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Current Driven Drift Instability in Both Self-Induced and External Magnetic Shear Field

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Abstract

The current-driven universal drift instability in a self-consistent magnetic shear field is analyzed. We develop a novel and simple method to determine the eigen value of the mode, recovering previous results. It is found that even in a fairly strong shear residual modes remain unstable in the presence of the parallel current. We obtain the confinement scaling law $\tau \propto na^2Rq/\sqrt{T}$ for high field tokamaks.

Recently the current-driven drift instability has been remarked in the view point of the anomalous plasma transport in tokamaks. The tokamak plasma is carrying its own current which generates the rotational transform and the magnetic shear. Therefore the critical shear for the stability of the drift wave should be consistently obtained, considering the competition between the stabilizing and destabilyzing effects.

In this letter we develop a novel and simple method to analyze drift instabilities in the sheared magnetic field and obtain the critical shear for the current-driven universal mode. The transport scaling law due to this mode is given $\tau \propto na^2Rq/\sqrt{T}$ (n: the plasma density, a: the radius of the plasma column, R: the major radius, q: the safety factor and T: the temperature).

In the magnetic sheared system, the drift wave resonates with electrons close to the rational surface and propagates across the magnetic surfaces until the wave damps off by the ion Landau damping. Until Rosenbluth and Catto[1] improved the calculation of Pearlstein and Berk[2] on the universal mode, WKB method has been employed even though the condition of the WKB method is violated. We study the electrostatic current-driven universal mode in both self-induced and the external magnetic shear, utilizing the complete orthonormal set to obtain the eigen value of this mode. Recovering the previous elaborate works in the two limiting cases, that is J (the longitudinal current) = 0[1] or $\omega = \omega_*$ (the drift frequency)[3], our results clarify the transition from the universal mode to the current driven mode.

The geometry we use is a slab and the x-axis is taken in the direction of the density gradient, $\nabla n = -\kappa n\hat{x}$, the magnetic field is given as $\vec{B} = (0, x/L_s, 1)B$ (x = 0 corresponds to the rational surface and L_s is the shear length). The longitudinal current is carried by electrons, so that we use the relation J = neu. As equilibrium distribution functions the shifted Maxellian distributions[1] are used. In the following, subscripts i and e denote ion and electron respectively, m and v are the mass and the thermal speed, $\tau = T_e/T_i$. We consider a potential perturbation of the form $\phi(x)\exp[i(ky-\omega t)]$. The equation of the dimensionless type is well known[1]

$$\frac{\partial^2\phi}{\partial\zeta^2} + [\ \lambda + \mu^2\zeta^2 + \eta(\zeta)\ ]\phi = 0 \tag{1}$$

$$\zeta = x/\tilde{\rho}_i, \quad \tilde{\rho}_i^2 = -\rho_i^2(\ln\Lambda)', \quad \Lambda = I_0(b)e^{-b}, \quad b = (k\rho_i)^2,$$

$$\zeta_e = \left|\frac{L_s}{k}\right|\frac{\omega}{v_e\tilde{\rho}_i}, \quad \mu = \left(\frac{\omega(\tau+1)}{\omega\tau+\omega_*}\right)^{1/2}\left|\frac{k}{L_s}\right|\frac{v_i\tilde{\rho}_i}{\omega}, \quad \lambda = \Lambda - \frac{\omega(\tau+1)}{\omega\tau+\omega_*},$$

$$\eta(\zeta) = -i\left[\sqrt{\frac{\pi}{2}}\frac{(\omega-\omega_*)\zeta_e}{|\zeta|(\omega\tau+\omega_*)} - \sqrt{\frac{\pi}{2}}\frac{\zeta}{|\zeta|}\frac{\omega}{(\omega\tau+\omega_*)}\frac{u}{v_e}\right] e^{-\frac{(\zeta_e-\zeta u/v_e)^2}{2\zeta^2}}.$$

where ρ is the Larmour radius and I_0 is the 0th order modified Bessel function.

