INSTITUTE OF PLASMA PHYSICS
NAGOYA UNIVERSITY

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Solitons, Envelope Solitons in Collisonless Plasmas

Yoshi H. Ichikawa and S. Watanabe*

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

Permanent address :

* Research Institute for Energy Materials, Yokohama National University, Yokohama, Japan.

Abstract ——— A review is given to extensive development of theoretical, computational and experimental studies of nonlinear wave propagation in collisionless plasmas. Firstly, the historical experiment of Ikezi et al. is discussed in comparison with theoretical analysis based on the Korteweg-de Vries equation. Systematic discrepancy between the observation and the theoretical prediction suggests that it is necessary to examine such as higher order mode coupling effect and contribution of trapped particles. Secondly, effects of the nonlinear Landau damping on the envelope soliton of ion plasma wave is discussed on the basis of theoretical study of Ichikawa-Taniuti, experimental observation of Watanabe and numerical analysis of Yajima et al. Finally, a new type of evolution equation derived for the Alfvén wave is examined in some detail. The rigorous solution obtained for this mode represents a new kind of envelope soliton, in which both of its phase and amplitude are subject to modulation of comparable spatial extension. In conclusion, the emphasis will be placed on the fact that much more intensive experimental researches are expected to be done, since the powerful methods to disentangle various nonlinear evolution equations are now available for theoretical approach.

I. Introduction. ——— Studies of nonlinear wave phenomena in collisionless plasmas provide a firm base not only for exploring fundamental researches on nonlinear physics, but also for developing practical applications in controlled nuclear fusion technology. Problems of laser-plasma interaction, anomalous transport and radio-frequency confinement are typical examples having strong motivation in the latter connection. In this paper, however, we will put our emphasis on the fact that recent advancement in understanding of nonlinear wave phenomena opens the way to establish physics of nonlinear phenomena in nature.

In the month of August 1834, Scott-Russel [1] had the first chance to observe a rounded, smooth and well defined heap of water continued its course along the channel appearently without change of form or diminution of speed. This solitary wave propagated about one miles at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. In 1895, analyzing competing process of dispersive effect and nonlinear steepning effect in the shallow water wave propagation, Korteweg and de Vries [2] have derived a nonlinear partial differential equation to explain the properties of the solitary wave. This equation is now called by their names.

Since Gardner and Morikawa [3] have rediscovered that the Korteweg-de Vries equation valids also for nonlinear magneto-hydrodynamic wave propagating perpendicular to the external magnetic field, refreshed interests have been stirred up on the studies of nonlinear wave phenomena in the field of plasma physics. Theoretical prediction of Washimi and Taniuti [4] on the possibility of the ion-acoustic solitary wave has been confirmed experimentally by Ikezi, Taylor and Baker [5], [6]. Reinforcement of the genius invention of the inverse scattering method of solving nonlinear evolution equations [7], [8] has encouraged us to endeavor to disentangle complicated nonlinear wave phenomena on the firm theoretical ground.

We begin our discussion on the historical experiment of Ikezi et al [5], [6] on the ion-acoustic soliton in section II, and then proceed to discuss recent theoretical development on the properties of solitons associated with the weakly dispersive system in section III. In section IV, we discuss theoretical and experimental aspects of the nonlinear wave modulation in the strongly dispersive region. We present in section V a new type of evolution equation derived for the Alfven wave, propagating along the magnetic field, and discuss its analytic steady state solution in some details. As concluding remarks, we mention briefly potential importance of the studies of nonlinear wave phenomena on understanding of behaviour of plasmas which are expected to be produced in controlled thermonuclear fusion devices.

II. Ion-acoustic solitons. ——— Firstly, let us derive the Korteweg-de Vries equation for the ion-acoustic wave on the basis of the reductive perturbation theory developed by Taniuti and his collaborators during the years of

1968∿1974, [9]. For a collisionless plasma composed by cold ions and warm electrons, the basic set of equations may be expressed as (in a dimensionless form),

$$\frac{\partial}{\partial t} n + \frac{\partial}{\partial x} (nu) = 0 \ , \qquad (1.a)$$

$$\frac{\partial}{\partial t} u + u \frac{\partial}{\partial x} u = - \frac{\partial}{\partial x} \psi \ , \qquad (1.b)$$

$$\frac{\partial^2}{\partial x^2} \psi = n_e - n \ , \qquad (1.c)$$

$$n_e = \exp \psi \qquad (1.d)$$

where $n=\tilde{n}_i/n_0$, $n_e=\tilde{n}_e/n_0$, $u=\tilde{u}(\kappa T_e/M)^{-1/2}$ and $\psi=\tilde{\psi}(\kappa T_e/e)^{-1}$ are the dimensionless ion number density, electron number density, ion velocity and electro-static potential, respectively. Dimensionless space-time variable (x,t) are measured by the Debye distance $(\kappa T_e/4\pi e^2 n_0)^{1/2}$ and the ion plasma frequency $(4\pi e^2 n_0/M)^{1/2}$. Needless to say, we are considering one dimensional wave motion in the system.

Imposing the boundary condition,

$$n=1, \quad \psi=0, \quad u=1 \quad \text{as} \quad |x| \to \infty \ , \qquad (2)$$

we introduce the stretched variables

$$\xi = \varepsilon^{1/2} (x-t), \qquad (3.a)$$

$$\tau = \varepsilon^{3/2} t \ . \qquad (3.b)$$

With these re-scaling of the independent variables, the basic equations (1.a)∿(1.d) are transformed as follows,

$$\varepsilon \frac{\partial}{\partial \tau} n - \frac{\partial}{\partial \xi} n + \frac{\partial}{\partial \xi} (nu) = 0 \tag{4.a}$$

$$\varepsilon \frac{\partial}{\partial \tau} u - \frac{\partial}{\partial \xi} n + u \frac{\partial}{\partial \xi} u = - \frac{\partial}{\partial \xi} \psi \tag{4.b}$$

$$\varepsilon \frac{\partial^2}{\partial \xi^2} \psi = \exp \psi - n \tag{4.c}$$

Substituting power series expansions of n, u and ψ,

$$n=1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + \cdots \tag{5.a}$$

$$u= \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \cdots \tag{5.b}$$

$$\psi= \varepsilon \psi^{(1)} + \varepsilon^2 \psi^{(2)} + \cdots \tag{5.c}$$

into eqs.(4.a)$\sim$(4.c), we can establish relationship among the first order quantities as

$$\psi^{(1)} = n^{(1)} = u^{(1)} = n_e^{(1)} \quad , \tag{6}$$

in the lowest order expansion of eqs.(4.a)$\sim$(4.b). Their explicit (ξ,τ)-dependence is determined through the Korteweg-de Vries equation

$$\frac{\partial}{\partial \tau} \psi^{(1)} + \frac{1}{2} \frac{\partial^3}{\partial \xi^3} \psi^{(1)} + \psi^{(1)} \frac{\partial}{\partial \xi} \psi^{(1)} = 0 \quad , \tag{7}$$

which is derived as the compatibility condition of the second order components of eqs.(4.a)$\sim$(4.c).

Although the Korteweg-de Vries equation can be solved analytically for an arbitrary initial value with the help of the inverse scattering method, here we present a steady state one-soliton solution of (7) as

$$\psi = A \operatorname{sech}^2[D(\xi - \lambda\tau)] \quad , \tag{8}$$

with

$$\lambda = \frac{1}{3} A \quad , \tag{9.a}$$

$$D = (\lambda/2)^{1/2} \quad . \tag{9.b}$$

Namely, the one soliton runs with velocity faster than the ion acoustic speed by the amount proportional to one third of its amplitude. The width of soliton is inversely proportional to the square root of its amplitude. Fig.1 illustrates nonlinear evolution of the large amplitude perturbation excited in the double plasma device, having the following parameters $n_0=(1\sim2)\times10^9$ cm^{-3}, $T_e=2\sim3$ eV, $T_e/T_i \sim 10$ in low pressure Argon gas with $(2\sim5)\times10^{-4}$ Torr in typical operation conditions. The large amplitude perturbation is decomposed into several peaks in the course of its propagation. The first small peak is a precursor consisted with ions reflected back from the large potential barrier, of which properties have been examined theoretically by Kato et al [10].

We may summarize the experimental results as follows:

1) The velocity of the soliton is approximately in accord with the theoretical value predicted by (9.a) but the observed velocity is faster than the velocity of the Korteweg-de Vries soliton.

2) The width of the soliton is in rough agreement with the theoretical value of (9.b), but it is narrower than the width of the Korteweg-de Vries soliton.

3) The number of solitons is in agreement with the value predicted by the analytic solution given by the

inverse scattering method.

4) The recurrence to its initial form of perturbation has been demonstrated.

Systematic discrepancy between the experimental observation and the theoretical prediction calls for refinement of simplified Korteweg-de Vries soliton description. As an improvement of the model, effects of finite ion temperature have been examined by Kato et al [10], Tappert [11] and Tagare [12]. With regards the large amplitude effects, Schamel [13] has proposed a different type of the nonlinear equation with full account of the trapped particles by electrostatic potential of the wave, while Konno and Ichikawa [14] have shown that contribution of three-wave interaction, with account of the finite ion temperature effect, removes the discrepancy between the theory and experiment considerably, (Fig.2). We should, however, emphasize that none of these can discriminate the others, conclusively. Experimental investigation for various electron-ion temperature ratio will be useful to draw definite conclusions.

III. Higher order perturbation and dressed soliton.

——— Besides the above mentioned refinements from the physical consideration, we may ask how contributions of higher order perturbation terms modify basic properties of the Korteweg-de Vries soliton within the mathematical framework of the model system described by eqs.(1.a)∿(1.d). We have undertaken the analysis of higher order terms of eqs.(4.a)∿(4.c), [15]. The second order quantities $n^{(2)}$ and

$u^{(2)}$ are expressed as

$$n^{(2)} = \psi^{(2)} + \frac{1}{2}\psi^{(1)}\psi^{(1)} - \frac{\partial^2}{\partial\xi^2}\psi^{(1)} , \qquad (10.a)$$

$$u^{(2)} = \psi^{(2)} - \frac{1}{2}\frac{\partial^2}{\partial\xi^2}\psi^{(1)} , \qquad (10.b)$$

while behaviour of the second order potential $\psi^{(2)}$ is determined from the following equation,

$$\frac{\partial}{\partial\tau}\psi^{(2)} + \frac{1}{2}\frac{\partial^3}{\partial\xi^3}\psi^{(2)} + \frac{\partial}{\partial\xi}(\psi^{(1)}\psi^{(2)}) = S(\psi^{(1)}), \qquad (11.a)$$

where

$$S(\psi^{(1)}) = -\frac{3}{8}\frac{\partial^5}{\partial\xi^5}\psi^{(1)} + \frac{1}{2}\psi^{(1)}\frac{\partial^3}{\partial\xi^3}\psi^{(1)} - \frac{5}{8}\frac{\partial}{\partial\xi}(\frac{\partial}{\partial\xi}\psi^{(1)})^2. \qquad (11.b)$$

Thus, the Korteweg-de Vries equation (7) and the linear inhomogeneous equation (11.a) with (11.b) describe nonlinear ion acoustic wave propagation in the second order.

