, $v_b/v_t = 9.9$ and $\delta\omega/\omega_{pt} = 0$.

Before the point $\eta = 12$, the collisional effect does not significantly alter the spatial evolution of the wave except that the wave amplitude saturates at a slightly lower level than the wave does in collisionless case. After this point, however, two curves begin to depart from each other. The wave amplitude decreases drastically much smaller than that of the collisionless case and fails to regrow to the initial saturation level.

The corresponding evolution of the beam in the collisional case is summarized in the sequence of phase space loci shown in Fig.2(B). Each locus is composed of the phase points of the 200 beam electrons at various positions denoted in Fig.2(A). In accordance with the amplitude oscillation, the sloshing back and forth of the trapped beam in the wave trough appears. Though the motion of the beam electrons in phase space is still a reversible process, after saturation its behavior changes gradually from that of the collisionless case. Near the point η_3, the locus splits into two parts and forms the double structure of vortex. Moreover, the beam electrons begin to be smeared out in an irreversible manner after the point η_4.

Numerical results described above agree with the results of particle simulation. Here we estimate the collision frequency in the simulation: Collision frequency ν of the one-dimensional particle simulation has the relationship[4] with the plasma density n_p and the Debye length λ_D, i.e. $\omega_p \nu^{-1} = \alpha (n_p \lambda_D)^2$, where α is a numerical factor and is about 0.1, and $n_p \lambda_D \cong 90$ in this simulation.

Then the calculated value of ν/ω_p is about 1.2×10^{-3}, the value $\nu/\omega_{pt} = 2\times10^{-3}$ assumed in Fig.2 is reasonable.

From the numerical calculations, it is clear that collisions play an important role in the nonlinear stage of beam-plasma instability. The features in Figs.1 and 2 are well explained as follows: Plasma electrons support the unstable single wave as a linear dielectric medium. On the analogy of an electric circuit, collisions of plasma electrons act as a phase shifter. A spatial evolution of the wave phase is shown in Fig.3. Near the point $\eta = 12.5$, the wave undergoes a rapid phase shift as well as a strong damping of the amplitude. According to our detailed analysis of the phase space loci, the beam electrons spill into adjacent wave troughs and spread in phase space due to the reduced amplitude and the phase shifts, as pointed out by Dimonte and Malmberg.[6] After η_3, numerical calculations also show that the kinetic energy of the beam electrons remains at a some lower level than the initial beam energy. It can be considered that there is nearly as many particles being accelerated as there is being decelerated, and that little net energy exchange occurs and the wave remains at a low level.

In Ref.6, oscillations could be destroyed as a result of particle phase-mixing by either (a)wave damping or (b)modulation of the main wave by unstable sidebands. In the former case, a catastrophic effect on the oscillation is expected to be similar to the results presented in Fig.2. They also obtained the similar result as (b) by launching the main wave and broad-band noise. We consider the case where a small collision and broad-band noise of low level coexist. The destruction of the amplitude oscillation may occur more easily, if the amplitude of the first minimum can decrease near to the noise level by a small collision. In this case, the destruction can occur after the first minimum. The tendency like this appears in the simulation result in Fig.1 and may occur in laboratory experiments when the saturation level is not much larger than the noise level.