Using the complete rothonormal set $\{\ \phi_n\ \}$ with the boundary condition for the waves to be out-going, we express the potential perturbation as $\phi(\zeta) = \sum^{\infty} a_n\phi_n(\zeta)$,

$$\phi_n(\zeta) = \left(\frac{i\mu}{2\pi}\right)^{1/4}\frac{1}{\sqrt{n!}} H_n(\sqrt{i\mu}\zeta)\ e^{-i\mu\zeta^2/2} \tag{2}$$

where H_n is the Hermite function and ϕ_n satisfies

$$\left[\ \frac{\partial^2}{\partial\zeta^2} + \mu^2\zeta^2 + \lambda_n\ \right]\phi_n = 0, \quad \lambda_n = i\mu(2n+1). \tag{3}$$

Multiplying ϕ_m from the left side of Eq.(1) and integrating it over ζ, we rewite Eq.(1) into the series of equations of 1st degree.

$$\sum_n V_{mn} a_n = (\lambda_m - \lambda) a_m \tag{4}$$

where

$$V_{mn} = \int_{\infty}^{\infty} \phi_m(\zeta)\eta(\zeta)\phi_n(\zeta)d\zeta \tag{5}$$

$$\equiv < m|\eta|n > , \qquad < m|1|n > \equiv \delta_{mn} .$$

When we take only a_0 in Eq.(4), that is m=n=0, and demand Im λ = 0, we obtain the critical shear given in ref.(1). For the current-driven drift mode, as remarked in ref.(3), $\eta(\zeta)$ has essentially an asymmetrizing effect on ϕ with respect to ζ. In order to include this effect we take (a_0, a_1) set and solve Eq.(4) in 2 x 2 truncated matrix form. Since the higher m mode are strongly damped ($\lambda_m \propto 2m + 1$), the coupling between higher m mode is negligible. As the smallness parameter we take $\varepsilon \equiv \sqrt{\mu}\zeta_e$ and the truncation of the matrix Eq.(4) is consistent with this ordering. We retain terms in V_{mn} up to the 2nd order of u/v_e, which is also the expansion parameter.

After some calculations we obtain

$$\lambda = [(\lambda_0 - V_{00} + \lambda_1 - V_{11}) \pm \sqrt{(\lambda_0 - V_{00} - \lambda_1 + V_{11})^2 + 4V_{10}{}^2}]/2 \tag{6}$$

$$V_{00} = \frac{i\sqrt{i}\varepsilon}{\omega\tau+\omega_*} K_0(\sqrt{2i}\varepsilon)[(\omega_*-\omega)-(3\omega-\omega_*)(u/v_e)^2/2] + \cdots \tag{7-1}$$

$$\sim \frac{(1-i)\ \varepsilon}{\sqrt{2}(\omega\tau+\omega_*)} \ln(\varepsilon)[(\omega_*-\omega)-(3\omega-\omega_*)(u/v_e)^2/2],$$

$$V_{11} = \frac{-\varepsilon^2}{\sqrt{2}(\omega\tau+\omega_*)} K_1(\sqrt{2i}\varepsilon)\left[(\omega_*-\omega)-(3\omega-\omega_*)\frac{u^2}{2v_e^2}\right], \quad (7\text{-}2)$$

$$V_{10} = V_{01} = \frac{i\sqrt{i}\ \omega}{\sqrt{2}(\omega\tau+\omega_*)} \frac{u}{v_e} \varepsilon K_1(\sqrt{2i}\varepsilon) . \quad (7\text{-}3)$$

The condition Im $\lambda = 0$ in Eq.(6) gives the equation for the critical shear stabilization,

$$192\mu^3 - 16\mu\left\{\frac{\omega}{\omega\tau+\omega_*}\right\}^2\frac{u^2}{v_e^2} + \frac{\sqrt{2}\varepsilon\ln(\varepsilon)}{\omega\tau+\omega_*}\left[52\mu^2(\omega_*-\omega) - \left\{\frac{\omega(\omega_*-\omega)}{\omega\tau+\omega_*} - 26\mu^2(3\omega-\omega_*)\right\}\frac{u^2}{v_e^2}\right] = 0 \ (\geqq 0 \text{ for stability }), \quad (8)$$

where we expand Bessel functions K_0 and K_1 and retain the lowest order of ε. Linearizing λ with respect to γ/ω_*, (γ: the linear growth rate), we have $\gamma/\omega_* = (\omega\tau+\omega_*)^2\mathrm{Im}\lambda/(\omega+1)\omega_*^2$.