Seeking a type of solutions $\psi^{(1)}(\eta)$ and $\psi^{(2)}(\eta)$ with argument

$$\eta = \xi - \lambda\tau , \qquad (12)$$

we have obtained a steady one soliton solution of the coupled set of eqs.(7) and (11.a) with (11.b) as follows, letting ordering parameter $\varepsilon \to 1$,

$$\psi(\eta) = \psi^{(1)}(\eta) + \psi^{(2)}(\eta) , \qquad (13.a)$$

$$\psi^{(1)}(\eta) = 3\lambda \ \mathrm{sech}^2(D\eta) , \qquad (13.b)$$

$$\psi^{(2)}(\eta) = \frac{9}{4}\lambda^2\mathrm{sech}^2(D\eta)\{2D\eta\tanh(D\eta) - 8 + 7\ \mathrm{sech}^2(D\eta)\} , \qquad (13.c)$$

with

$$D = (\lambda/2)^{1/2} \quad . \tag{13.d}$$

The perturbed potential $\psi(\eta)$ can be regarded as the dressed soliton, of which velocity λ is given by the amplitude A of the ion acoustic potential perturbation as

$$\lambda = \frac{1}{3} A + \frac{1}{12} A^2 \quad . \tag{14}$$

We have observed numerically that the steady state clouds (13.c) moves stably with the Korteweg-de Vries soliton core (13.b).

We have also examined numerically the collision processes of the dressed solitons, [16]. As a solution of the Korteweg-de Vries equation (7), we take the well-known two-soliton solution,

$$\begin{aligned}\psi^{(1)}(\xi,\tau) &= 24[1 + \exp(2D_1\eta_1) + \exp(2D_2\eta_2) \\ &+ \exp(2D_1\eta_1 + 2D_2\eta_2 + \delta_3)]^{-1}[D_1{}^2\exp(2D_1\eta_1) \\ &+ D_2{}^2\exp(2D_2\eta_2) + 2(D_1 - D_2)^2\exp(2D_1\eta_1+2D_2\eta_2) \\ &+ D_2{}^2\exp(4D_1\eta_1+ 2D_2\eta_2 +\delta_3) + D_1{}^2\exp(2D_1\eta_1 + \\ &4D_2\eta_2 + \delta_3)] \ ,\end{aligned}$$

where

$$\eta_1 = \xi - \lambda_1\tau - \delta_1 \quad , \tag{16.a}$$

$$\eta_2 = \xi - \lambda_2\tau - \delta_2 \quad , \tag{16.b}$$

$$D_1 = (\lambda_1/2)^{1/2} \quad , \tag{16.c}$$

$$D_2 = (\lambda_2/2)^{1/2} \quad , \tag{16.d}$$

$$\delta_3 = \log\left[\frac{(D_1 - D_2)^2}{(D_1 + D_2)^2}\right] \quad . \tag{16.e}$$

The parameters δ_1 and δ_2 denote initial positions of the two solitons. We describe an initial state of the binary system of the dressed solitons approximately by superposing the steady state second order clouds (13.c) with the amplitudes given by λ_1 and λ_2 at the positions of δ_1 and δ_2, respectively. Fig.3 presents the temporal evolution of the shape of two dressed solitons with the values of λ_1=0.3 and λ_2=0.24. The thin line represents the Korteweg-de Vries soliton core, while the broken line at time τ=0 represents the steady state second order clouds associated with each soliton core. In the course of collision process, we observe that the clouds associated with the binary soliton core redistribut themselves in such a way to equalize their amplitude after the collision.

Concerning with the structure of the dressed soliton, Sugimoto and Kakutani [17] have remarked that the term with $D\eta \tanh(D\eta)$ implies the fact that the reductive perturbation expansion carried up to the second order is not free from the secularity. They have proposed to eliminate this term by the method of multiple space-time variables. Introducing the following multiple space-time variables,

$$\xi = \varepsilon^{1/2} (x - t) , \qquad \tau = \varepsilon^{3/2} t , \tag{17.a}$$

$$\xi_2 = \varepsilon^{3/2} (x - t) , \qquad \tau_2 = \varepsilon^{5/2} t , \tag{17.b}$$

they have obtained the following equation for the place of (11.a),

$$\frac{\partial}{\partial \tau} \psi^{(2)} + \frac{1}{2} \frac{\partial^3}{\partial \xi^3} \psi^{(2)} + \frac{\partial}{\partial \xi}(\psi^{(1)} \psi^{(2)}) = S(\psi^{(1)})$$
$$- [\frac{\partial}{\partial \tau_2} \psi^{(1)} - \frac{1}{2} \frac{\partial}{\partial \xi_2}(\psi^{(1)})^2 - \frac{3}{2} \frac{\partial^3}{\partial \xi_2 \partial \xi^2} \psi^{(1)}] , \tag{18}$$

where $S(\psi^{(1)})$ is given by (11.b). As for the steady one soliton solution of eqs.(11.a) and (18), taking a form of

$$\psi^{(1)}(\eta;\tau_2,\xi_2)=6D(\tau_2,\xi_2)^2\operatorname{sech}^2[D(\tau_2,\xi_2)(\eta+\theta(\tau_2,\xi_2))], \tag{19}$$

where η is given by (12), and D is defined by (13.d), they have obtained the following set of equations,

$$\frac{\partial}{\partial\tau_2} D = 0 \quad , \qquad \frac{\partial}{\partial\xi_2} D = 0 \quad , \tag{20.a}$$

$$\frac{\partial}{\partial\tau_2} \theta = 3D^4, \qquad \frac{\partial}{\partial\xi_2} \theta = -\frac{3}{2} D^2 \ , \tag{20.b}$$

as conditions to eliminate the secular term. Hence, slow variation of the phase $\theta(\tau_2,\xi_2)$ is given by

$$\theta(\tau_2,\ \xi_2) = -\frac{3}{2}D^2(\xi_2 - \lambda\tau_2) = -\varepsilon\frac{3}{2} D^2\eta, \tag{21}$$

where $D = (\lambda/2)^{1/2}$ is a constant. Solving $\psi^{(2)}$ from (18) with (19) and (21), one can easily write down the perturbed potential up to the second order terms as

$$\tilde{\psi}(\eta) = 3\lambda \operatorname{sech}^2(\tilde{D}\eta) + \frac{9}{4}\lambda^2\operatorname{sech}^2(\tilde{D}\eta)\cdot$$

$$[-8 + 7\operatorname{sech}^2(\tilde{D}\eta)] \quad , \tag{22}$$

with the definition of

$$\tilde{D} = (\lambda/2)^{1/2}(1 - \frac{3}{4}\lambda) \quad . \tag{23}$$

It should be noticed here that the velocity of soliton λ is given by (14) expressed in terms of the maximum soliton amplitude A as before, but structure of the renormalized soliton is now given as

$$\tilde{\psi}(\eta) = \tilde{\psi}_{co}(\eta) + \tilde{\psi}_{cl}(\eta) \quad , \tag{24.a}$$

$$\tilde{\psi}_{co}(\eta) = 3\lambda(1 - \frac{3}{4}\lambda)\ \mathrm{sech}^2(\tilde{D}\eta) \quad , \tag{24.b}$$

$$\tilde{\psi}_{cl}(\eta) = -7\cdot\frac{9}{4}\lambda^2\mathrm{sech}^2(\tilde{D}\eta)\ \tanh^2(\tilde{D}\eta) \quad , \tag{24.c}$$

where $\tilde{\psi}_{co}(\eta)$ is the second order renormalized soliton core, and $\tilde{\psi}_{cl}(\eta)$ represents the second order cloud surrounding the core. In Fig.4, we illustrate the structure of the renormalized soliton.

Now, Kodama and Taniuti [18] have developed an elegant analysis of the renormalization procedure in carrying out the reductive perturbation to arbitrary higher order terms. They have reduced a set of equations for a model system to a renormalized Korteweg-de Vries equation, and have shown explicitly that the renormalization can be carried out not only for the one soliton state, but also for the system with an arbitrary number of solitons. Thus, we are now standing at a position where we can investigate dynamical properties of the renormalized soliton systems on firm ground.

IV. Self-modulation of strongly dispersive waves. ——— We now turn our interests to phenomena of self-modulation of a quasi-monochromatic wave in strongly dispersive region such as the electron Langmuir wave, the ion plasma wave and the whistler wave in magnetized plasmas. The problem has close connection with such phenomena of self-focusing and self-contraction of wave packets in nonlinear optics, and modulation-instability of the gravity waves on water. In collisionless plasmas, it is well awared by us that the resonant wave-particle interaction at the phase velocity causes very different nonlinear modulation associated with the trapped particles. Nevertheless, we develope our disscussion for a special case in which the trapped particles do not give rise to appreciable effects.

Taniuti et al [19] have presented a systematic analysis of the nonlinear modulation of a quasi-monochromatic wave by examining a system of equations

$$\frac{\partial}{\partial t} U + A(U) \frac{\partial}{\partial x} U + B(U) = 0 \quad , \tag{25}$$

where U is a column vector with n-components u_1, u_2, $\cdots$ u_n and A an n × n matrix and B a column vector. The set of eqs.(1.a)∿(1.d) can be reduced to the standard form by setting $-\partial\psi/\partial x = E$. It is assumed that (25) has a constant state solution U_0, which satisfies

$$B(U_0) = 0 \quad . \tag{26}$$

Considering a plane wave of infinitesimal amplitude propagating in the constant state U_0, we assume that U can be expanded about U_0 as

$$U = U_0 + \sum_{\ell=-\infty}^{\infty} \sum_{n=1}^{\infty} \varepsilon^n U_\ell^{(n)}(\xi,\tau)\exp[i\ell(kx-\omega t)] , \tag{27}$$

where ε measures the size of perturbed amplitude and (ξ,τ) are the stretched space-time variables defined as

$$\xi = \varepsilon(x - \lambda t) \quad , \tag{28.a}$$

$$\tau = \varepsilon^2 t \quad . \tag{28.b}$$

Substitution of eqs.(28.a)$\sim$(28.b) with (27) into the original equation (25) yields a set of equations corresponding to the each order of powers of ε and the ℓ-th harmonic component. In the first order of ε, the linear dispersion relation

$$\det[\pm i(\omega I - k\,A(U_0)) + \nabla_u B(U)_{u=u_0}] = 0 \quad , \tag{29}$$

assures that $U_{\pm i}^{(1)}$ can be expressed as

$$U_{\pm 1}^{(1)}(\xi,\tau) = \psi(\xi,\tau)R \tag{30}$$

with the right eigenvector R given by

$$[\mp i(\omega I - k\,A(u_0))] + \nabla_u B(u)_{u=u_0}]R = 0 \quad , \tag{31}$$

and $U_\ell^{(1)} = 0$ for $|\ell| \neq 1$. In the second order of ε, the ℓ=1 component yields a condition

$$\lambda \equiv \frac{\partial\omega(k)}{\partial k} \tag{32}$$

to deal with nontrivial case $\partial U_{\pm 1}^{(1)}/\partial\xi \neq 0$. The ℓ=2 and ℓ=0 component of the second order equation determines the second order beat wave $U_2^{(2)}$ and $U_0^{(2)}$, respectively. Finally, at the third order of ε, the ℓ =1 component gives rise to the nonlinear Schrödinger equation,

$$i\frac{\partial}{\partial\tau}\psi + p\frac{\partial^2}{\partial\xi^2}\psi + q\,|\psi|^2\psi = 0 \quad , \tag{33}$$

where $p=(1/2)\partial^2\omega/\partial k^2$ represents the dispersion effect, while q measures the strength of nonlinearity. For the ion plasma wave propagating in a system described by eqs.(1.a)$\sim$(1.d), we get [20]

$$p = -\frac{3}{2}\omega\left(\frac{\omega}{k}\right)^4 \quad , \tag{34.a}$$

$$q = \frac{\omega}{2}\left[-\frac{1}{k^2} - \frac{k^2}{2\omega^2} + \frac{\omega^4}{2k^4} + \frac{\omega^4}{3k^6} - \frac{\omega^6}{3k^6}\right.$$

$$\left. + \frac{1}{1-(\omega/k)^3}\frac{\omega^2}{k^2}\left(1+\frac{\omega^2}{k^2}\right)^2\right] . \tag{34.b}$$