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Figure captions

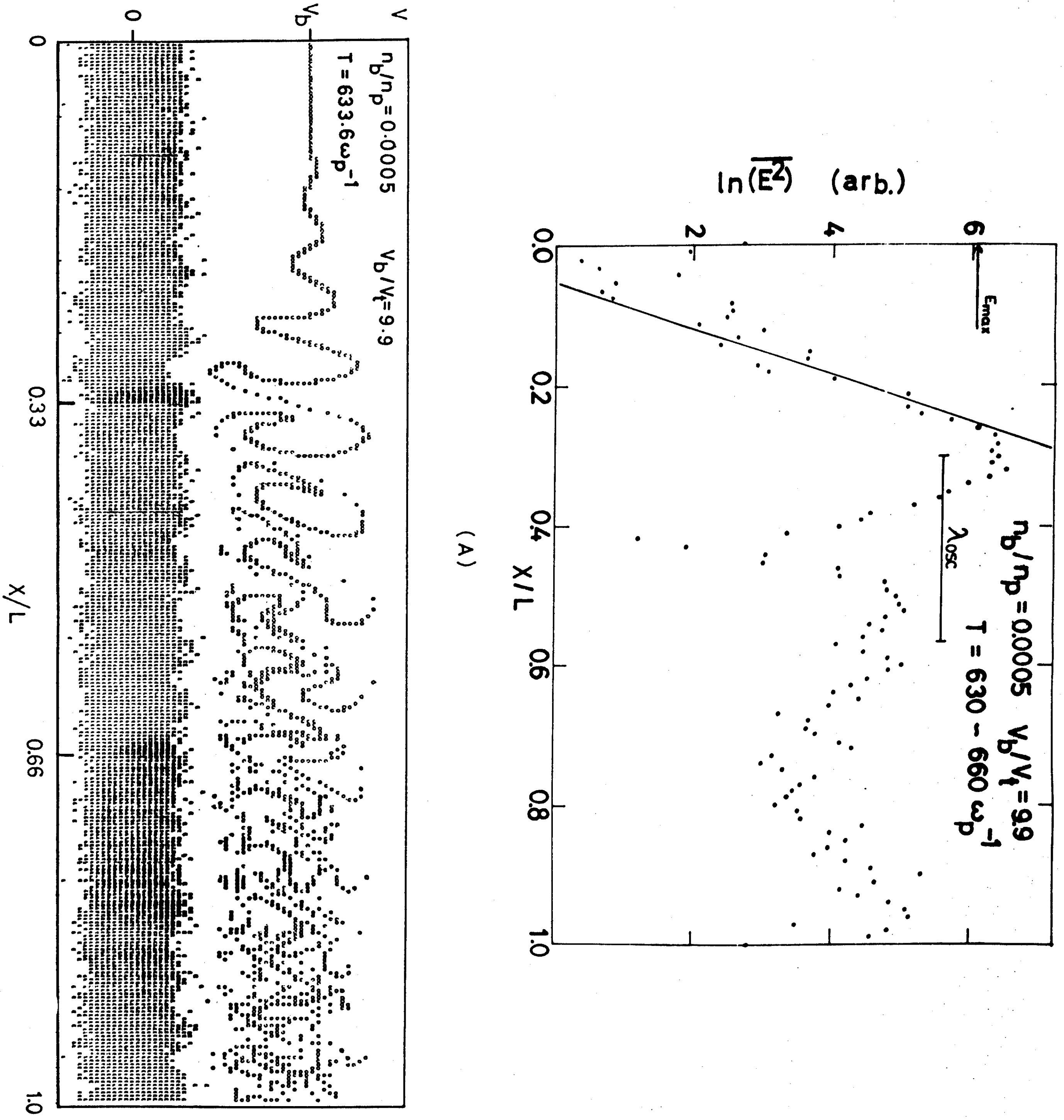
Fig.1. (A) A spatial evolution of the wave power averaged in a term of 630~660 ω_p^{-1}. The solid line shows the theoretical linear growth rate. Both E_{max} and λ_{osc} are the calculated values for the maximum amplitude of the wave and the period of the amplitude oscillation, respectively, from O'Neil's theory.

(B) The motion of the beam particles in phase space.

Fig.2. (A) Comparison of the calculated spatial evolution of the single wave for collisionless case, the dashed curve, and for collisional case, the solid curve.

(B) Phase space loci for the beam particles at various positions denoted in Fig.2(A). Each point gives the velocity x/v_b in the laboratory frame and the normalized spatial coordinate ξ for one of the beam particles. Only every second particle is plotted.