In the slab geometry, the shear length and the current density has the relation through Maxwell's equation $\nabla\times\vec{B} = 4\pi\vec{J}/c$, that is $u/v_e = -\sqrt{m_e/m_i}\rho_i/\beta_i L_s$, and the shear stabilization effect is reduced or annihilated according to the value of β_i [3]. In Fig.(1) we show the critical shear v.s. $\sqrt{m_e/m_i}\rho_i\kappa/\beta_i$. The hatched portion is unstable and the dashed vertical line shows the criterion in ref.(3) in which the finite Larmour radius effect and the $\varepsilon\ln\varepsilon$ term are neglected. The mass ratio m_i/m_e is taken 1836. In the current carrying cylindrical plasma, the shear parameter is determined not by the local current density but by the shape of the current profile. In this situation the relation $u/v_e = -\sqrt{m_e/m_i}\rho_i/\beta_i L_s$ does not hold. In Fig.(2) and Fig.(3), the critical shear and the growth rate (for fixed shear parameter) are shown v.s. b for various values of u/v_e. We see that even in the fairly strong shear modes with wide range of b become unstable in the presence of the current. The similar result to Fig.(2) has been obtained in ref.(4) although they have used the WKB method.

From these results we discuss about the transport scaling law due to the current-driven universal mode. The anomalous diffusion coefficient is expressed as[5] $D = \sum_k (\tilde{n}_k/n)^2 \gamma/\kappa^2$. We evaluate $(\tilde{n}_k/n)^2 = \kappa^2/k_\perp^2 = \kappa^2/(k^2+k_x^2)$.[5] The fact that the growing region along x of this mode is wider than the wave localization width indicates $k_x^2 = \mu\tilde{\rho}_i^{-2}/2$. For simplicity, we take $\gamma = \omega(\omega\tau+\omega_*)u/2v_e(1+\tau)^2\omega_*$ and $\tau = 1$, and we see

$$D = \sum_k \frac{k}{(k^2+k_x^2)} \frac{\kappa T_e}{eB} \frac{\Lambda}{2(2-\Lambda)^2} \frac{u}{v_e} \tag{9}$$

Replacing the summation by an integral, noting that $k = m/r$, where m is the poloidal mode number and r is the radius from the center of the plasma column, we get

$$D = \frac{r\kappa T_e}{4eB} \frac{u}{v_e} \int_0^\infty \frac{db}{(k_x^2\rho_i^2+b)} \frac{\Lambda}{(2-\Lambda)^2} \tag{10}$$

$$k_x^2\rho_i^2 = (2-\Lambda)\sqrt{I_0/\Lambda(I_0-I_0')}/2\kappa L_s \ .$$

The particle confinement time τ_p may be given as $\tau_p \simeq 0.5 \times 10^{-5}$ $na^2Rq/\sqrt{T}\ F$, where F is the slowly varying function of the shear parameter and is order of unity. a, R, n, T and B are measured in cm, m, 10^{13}/cc, keV and T respectively. Assuming $\tau_p \simeq \tau_E$, and from the relations $\eta J^2 = nT/\tau_E$ and $\eta \propto Z_{eff}T^{-3/2}$,

$$\tau_{p,E} \simeq 3 \times 10^5 \; na^2 Rq\left[\frac{qR}{a^2B^2}\right]^{1/6} \quad . \tag{11}$$

the confinement time $\tau_{p,E}$ is approximately proportional to the total particle number.

To derive this scaling law we assume that the intervals between the rational surfaces of the neighbouring modes are wider than each mode localization width, otherwise, the couplings with neighbouring modes should not be neglected. Our analysis indicates that the plasma may suffer from this mode in high field tokamaks unless the plasma density is increased, because u/v_e is proportional to B/n for the fixed q(r). On the other hand in high density tokamak where $\nu_e/\omega_{*max} > 1$ holds, (ν_e is the electron collision frequency, and ω_{*max} is the maximum drift frequency), the collisional drift wave dominates and we may have the other transport scaling law[6].