Since q given by (34.b) is positive, the coefficients p and q take the opposite sign.

When $pq < 0$, finite amplitude plane wave is stable against modulation. For this case, setting

$$\psi = \sqrt{\rho(\xi,\tau)}\,\exp\left[\frac{i}{2p}\int^{\xi}\sigma(\xi',\tau)\,d\xi'\right] \quad , \tag{35}$$

we obtain the following soliton solution,

$$\rho(\xi,\tau) = \rho_0\left[1 - A\,\mathrm{sech}^2\left\{\frac{C}{2p}A^{1/2}(\xi-\lambda_{\pm}\tau)\right\}\right] \quad , \tag{36.a}$$

$$\sigma(\xi,\tau) = \lambda_{\pm} \mp C(1-A)^{1/2}\left[1-A\,\mathrm{sech}^2\left\{\frac{C}{2p}A^{1/2}(\xi-\lambda_{\pm}\tau)\right\}\right]^{-1}, \tag{36.b}$$

with

$$\lambda_{\pm} = \sigma_0 \pm C(1-A)^{1/2} \quad , \tag{36.c}$$

$$C = (-2pq\,\rho_0)^{1/2} \quad . \tag{36.d}$$

This type of envelope soliton is called as a dark soliton reffering to the nonlinear optics.

On the other hand, when the coefficients p and q take the same sign, i.e., $pq > 0$, the wave is modulationally unstable in the sense that the finite amplitude plane wave breaks up to a train of solitons. For this case, eq.(33) has a envelope-soliton solution, which satisfies the boundary condition that $\psi(\xi,\tau)$ and its derivatives vanish at $\xi=\pm\infty$,

$$\psi(\xi,\tau) = A \operatorname{sech} [(\frac{q}{2p})^{1/2} A(\xi - V\tau)] \cdot$$

$$\exp[i(\frac{V}{2p} \xi - \frac{V^2}{4p}\tau + \frac{1}{2} q A^2\tau)] , \qquad (37)$$

where an arbitrary constant V defines the velocity of the envelope soliton.

Now, it has been emphasized by Ikezi and Kiwamoto [21] that nonlinear Landau damping processes play important part in the phenomena of nonlinear propagation of the ion plasma wave. Therefore, we have examined carefully contribution of the resonance particles at the group velocity by formulating the problem on basis of the Vlasov description of collisionless plasmas, [22]. It has been found that the wave-wave-particle resonant interaction modifies drastically contribution of the slow beat wave, i.e., the second order $\ell=0$ component in the expansion scheme of eq.(27), and gives rise to the modified nonlinear Schrödinger equation with a nonlocal-nonlinear integral term,

$$i \frac{\partial}{\partial\tau} \psi + p \frac{\partial^2}{\partial\xi^2} \psi + q|\psi|^2\psi + r \frac{P}{\pi}\int \frac{|\psi(\xi',\tau)|^2}{\xi - \xi'} d\xi'\psi = 0 . \qquad (38)$$

The linear stability of (38) can be examined by linearization of (35) given as

$$\rho = \rho_0 + \{\delta\rho\exp(i(K\xi - \Omega\tau)] + \text{complex conjugate}\} , \qquad (39.a)$$

$$\sigma = \{\delta\sigma\exp[i(K\xi - \Omega\tau)] + \text{complex conjugate}\} , \qquad (39.b)$$

The dispersion relation reduced from (38) determines

$$\Omega = \Omega_r(K) + i\,\Gamma(K) \quad , \tag{40.a}$$

where

$$\Omega_r = \pm\frac{1}{\sqrt{2}}[\{(p^2K^2-2pq\rho_0)^2+(2pr\rho_0)^2\}^{1/2}+(p^2K^2-2pq\rho_0)]^{1/2}|K| \quad , \tag{40.b}$$

$$\Gamma = \mp\frac{1}{\sqrt{2}}[\{(p^2K^2-2pq\rho_0)^2+(2pr\rho_0)^2\}^{1/2}-(p^2K^2-2pq\rho_0)]^{1/2}|K| \quad , \tag{40.c}$$

In the small amplitude limit $|p/2q|K^2 >> \rho_0$, this is reduced correctly to the nonlinear Landau damping process, in which the wave energy is transformed from the higher frequency side band to the lower frequency side band.

When $pq > 0$, eq.(40.c) takes maximum growth rate

$$\Gamma_m = \mp (q^2 + r^2)^{1/2}\rho_0 \tag{41.a}$$

with the maximum frequency shift

$$\Omega_m = \pm\frac{r}{q}(q^2 + r^2)^{1/2}\rho_0 \tag{41.b}$$

for the value of wave number

$$K_m = \{(q^2 + r^2)/pq\}^{1/2}\sqrt{\rho_0} \tag{41.c}$$

On the other hand, if $pq < 0$, in the large amplitude limit of $\rho_0 >> |p/2q|K^2$, eqs.(40.b) and (40.c) take the asymptotic values

$$\Omega_r = \pm\{(q^2+r^2)^{1/2}+|q|\}^{1/2}\sqrt{|p|K^2\rho_0} \tag{42.a}$$

$$\Gamma = \mp\{(q^2+r^2)^{1/2}-|q|\}^{1/2}\sqrt{|p|K^2\rho_0} \tag{42.b}$$

Above analysis illustrates that the contribution of wave-wave-particle resonance at the group velocity leads to modulational instability regardless the sign of pq.

Experimental investigations of the modulational instability was carried out for the ion acoustic wave [23]. Fig.5 shows a transition from the linear to nonlinear propagation of ion acoustic wave packets in the typical low pressurs Argon plasma, $(1\sim2)\times10^{-4}$ Torr, with $n_e=(1\sim2)\times10^9$ cm^{-3}, $T_e=(1.5\sim2.0)$ eV and $T_e/T_i=10\sim12$. The initial profile of an envelope has 30 μsec duration, in which the amplitude increases linearly in the first 10 μsec, then is kept constant for the sussequent 10 μsec and falls down linearly to zero in the last 10 μsec. The carrier frequency is about 0.5 ω_{pi}. Wave packets are excited in a plasma by a conventional grid exciter and are recieved by a plane probe at 9 cm from the grid. For a small amplitude of exciting voltage, V_{ex} = 1.5 V, the recieved wave form resembles the input one, except that the frequency in the envelope tail is slightly higher than that in the front, indicating manifestation of the dispersion effect. The profile of the envelope changes drastically when the amplitude of wave packets increases. That is, at V_{ex} = 2.0 V the wave front steepens, and then the modulational instability sets in as can be seen from the wave patterns at V_{ex} = 3.0 V and V_{ex} = 4.0 V. In the bottom trace, the initial wave packet is divided into three parts and the amplitude of the first region is largely enhanced.

Let us make an important remark on the largest wave packet in the bottom trace. We recognize that the frequency in the region A where the amplitude builds up with time is higher than the frequency in the region B where the amplitude diminishes with time. This shift in frequency indicates that the large amplitude wave propagates more rapidly than the small amplitude wave, providing an evidence for the nonlinear dispersion effect. The group velocity dispersion of the ion wave, on the other hand, makes the velocity of the high frequency part (region A) slower than that of the low frequency part (region B). Thus, as a result of competition between nonlinear dispersion and group velocity dispersion, the modulational instability takes place in the ion wave propagation. This is the reason why we have observed the modulational instability of the ion wave. In the bottom trace, the frequency shift is found to be $|\Delta\omega/\omega_0|=0.15$, which is about two times larger than the shift calculated from eq.(40). This discrepancy is plausible, because, in the experiment, the envelope amplitude damps spatially and there exists ambiguity in determining the amplitude experimentally.

Recently, Ikezi et al [24] have examined the modulation of ion waves, and concluded that the modulational instability does not take place, but effects of trapped particles are essential. In order to clearify the discrepancy between their observation and our results, it is necessary to re-examine experimental condition such as effects of ion collision, presence of noise in a plasma.

Having shown an experimental evidence of the nonlinear wave modulation of the ion plasma wave, we close discussion of the present section by illustrating results of numerical analysis of eq.(38) carried out by Yajima et al, [25]. Restricting our interest to the case of $pq > 0$, we examine how the envelope soliton given as (37) deforms under the action of the nonlocal-nonlinear integral term of eq.(38). Fig.6 shows the numerical solutions of eq.(38) with initial value

$$\psi(\xi,\ \tau=0) = A\ \mathrm{sech}[(\frac{q}{2p})^{1/2} A\ \xi], \tag{43}$$

where we take A=q=2p=1 for simplicity. The value of r is arbitrary chosen to be r=0.5. We can see that the soliton deforms in asymmetric way and comes to run towards the positive direction. Fig.7 shows effects of nonlinear Landau damping on bound state of envelope solitons. Eq. (33) has a solution

$$\psi(\xi,\tau)=4\ A\ \exp[iqA^2\tau/2]\ \frac{\cosh(3B\xi)+3\cosh(B\xi)\exp(4iqA^2\tau)}{\cosh(4B\xi)+4\cosh(2B\xi)+3\cos(4qA^2\tau)} \tag{44}$$

which satisfies the initial condition

$$\psi(\xi,\ \tau=0) = 2\ A\ \mathrm{sech}(B\xi) \tag{44.a}$$

$$B = (q/2p)^{1/2}\ A. \tag{44.b}$$

This solution does not decay into a train of solitons, but pulsates with a period $\pi/(2qA^2)$. Numerical solutions of eq.(38) for the initial condition (44.a) indicate that solitons bounded in its initial state are made to be free, and each solitons travels with changing their shape and velocity. Associated with the gain of velocity of soliton, the resonant particles at the group velocity will be ejected to the opposite direction as a bunch of particles.