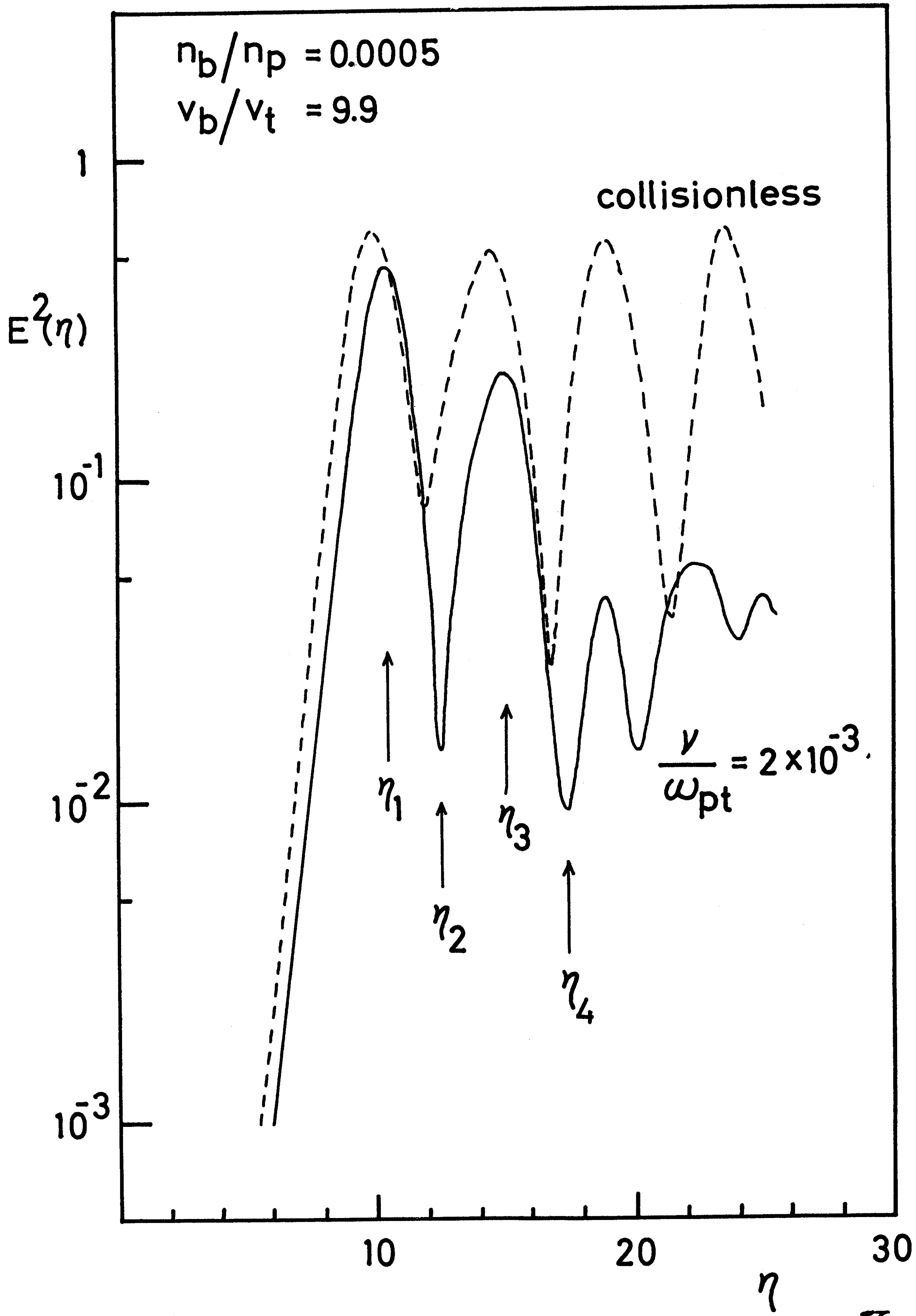
Fig.3. A spatial evolution of the wave phase for (A) a collisionless case and (B) a collisional case. The solid curve and the dashed curve are the real part and the imaginary part of the electric field of the wave, respectively.



ln$(\overline{E^2})$ (arb.)
$n_b/n_p = 0.0005$ $V_b/V_t = 9.9$
$T = 630 - 660\ \omega_p^{-1}$
E_{max}
λ_{osc}
X/L
(A)
$n_b/n_p = 0.0005$ $V_b/V_t = 9.9$
$T = 633.6\omega_p^{-1}$
V
V_b
X/L
(B)