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References

1] Rosenbluth, M.N., and P.J.Catto, Nucl. Fusion, 15 573 (1975).

2] Pearlstein, L.D., and H.L.Berk, Phys. Rev. Letters, 23 220 (1969).

3] Rosenbluth, M.N., and C.S.Liu, Phys. Fluids, 15 1801 (1972).

4] Gladd, N.T., and W.H.Horton, Phys. Fluids, 16 879 (1973).

5] Yoshikawa, S., Phys. Fluids, 16 1749 (1973). See also Tange, T., K.Itoh, S.Inoue and K.Nishikawa, J. Phys. Soc. Japan, 42 2006 (1977).

6] Inoue, S., K.Itoh and S.Yoshikawa, to be published.

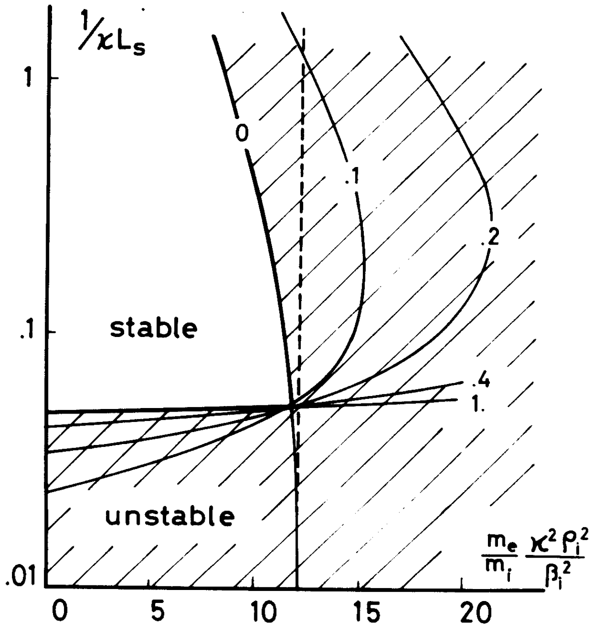
Figure Captions

Fig.1 The critical shear v.s. $\sqrt{m_e/m_i}\rho_i\kappa/\beta_i$ in the slab geometry is shown for various values of b. The value of the parameter b is indicated on each line.

Fig.2 The critical shear v.s. b for various value of $(u/v_e)^2$ (indicated on each line) is shown. $m_i/m_e = 1836$.

Fig.3 The growth rate for the fixed shear parameter v.s. b is shown for various values of $(u/v_e)^2$ (indicated on each line). $m_i/m_e = 1836$.

Fig.1



$1/\varkappa L_s$
1
.1
.01
stable
unstable
0
.1
.2
.4
1.
0
5
10
15
20
$\frac{m_e}{m_i}\frac{\varkappa^2\rho_i^2}{\beta_i^2}$

Fig.2

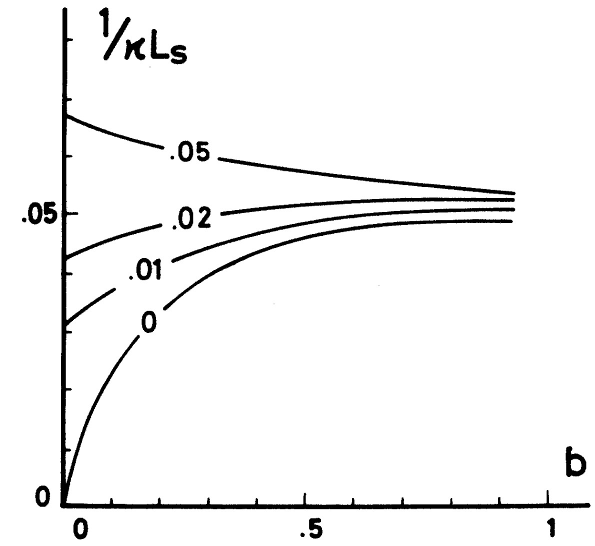


Fig.3