V. Circular poralized nonlinear Alfvén waves. ——— Investigation of proporties of the Alfvén waves in a gaseous plasma attracts particular interests in connection with search for useful methods to heat a plasma, [26]. In the problems of space physics, large amplitude incompressible magnetic field perturbation observed in the solar wind has been attributed to propagation of the Alfvén wave, [27], and has inspired theoretical analysis of possible existence of an exact solitary Alfvén wave, [28].

In their systematic analysis of nonlinear hydromagnetic waves, Kakutani and Ono [29] have shown that, as far as the waves are propagating at an angle with a uniform external magnetic field, the nonlinear magneto-acoustic wave is described by the Korteweg-de Vries equation, while propagation of the nonlinear Alfvén wave is described by the modified Korteweg-de Vries equation. However, it has been noticed firstly by Kawahara [30] that when the hydromagnetic wave is propagating along the external magnetic field these equations cease to be valid, because the dispersion relations for the magneto-acoustic wave and the Alfvén wave are degenerate in the long wave length limit for the parallel propagation. He obtained a modified type of nonlinear Schrödinger equation. Since the equation derived by Kawahara represents a new type of evolution equation, we will describe briefly derivation of this equation, and then discuss its stationary exact solutions, which represent new types of envelop solitons.

Neglecting the effects of displacement current and charge separation, we can reduce the system of equation for cold plasma to the fundamental equations for one-dimensional propagation in dimensionless form,

$$\frac{\partial}{\partial t} n + \frac{\partial}{\partial x} (nu) = 0 \quad , \tag{45.a}$$

$$\frac{d}{dt} u + n^{-1} \frac{\partial}{\partial x} \{\tfrac{1}{2}(B_y{}^2 + B_z{}^2)\} = 0 \quad . \tag{45.b}$$

$$\frac{d}{dt} v - n^{-1} \frac{\partial}{\partial x} B_y = - R_e{}^{-1} \frac{d}{dt}(n^{-1} \frac{\partial}{\partial x} B_z) \quad , \tag{45.c}$$

$$\frac{d}{dt} w - n^{-1} \frac{\partial}{\partial x} B_z = R_e^{-1} \frac{d}{dt}(n^{-1} \frac{\partial}{\partial x} B_y) \quad , \tag{45.d}$$

$$\frac{d}{dt} B_y - \frac{\partial}{\partial x} v + B_y \frac{\partial}{\partial x} u = R_i^{-1} \frac{\partial}{\partial x}(\frac{d}{dt} w) \quad , \tag{45.e}$$

$$\frac{d}{dt} B_z - \frac{\partial}{\partial x} w + B_z \frac{\partial}{\partial x} u = - R_i^{-1} \frac{\partial}{\partial x}(\frac{d}{dt} v) \quad , \tag{45.f}$$

where $d/dt \equiv \partial/\partial t + u \cdot \partial/\partial x$, $\vec{v} = (u, v, w)$ denotes the velocity of electrons, n the density of electrons, $\vec{B} = (B_x=1, B_y, B_z)$ the magnetic induction vector, R_e and R_i represent ratios of the electron and the ion cyclotron frequencies to the characteristic frequency, respectively. The above system has a linear dispersion relation

$$\frac{\omega}{k} = 1 \pm \mu k \quad , \tag{46.a}$$

where

$$\mu = \frac{1}{2} (R_i^{-1} - R_e^{-1}) \quad . \tag{46.b}$$

The double sign ± designates the right (+) and left (-) polarized Alfvén waves, of which amplitudes are given as

$$\Phi_R = B_y - i\, B_y \quad , \tag{47.a}$$

$$\Phi_L = B_y + i\, B_z \quad , \tag{47.b}$$

respectively. As for stretching of the space-time variables is concerned, in accord with the linear dispersion relation (46.a), we introduce the stretched space-time variables

$$\xi = \varepsilon\ (x - t) \quad , \tag{48.a}$$

$$\tau = \varepsilon^2\ t \quad , \tag{48.b}$$

assuming $k \sim O(\varepsilon)$. We expand the variables, on the other hand, in accord with Kakutani and Ono as

$$n = 1 + \varepsilon\, n^{(1)} + \varepsilon^2 n^{(2)} + \cdots \tag{49.a}$$

$$u = \varepsilon\, u^{(1)} + \varepsilon^2 u^{(2)} + \cdots \tag{49.b}$$

$$v = \varepsilon^{1/2}\ (v^{(1)} + \varepsilon\, v^{(2)} + \cdots\ ) \tag{49.c}$$

$$w = \varepsilon^{1/2}\ (w^{(1)} + \varepsilon\, w^{(2)} + \cdots\ ) \tag{49.d}$$

$$B_y = \varepsilon^{1/2}\ (B_y{}^{(1)} + \varepsilon\, B_y{}^{(2)} + \cdots) \tag{49.e}$$

$$B_z = \varepsilon^{1/2}\ (B_z{}^{(1)} + \varepsilon\, B_z{}^{(2)} + \cdots) \tag{49.f}$$

Substituting eqs.(49.a)∿(49.f) with the transformation of eqs.(48.a)∿(48.b), we get the following relationships among the first order quantities,

$$v^{(1)} = -\ B_y{}^{(1)} \ , \qquad w^{(1)} = -\ B_z{}^{(1)} \tag{50.a}$$

$$n^{(1)} = u^{(1)} = \frac{1}{2}\ \{B_y{}^{(1)2} + B_z{}^{(1)2}\} \tag{50.b}$$

Eliminating the second order quantities from set of equations at the order of $\varepsilon^{5/2}$, we obtain simply the nonlinear evolution equation for the right polarized Alfvén wave

$$\frac{\partial}{\partial\tau}\ \phi_R + \frac{1}{4}\frac{\partial}{\partial\xi}\{|\phi_R|^2\ \phi_R\}\ - i\mu\frac{\partial^2}{\partial\xi^2}\ \phi_R = 0\ , \qquad (51.a)$$

and for the left poralized Alfvén wave,

$$\frac{\partial}{\partial\tau}\ \phi_L + \frac{1}{4}\frac{\partial}{\partial\xi}\{\ |\phi_L|^2\ \phi_L\}\ + i\mu\frac{\partial^2}{\partial\xi^2}\ \phi_L = 0\ . \qquad (51.b)$$

where ϕ_R and ϕ_L are the first order amplitude of eqs. (47.a) and (47.b), respectively. These nonlinear evolution equations have been rederived by Mio et al, [31]. They have carried out analysis of the modulation instability of the Alfvén wave on the basis of eqs.(51.a) and (51.b).

It would be worthwhile, however, to present here exact steady state solution of eqs.(51.a) and (51.b) obtained by Wadati et al, [32]. Substitution of a form

$$\phi_R(\xi,\tau) = \sqrt{8}\ \psi(\xi,\tau)\ \exp\{\ i\chi(\xi,\tau)\} \qquad (52)$$

with real functions ψ and χ into eq.(51.a) yields a pair of coupled equations for ψ and χ:

$$\psi_\tau + 6\ \psi^2\psi_\xi + 2\ \mu\psi_\xi\chi_\xi + \mu\ \chi_{\xi\xi}\psi\ = 0\ , \qquad (53.a)$$

$$\chi_\tau\psi + 2\ \chi_\zeta\psi^3 - \mu\ \psi_{\xi\xi}\ + \mu\ \chi^2{}_\xi\psi = 0\ . \qquad (53.b)$$

We seek a solution in the following form,

$$\chi(\xi,\ \tau) = \mu^{-1}(K\xi\ - \ \Omega\tau) + \ \Theta(y)\ , \qquad (54.a)$$

$$\psi(\xi, \tau) = \psi(y) \quad , \qquad (54.b)$$

with

$$y = \mu^{-1} (\xi - \lambda\tau) \quad , \qquad (54.c)$$

where wave number K, frequency Ω and propagation velocity λ are constants to be determined from solutions of (53.a) and (53.b). Under the conditions of eqs.(54.a)∿(54.c), we can obtain

$$(\frac{d}{dy}\Phi)^2 = -\Phi^4 + 2\lambda\Phi^3 - 4\{\Omega + A + \frac{1}{4}(\lambda-2K)^2 - K^2\}\Phi^2$$

$$+ 4B\Phi - 4A^2 \qquad (55.a)$$

$$\frac{d}{dy}\Theta = \frac{A}{\Phi} + \frac{1}{2}(\lambda - 2K) - \frac{3}{2}\Phi \qquad (55.b)$$

where

$$\Phi(y) = \psi^2(y) \quad , \qquad (55.c)$$

and, A and B are integration constants. Restricting our interest to solitary wave solutions which satisfy the boundary conditions

$$\Phi(y) \to \Phi_0 = \psi_0{}^2 \quad , \qquad (56.a)$$

$$\frac{d}{dy}\Theta \to 0 \quad , \qquad \text{as } |y| \to \infty \quad , \qquad (56.b)$$

we can specify the integral constants and the shift of carrier frequency as

$$A = \frac{3}{2}\Phi_0{}^2 - \frac{1}{2}(\lambda - 2K)\Phi_0 \quad , \qquad (57.a)$$

$$B = 4\Phi_0{}^3 + \frac{1}{2}(12K - 5\lambda)\Phi_0{}^2 + 2(K - \frac{1}{2}\lambda)^2\Phi_0 \quad , \qquad (57.b)$$

$$\Omega = K^2 + 2K\Phi_0 \quad . \qquad (57.c)$$

Straightforward but lengthy calculation gives

$$\Phi(y) = \psi^2(y) = \Phi_0 + \frac{8\kappa\gamma^2}{\beta}[\kappa m + \cosh(2\gamma(y-y_0))]^{-1/2}, \tag{58.a}$$

$$\Theta(y) = \Theta(y_0) - 3\kappa \tan^{-1}\{\sqrt{\frac{1-\kappa m}{1+\kappa m}}\tanh(\gamma(y-y_0))\}$$
$$- \gamma\,\delta \tan^{-1}\{\sqrt{\frac{1-\kappa\ell}{1+\kappa\ell}}\tanh(\gamma(y-y_0))\} , \tag{58.b}$$

where

$$\kappa = \pm 1 , \tag{59.a}$$

$$\ell = \frac{\alpha}{\beta} + 8\frac{\gamma^2}{\beta\Phi_0} \quad \text{and} \quad m = \frac{\alpha}{\beta} , \tag{59.b}$$

$$\alpha = 2(2\Phi_0 - \lambda) . \tag{59.c}$$

$$\beta = 4\{(\Phi_0 + K)(\lambda - K - 2\Phi_0)\}^{1/2} , \tag{59.d}$$

$$\gamma = \frac{1}{2}\{(\lambda - \lambda_1)(\lambda_2 - \lambda)\}^{1/2} \tag{59.e}$$