Fig. 1

(A)



$n_b/n_p = 0.0005$
$v_b/v_t = 9.9$
1
collisionless
$E^2(\eta)$
10^{-1}
10^{-2}
10^{-3}
η_1
η_2
η_3
η_4
$\frac{\gamma}{\omega_{pt}} = 2\times10^{-3}$
10
20
30
η

Fig. 2

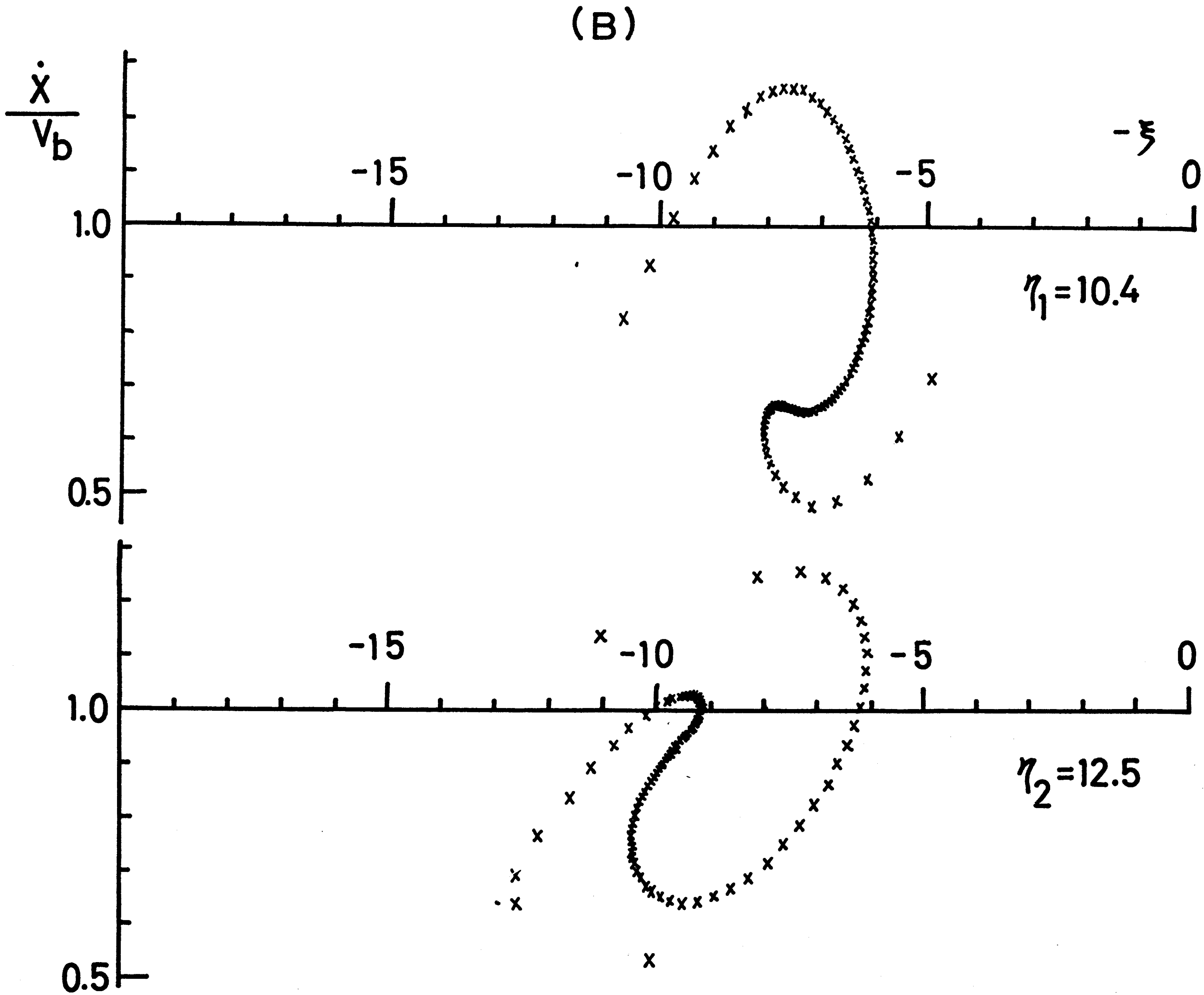


Fig. 2

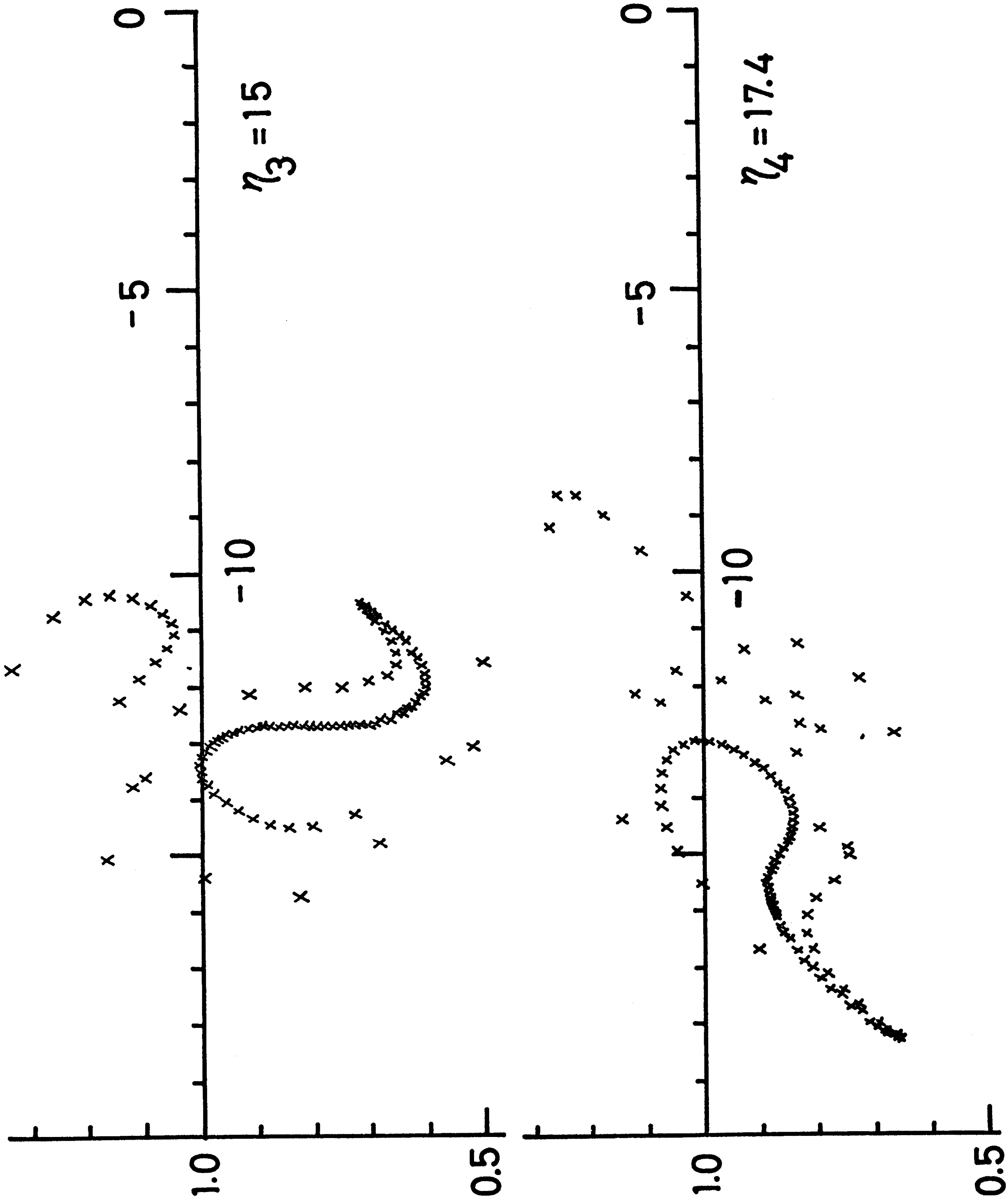


Fig. 2