$$\delta = \text{sign of } (3\Phi_0 - \lambda + 2K) \tag{59.f}$$

The propagation velocity λ is allowed to take a value in the region of

$$\lambda_1 < \lambda < \lambda_2 \tag{60.a}$$

where

$$\lambda_1 = 2(K + 2\Phi_0) - 2\sqrt{\Phi_0(\Phi_0 + K)} , \tag{60.b}$$

$$\lambda_2 = 2(K + 2\Phi_0) + 2\sqrt{\Phi_0(\Phi_0 + K)} . \tag{60.c}$$

A similar analysis is possible for the left polarized waves. In the case, solitary waves are obtained just by replacing

$$\Omega \rightarrow -\Omega \quad \text{and } K \rightarrow -K \tag{61}$$

in the above expressions, eqs.(59.a)∿(59.f) and eqs.(60.b) and (60.c). Then, we have an extra restriction on the wave number,

$$\Phi_0 > K \ . \tag{62}$$

As can be seen from eq.(58.a), $\kappa=+1$ designates bright modulation while $\kappa=-1$ dark modulation of the amplitude, respectively. Eqs.(58.a) and (58.b) represent that modulation of the amplitude is closely coupled to modulation of the phase. Furthermore, unlike the envelope soliton given by eq.(37), the propagation velocity λ of the solitary wave (58.a) and (58.b) is not an arbitrary constant, but it is restricted to a region defined by eqs.(60.a)∿(60.c). These properties are quite unique and could be detected by experiment. In Fig.8 referring to the right polarized mode, we illustrate the bright Alfvén solitary wave and the dark Alfvén solitary wave for arbitrary choosen parameters of μ , K and Φ_R.

Besides the solitary waves discussed above, eqs.(51.a) and (51.b) admit also algebraic solitary waves. For the right polarized Alfvén wave, we get

$$\Phi_R^{(a)}(y) = \Phi_0 + \frac{4\gamma}{4+\gamma^2(y-y_0)^2} \tag{62.a}$$

$$\Theta_R^{(a)}(y) = \Theta(y_0) + \delta\tan^{-1}(\varepsilon(y-y_0)) - 3\tan^{-1}(\tfrac{\gamma}{2}(y-y_0)) \tag{62.b}$$

where

$$\gamma = 4(\Phi_0 + K) + 4\,\delta\,\sqrt{\Phi_0(\Phi_0 + K)}\quad, \tag{63.a}$$

$$\varepsilon = \frac{\gamma}{2}\,[2\,\sqrt{\Phi_0 + K} + \delta\sqrt{\Phi_0}\,]^{-1}\,\sqrt{\Phi_0}\quad, \tag{63.b}$$

$$\delta = \begin{cases} +1 & \text{for} \quad \lambda_+ \\ -1 & \text{for} \quad \lambda_- \end{cases}\quad, \tag{63.c}$$

and the velocities $\lambda_\pm$ are defined as

$$\lambda_\pm = 2(2\Phi_0 + K) \pm 2\sqrt{\Phi_0(\Phi_0 + K)}\quad. \tag{64}$$

Fig.9 illustrates the algebraic solitary Alfvén wave with the right polarization at the velocity λ_+ and λ_-, respectively. For the left polarized Alfvén wave, we get

$$\Phi_L^{(a)}(y) = \Phi_0 + \frac{4\,\gamma}{4 + \gamma^2(y - y_0)^2} \tag{65.a}$$

$$\Theta_L^{(a)}(y) = \Theta(y_0) - \tan^{-1}(\varepsilon(y - y_0)) + 3\tan^{-1}(\frac{\gamma}{2}(y - y_0)) \tag{65.b}$$

where

$$\gamma = 4(\Phi_0 - K) + 4\sqrt{\Phi_0(\Phi_0 - K)} \tag{66.a}$$

$$\varepsilon = \frac{\gamma}{2}\,[2\sqrt{\Phi_0 - K} + \sqrt{\Phi_0}\,]^{-1}\,\sqrt{\Phi_0} \tag{66.b}$$

with the velocity

$$\lambda_L = 2(2\Phi_0 - K) + 2\sqrt{\Phi_0(\Phi_0 - K)}\quad. \tag{67}$$

An algebraic envelope of the left polarized solitary Alfvén wave is shown in Fig.10. The figure indicates that contribution of a term with sin $\chi(\xi,\tau)$ is relatively large for this mode.

We now close this section by emphasizing the above described peculiar properties of the solitary Alfvén waves are not known for any other types of nonlinear evolution equations.

VI. Concluding discussions ——— In the preceeding sections, we have discussed very fundamental aspects of solitons and envelope solitons in collisionless plasmas, restricting ourselves to their simplest forms. Our main purpose is firstly to emphasize that the theoretical studies of structure of the dressed solitons provide refined physical pictures on the nonlinear wave phenomena. Effects of the nonlinear wave interaction are classified to

1) the self-interaction effects, of which the lowest order terms are essential to realize the solitons or the envelope solitons, while the higher order terms should be renormalized so as to remove the secular behaviour, and

2) the nonlinear "mode"-"mode" interaction effects, which are responsible to characterize dynamical processes of the nonlinear wave "mode", such as the solitons or the envelope solitons.

Observed characteristics of the nonlinear wave phenomena will be subject to systematic analyses on the basis of conserved properties of the renormalized soliton cores and dynamical distortion of the coulds surrounding the cores.

Secondly, reffering to the ion plasma wave, we have discussed nonlinear wave modulation of the strongly dispersive waves with account of the effects of resonant particles at the group velocity. These particles are expected to play important role in heating processes of plasmas by large amplitude waves. Higher order effects on the envelope solitons have been discussed by us [33], and recently Kodama [34] has presented a renormalization procedure for a model system describing a strongly dispersive wave.

Thirdly, we have presented some detailed discussions on a new type of nonlinear evolution, which has been derived for the circular polarized Alfvén waves. Rigorous steady state solutions present quite exotic envelope solitons. Since this equation has not been examined so far on the frame-work of the inverse scattering method, we call attentions of theoretical researchers working in this field.

We conclude present paper that now the self-interaction effects of coherent nonlinear waves have been well understood owing to advancement of theoretical studies. These coherent nonlinear waves will be also playing crucial role in the anomalous transport processes encountered in various high temperature plasma devices, where the processes have been phenomenologically treated on a basis of the concept of quasi-linear theory. Although we admit practical convenience of these approaches for supplying a conceptual guidance, theoretical endeavor for deeper understanding of the fundamental properties of nonlinear wave phenomena is indispensable to establish the solid grouds for researches of such complicated nonlinear wave-particles system as collisionless plasmas.

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Caption of Figures

Fig.1. Propagation of nonlinear ion-acoustic wave. The top trace is an applied pulse. The lower traces represent subsequent decomposition of the induced perturbation into a precursor (indicated by arrows) and solitons at the distances indicated on the right.

Fig.2. The soliton velocity as a function of amplitude of the density perturbation $\delta n/n_0$. The bars are experimental results taken from the reference [5]. The broken line with dots is for the Korteweg-de Vries soliton. The dotted lines are curves Calculated for the reference [13] with arbitrary parameter of $\beta=T_e/T_t$, where T_t is temperature of trapped electrons. The heavy curves are results of the reference [14].

Fig.3. Collision process of two dressed solitons with $\lambda_1=0.3$ and $\lambda_2=0.24$, represented by the heavy lines. The thin lines indicate the Korteweg-de Vries soliton cores, while the dotted lines represent the second order clouds.

Fig.4. Structure of renormalized dressed soliton. The heavy line represents shape of eq.(24.a) for $\lambda = 0.3$, while the thin line is eq.(24.b) and the dotted line eq.(24.c), respectively.

Fig.5. Nonlinear modulation of ion-acoustic wave packet observed in the reference [23].

Fig.6. Temporal evolution of the envelope soliton under action of the wave-wave-particle resonant interaction.

Fig.7. Effect of the wave-wave-particle interaction upon the bound state of three envelope solitons. The left figure shows evolution of envelope of eq.(44), while the other two figures represent distortion of the envelope due to the nonlocal-nonlinear term of eq.(38).

Fig.8. Envelope solitons of the right polarized Alfvén wave for arbitrary values of parameters $\mu=0.5$, with $\Phi_0=0.5$ and $K=0.01$. The upper trace is for the bright ($\kappa=+1$) and the lower trace for the dark ($\kappa=-1$) envelope solitons, respectively. The dotted line represents the real part of $\psi(\xi,\tau)$ $\exp(i\chi(\xi,\tau))$.

Fig.9. Algebraic envelope solitons of the right Alfvén wave.

Fig.10. Algebraic envelope soliton of the left Alfvén wave.

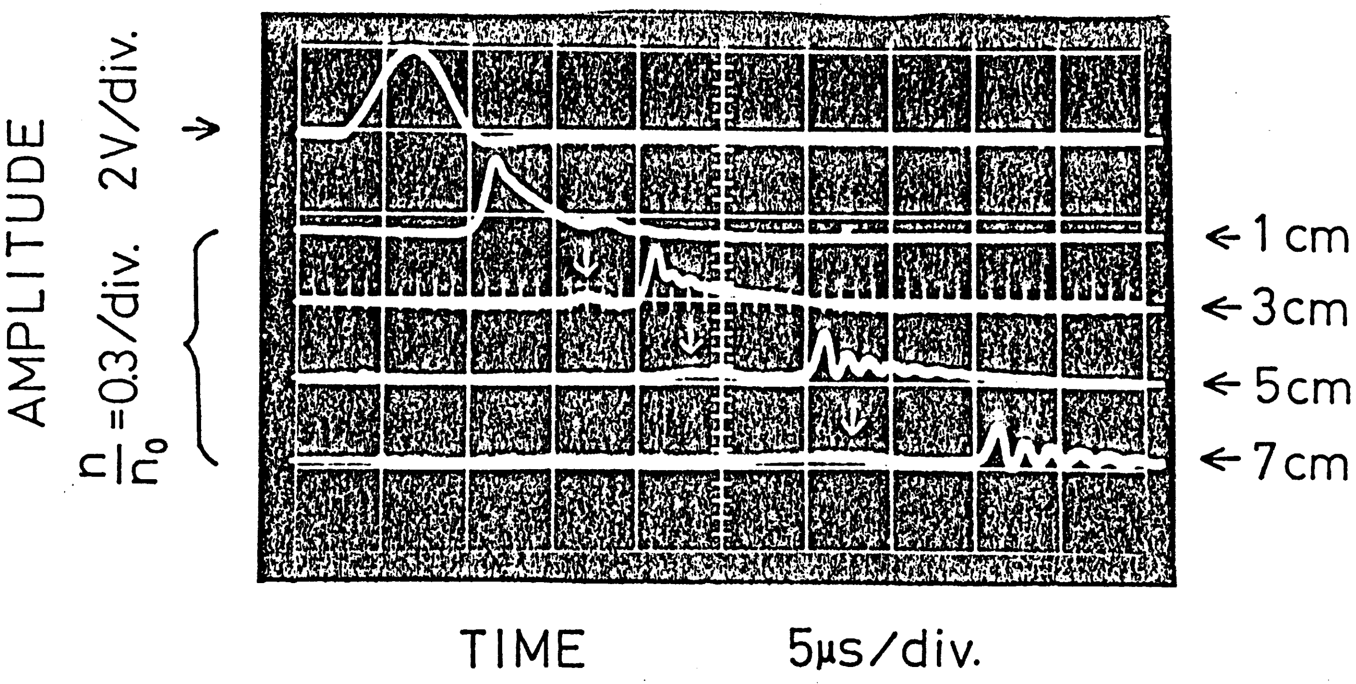


FIG. 1

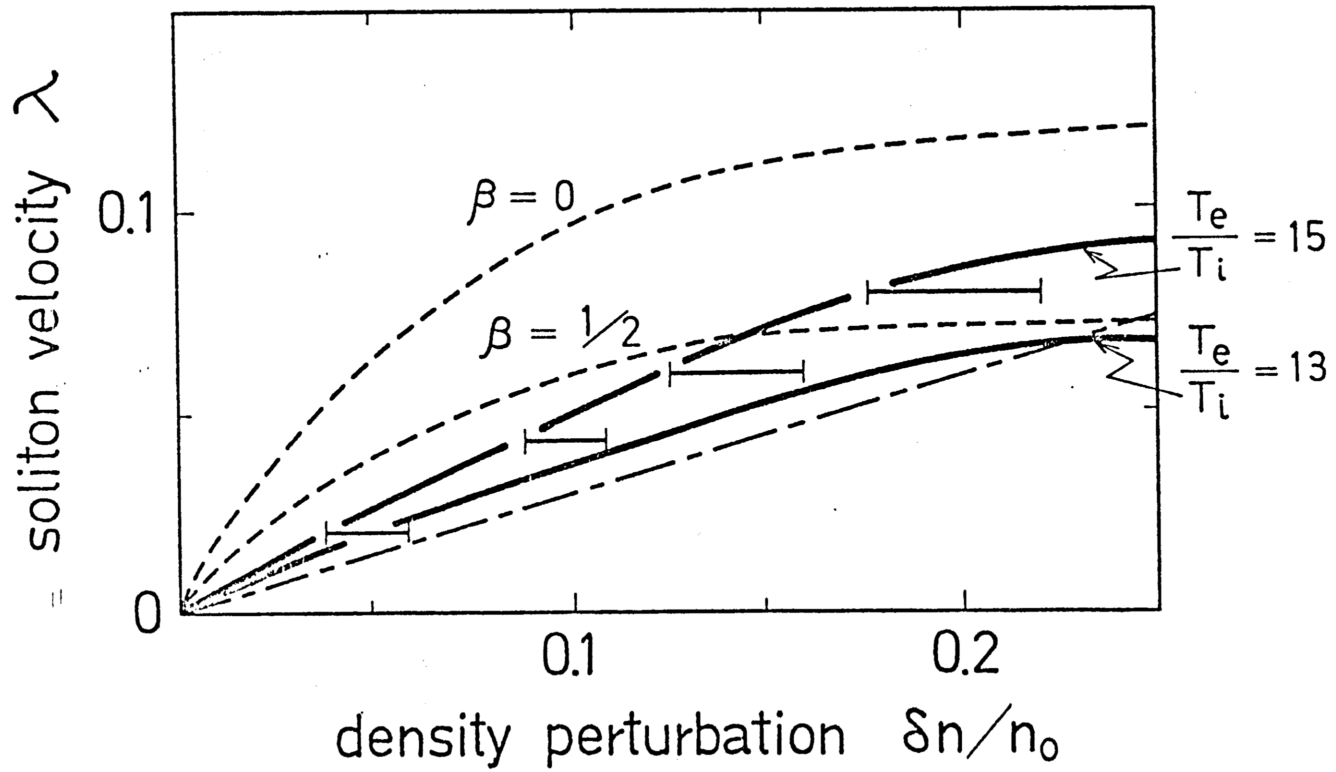

soliton velocity λ
0.1
0
β = 0
β = 1/2
Te/Ti = 15
Te/Ti = 13
0.1
0.2