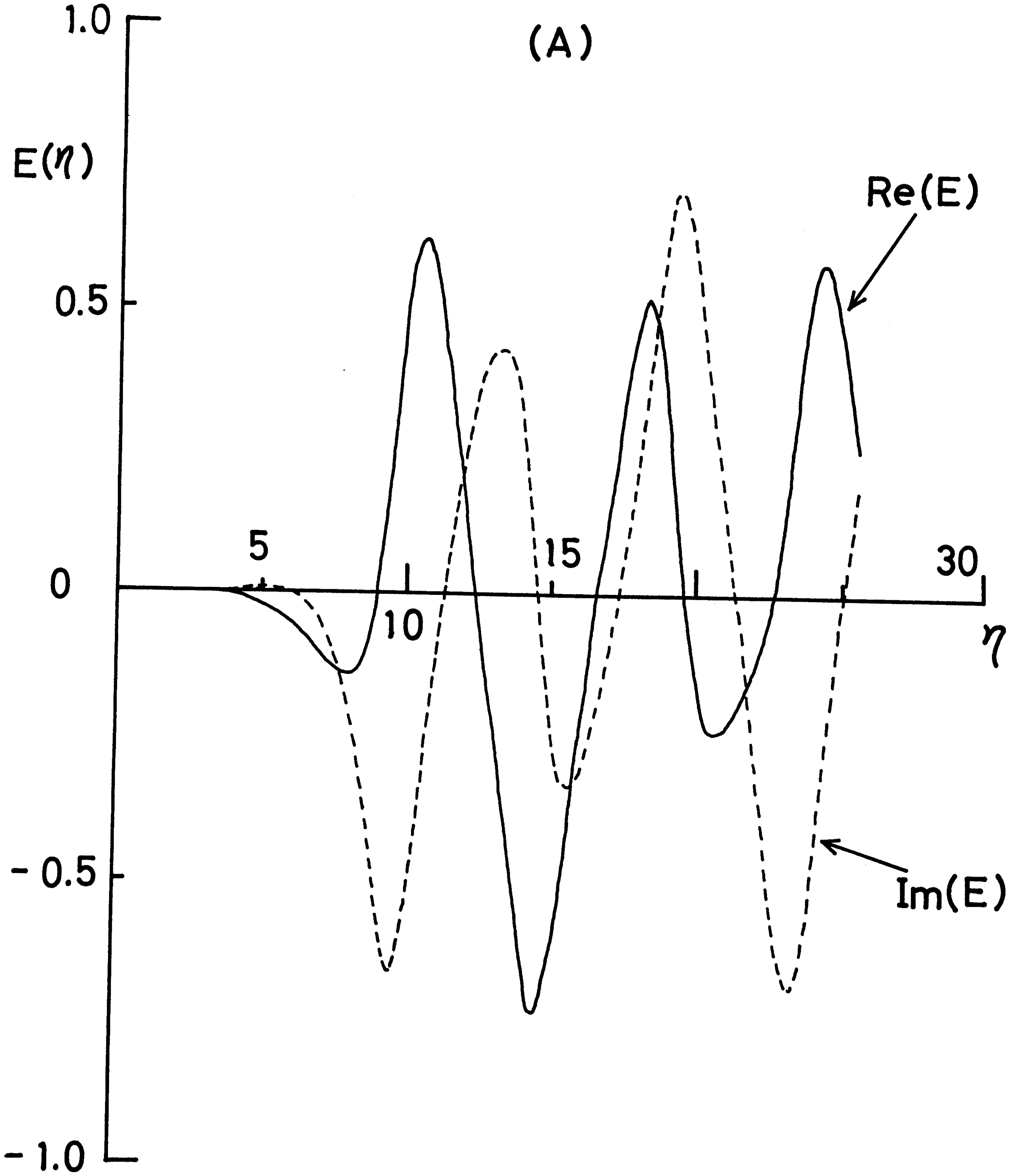


Fig. 3

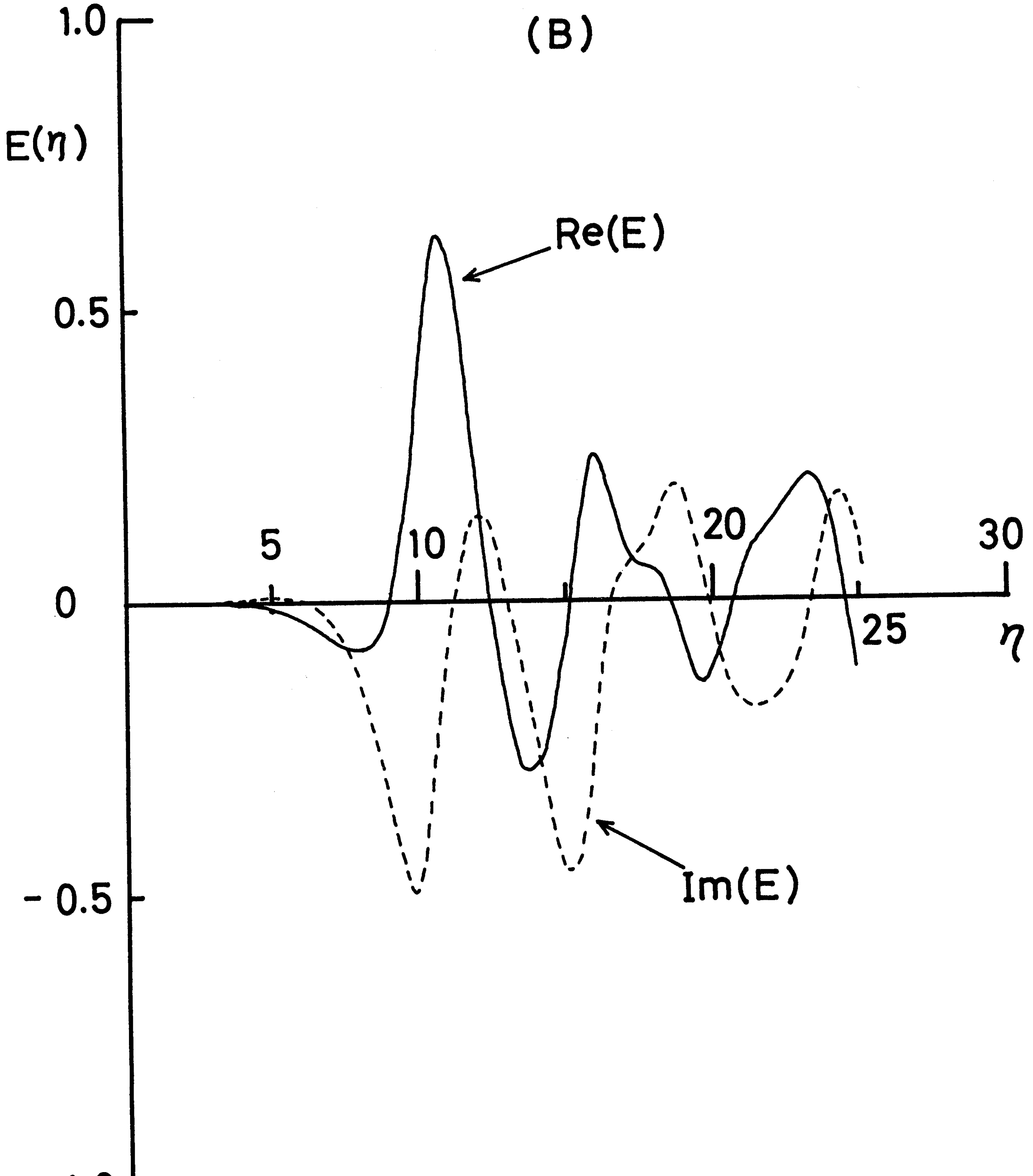


Fig. 3