density perturbation δn/n₀

Fig. 2

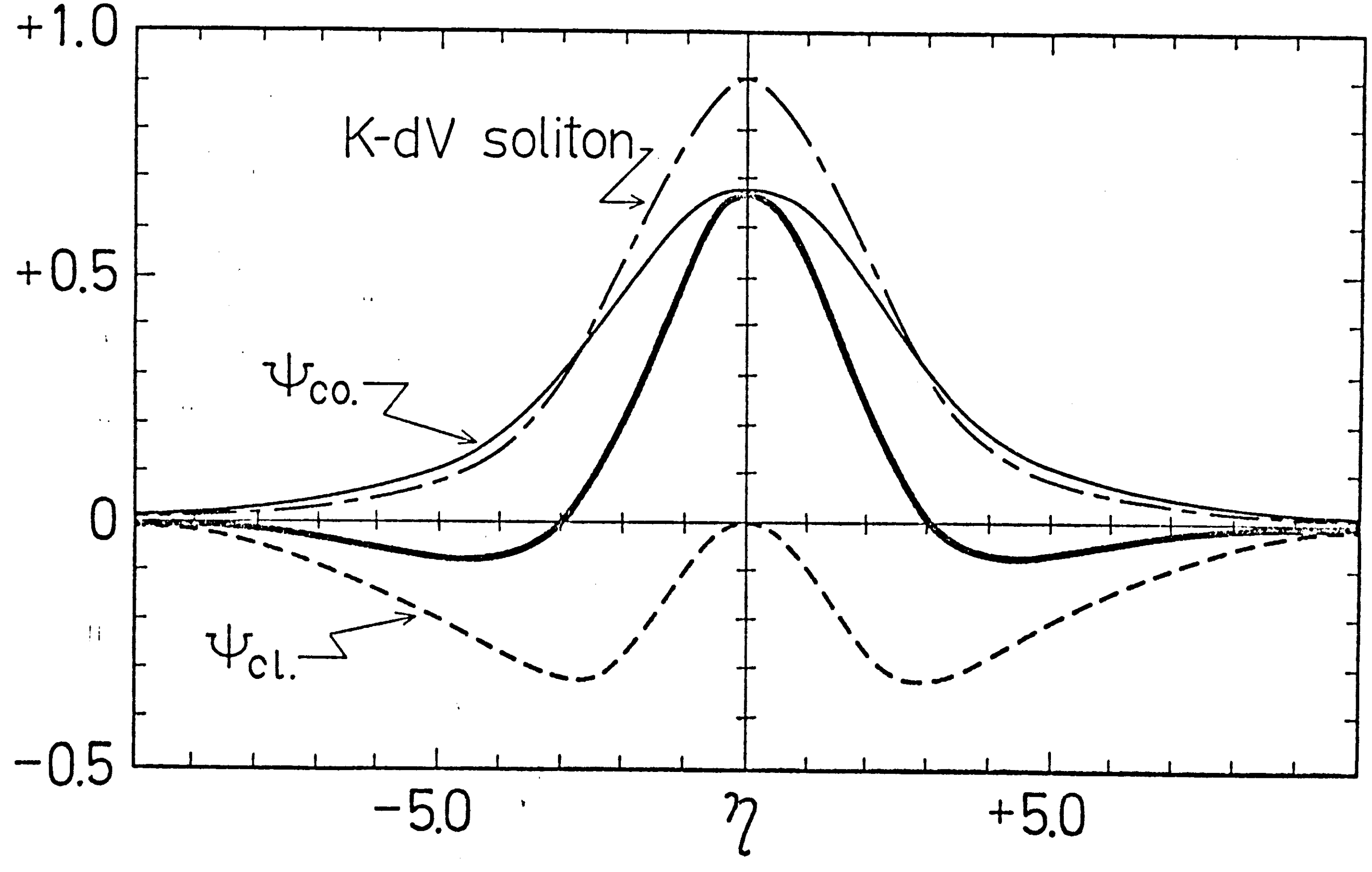
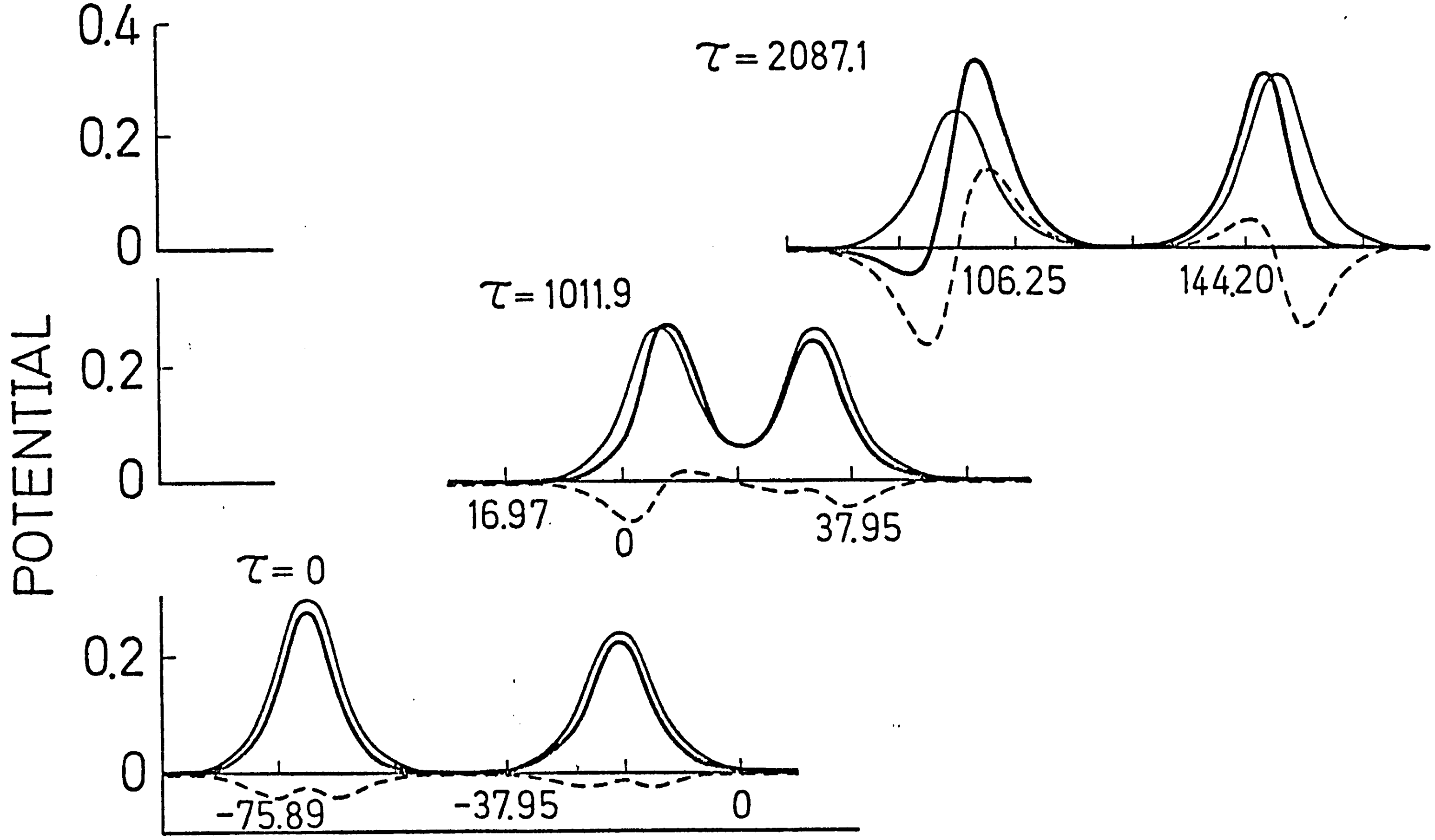
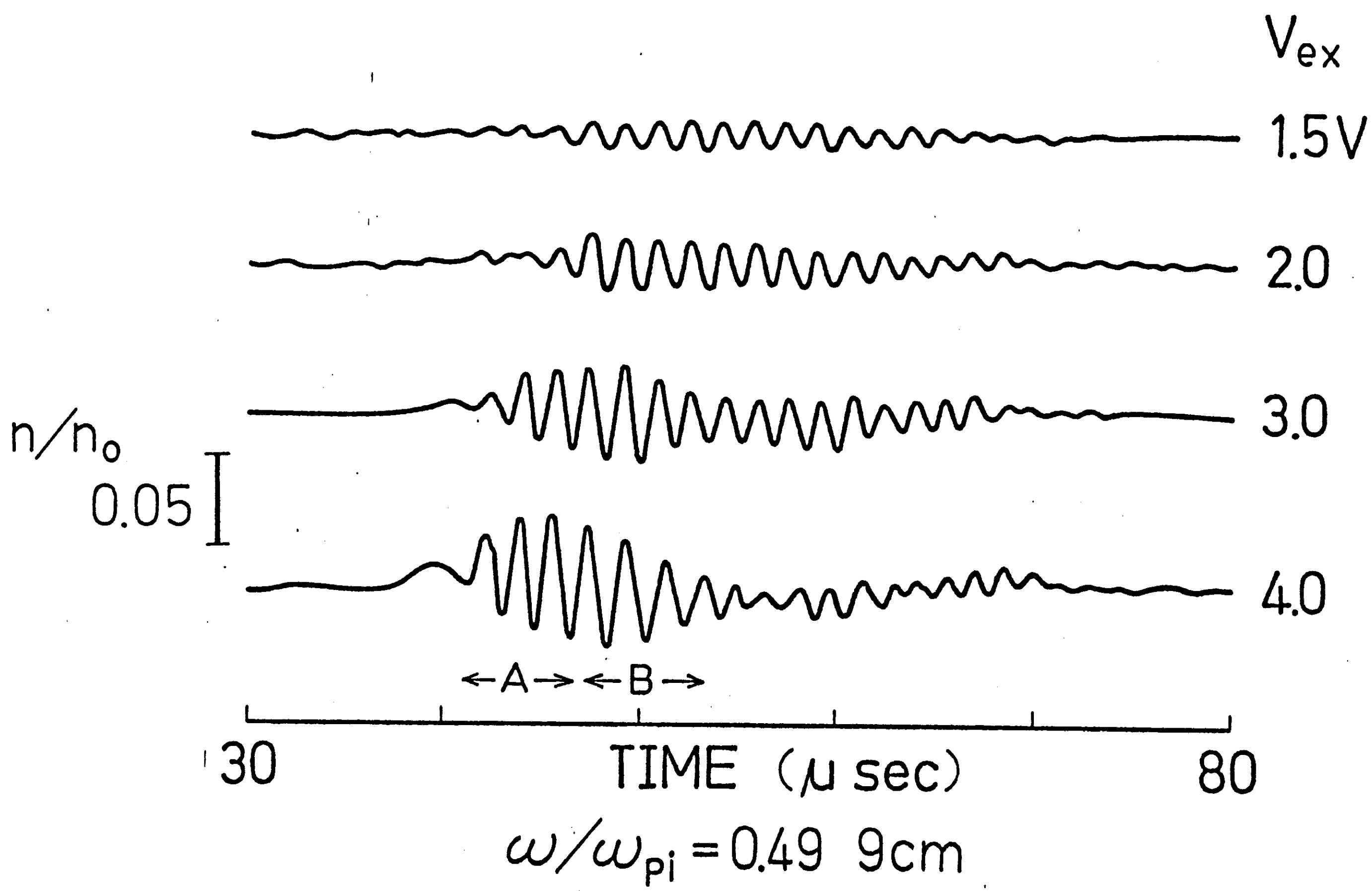


FIG. 3



POTENTIAL
0.4
0.2
0
0.2
0
0.2
0
τ = 2087.1
106.25
144.20
τ = 1011.9
16.97
0
37.95
τ = 0
-75.89
-37.95
0

Fig. 4



V_{ex}
1.5V
2.0
3.0
4.0
n/n_o
0.05
←A→←B→
30
TIME (μ sec)
80
$\omega/\omega_{pi} = 0.49$ 9cm

FIG. 5

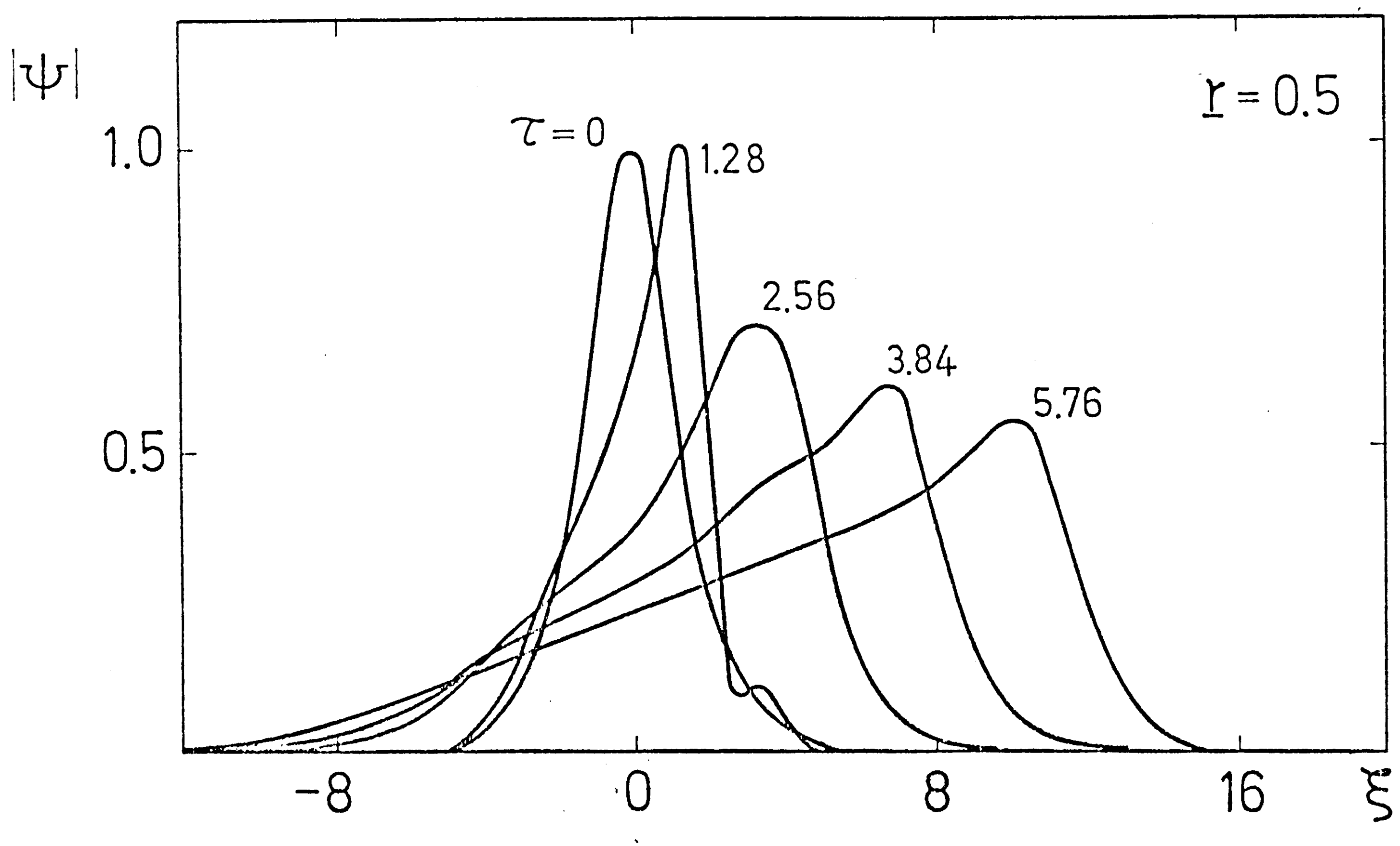


FIG. 6

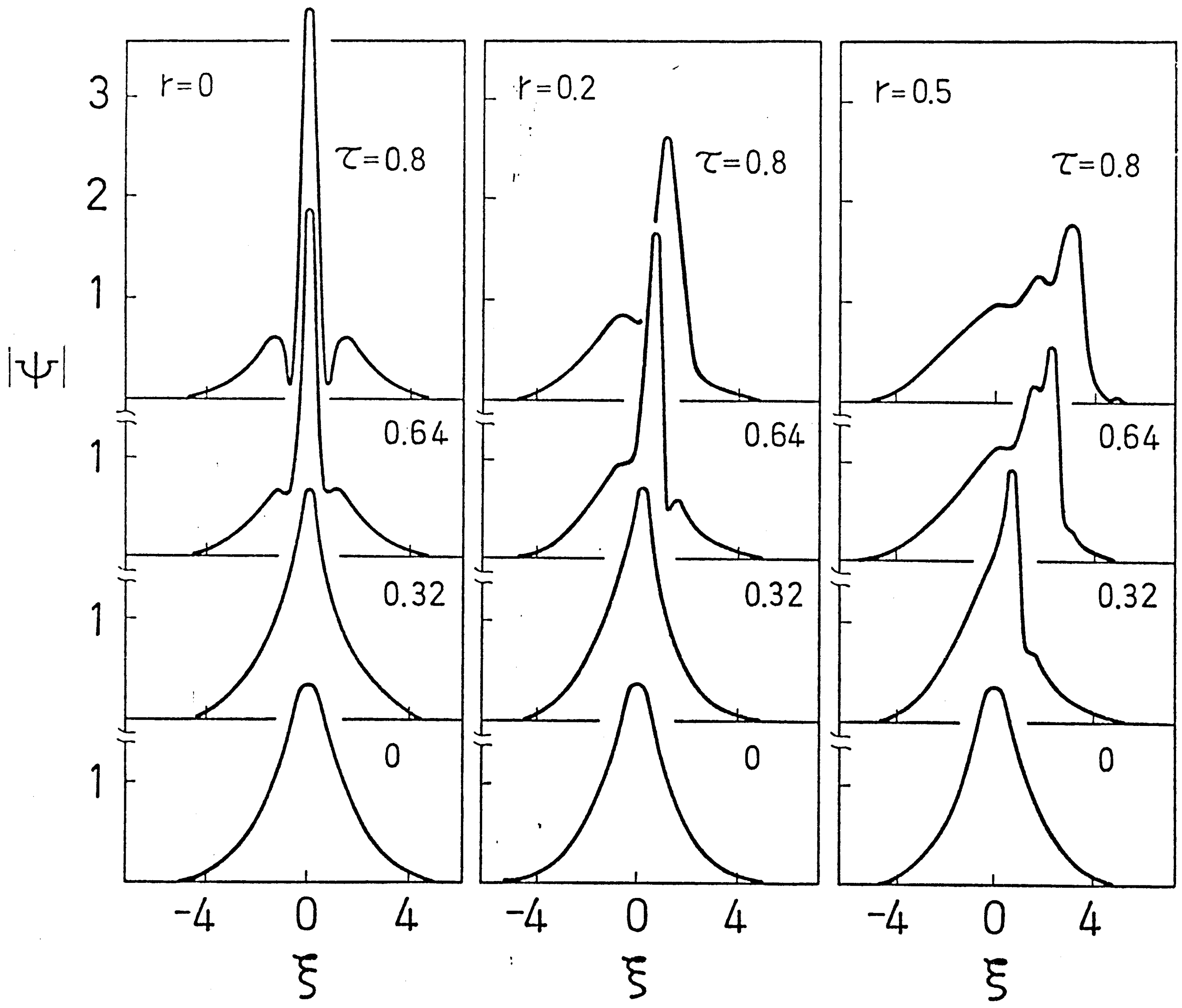


FIG. 7

$\mu = 0.5$

$\Phi_0 = 0.5, \quad K = 0.01, \quad \delta = -1,$

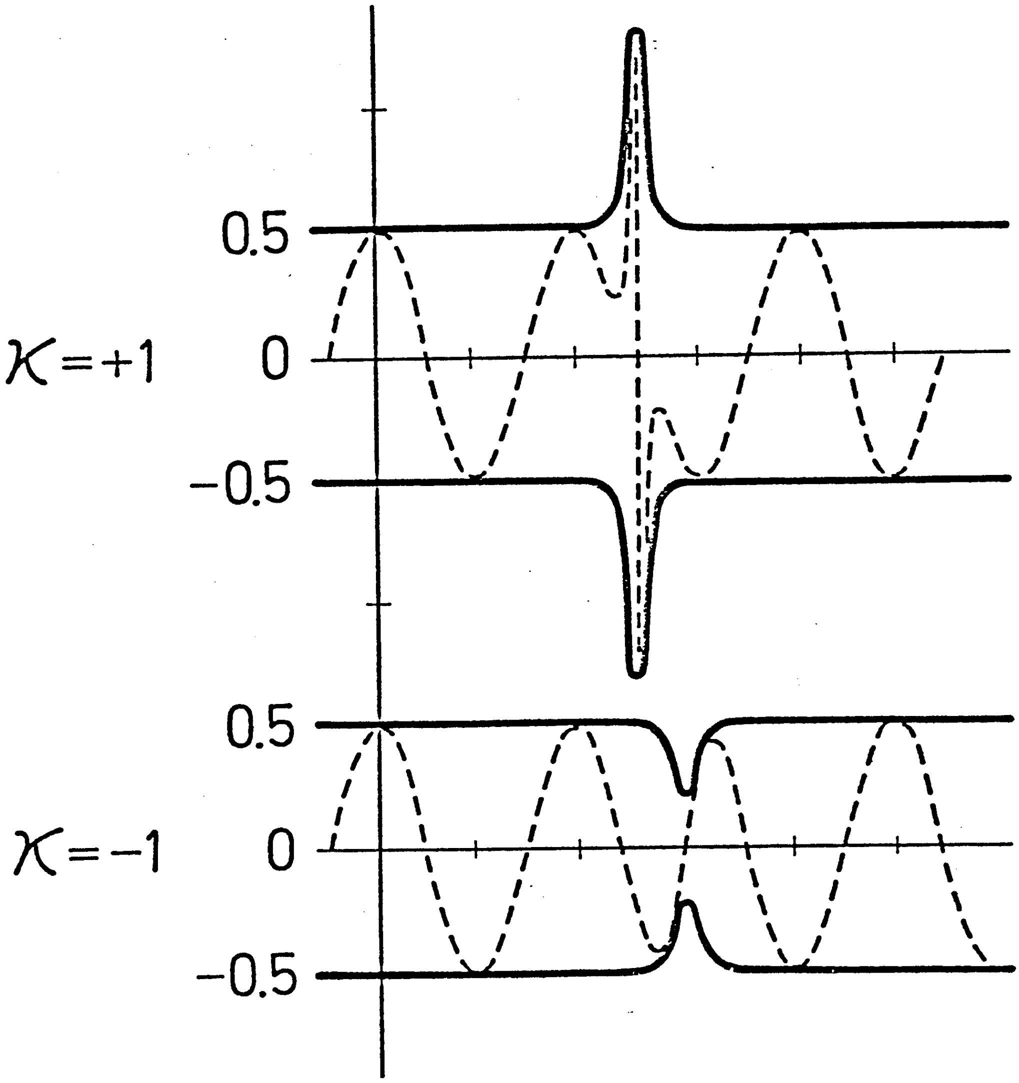


FIG. 8

$$\mu = 0.5$$

$$\Phi_0 = 0.5, \quad K = 0.01, \quad \delta = -1,$$

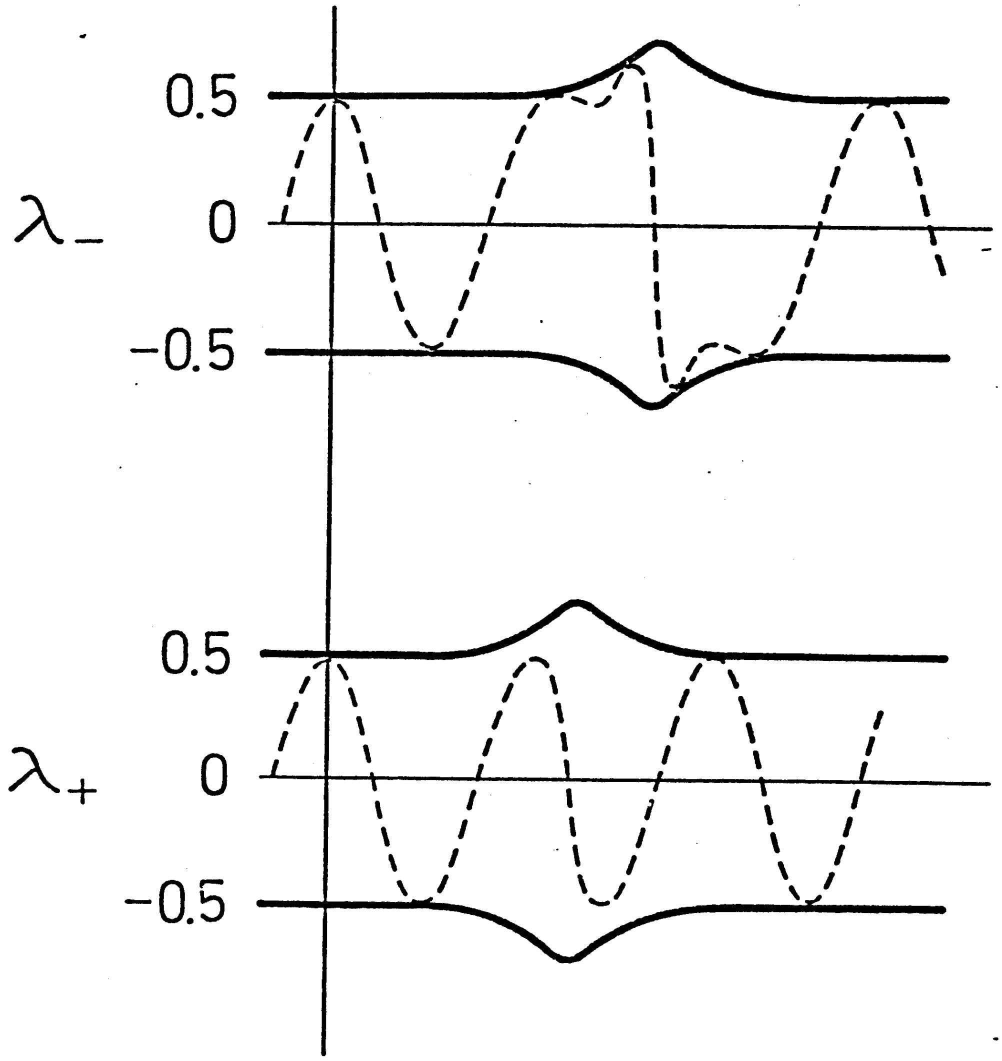


FIG. 9

$\mu = 0.5$

$\Phi_0 = 0.5, \quad K = 0.01,$

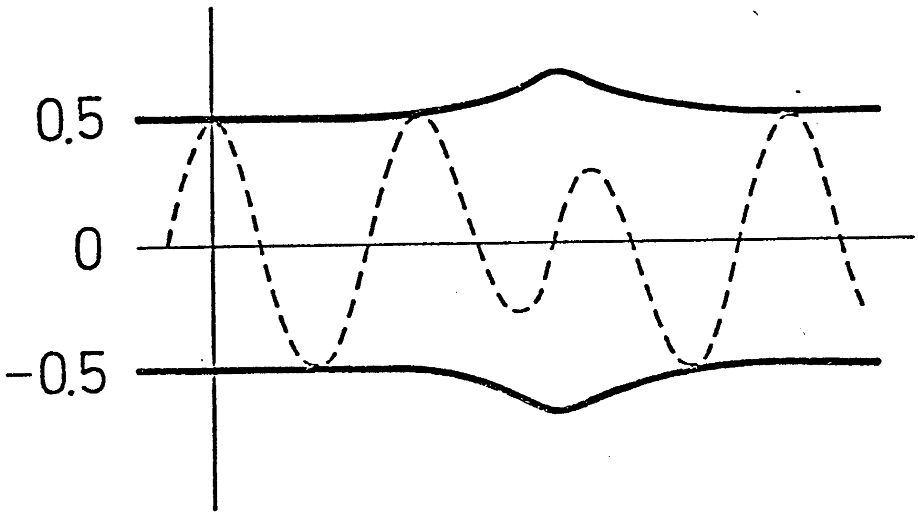


FIG. 10