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Shear Stabilization of Collisional Drift Instability

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Abstract

Using the normal mode expansion method, the collisional drift waves of the slab plasma in the sheared magnetic field are investigated. In the strong shear parameter regime , $1 \lesssim \kappa L_s << m_i/m_e$ (κ : density gradient, L_s : shear length and m_i/m_e : mass ratio), the growth rate is found to be proportional to $\sqrt{\nu/\omega_*}$. The diffusion coefficient is also estimated and the density limitation of the high density tokamak is shown.

Drift instabilities have been subject to exaustive investigations concerning about the anomalous loss of magnetically confined plasmas. Many driving mechanisms of the drift instabilities have been found in various parameter regimes[1]. Recently the plasma confinement scaling law, that is the confinement time τ_E is proportional to the plasma density n, has been adopted for tokamak plasmas, and efforts have been paid to increase the plasma density[2]. However, with the increment of the plasma density, the growth rate of the dissipative drift mode may increase as to restrict the plasma confinement. The stabilization is mainly due to the magnetic shear.

In this letter we investigate the collisional drift wave taking the shear effect correctly into consideration, and find that the growth rate $\gamma \propto \sqrt{\nu}$ instead of $\gamma \propto \nu$ (ν: electron collision frequency). We may say that even in fairly strong magnetic shear the collisional mode gives the density limitation to the high density tokamak plasma.

We use a slab model with sheared magnetic field in the following analysis. The x-axis is taken in the direction of the density gradient as $\nabla n = - \kappa n \hat{x}$. The magnetic field is given by $\vec{B} = (0, x/L_s, 1)B$. We consider an electrostatic fluctuation $\vec{E} = -\nabla\phi$ of the form $\phi(\vec{x},t) = \phi(x)\exp[i(ky-\omega t)]$ where we put $k_z = 0$ without a lack of generality. Other quantiites are also perturbed as $n_j = n_{0j} + \tilde{n}_j = n_{0j} + \bar{n}_j \exp[i(ky-\omega t)]$. The subscripts e and i stand for electrons and ions respectively.

We take the electrons collision into account using the linearized Vlasov equation with BGK collision model[3], that is,

$$[\frac{\partial}{\partial t} + \vec{v}\cdot\nabla - \frac{e}{m_e}(\vec{v}\times\vec{B})\cdot\nabla_v]\tilde{f}_e = \frac{e}{m_e}\tilde{E}\cdot\nabla_v f_{0e} - \nu\tilde{f}_e + \nu\frac{\tilde{n}_e}{n_{0e}}f_{oe}. \quad (1)$$

For ions we use the linearized Vlasov equation. We assume that the equilibrium state is local Maxwellian. The equilibrium distribution function is given as $f_{0j}(\vec{x},\vec{v},t) = (\sqrt{2\pi}v_j)^{-3}\exp[-\kappa(x+v_y/\Omega_j) - v^2/2v_j^2]$ (Ω_j: the cyclotron frequency, v_j: the thermal velocity $\sqrt{T_j/m_j}$). In the slab geometry, the magnetic shear is due to the plasma current J_z. Here we neglect the effect of the force free current[4] for simplicity. According to the well established method, the density fluctuations are given as

$$\frac{\bar{n}_e}{n_{e0}} = \frac{e\phi}{T_e}[1+(\frac{\omega-\omega_*}{\sqrt{2}|k_\parallel|v_e})Z(\xi)/\{1+\frac{i\nu}{\sqrt{2}|k_\parallel|v_e}Z(\xi)\}] \equiv \frac{e\phi}{T_e}(1+\chi_{res}), \quad (2)$$

$$\frac{\bar{n}_i}{n_{i0}} = -\frac{e}{T_i}[\phi + \frac{\omega-\omega_{*i}}{\sqrt{2}|k_\parallel|v_i}\{\Lambda\phi-\Lambda'\rho_i^2\frac{\partial^2\phi}{\partial x^2}\}Z(\frac{\omega}{\sqrt{2}|k_\parallel|v_i})], \quad (3)$$

where $Z(\xi)$ is the plasma dispersion function $\frac{1}{\sqrt{\pi}}\int\frac{e^{-t^2}}{t-\xi}dt$, $\xi = (\omega+i\nu)/\sqrt{2}|k_\parallel|v_e$, $k_\parallel = kx/L_s$, $\Lambda(b) = I_0(b)e^{-b}$, $b = k^2\rho_i^2$, and ρ_i is the ion Larmour radius. The Poisson's equation $\bar{n}_e = \bar{n}_i$ (for the Debye length $<< k^{-1}$) gives the differential equation as the dispersion relation. Since the instability source of the mode is localized in the narrow region near $x = 0$, we expand $Z(\omega/\sqrt{2}|k_\parallel|v_i)$ with respect to $kxv_i/L_s\omega$ and obtain

$$\frac{\partial^2\phi}{\partial\zeta^2} + [\ \lambda + \mu^2\zeta^2 + \eta(\zeta)\ ]\ \phi = 0 \qquad (4)$$

$$\zeta = x/\tilde{\rho}_i,\ \tilde{\rho}_i^2 = (-\Lambda'/\Lambda)\rho_i^2,\ \lambda = \Lambda - \omega(\tau+1)/(\omega\tau+\omega_*),$$

$$\mu^2 = \frac{\omega(\tau+1)}{\omega\tau+\omega_*}\cdot\frac{k^2 v_i^2}{\omega^2}\frac{\tilde{\rho}_i^2}{L_s^2},\ \eta = -\frac{\omega}{\omega\tau+\omega_*}\chi_{res},\ \tau = T_e/T_i.$$

The ion Landau damping does not appear explicitly in this equation, as in the case of the universal mode[4,5], but determines the boundary condition for the waves to be out-going as $x\to\pm\infty$. Since $dQ/d\zeta/Q > \sqrt{Q}$ ($Q = |\lambda-\mu^2\zeta^2+\eta|$) near $\zeta=0$, the WKB method is inapplicable to this problem. As in the case of the current driven mode[4], we take the complete orthonormal set of functions $\{\ \phi_n|\ \phi_n(\zeta) \equiv (i\mu/2\pi)^{1/4} H_n(\sqrt{i\mu}\zeta)e^{-i\mu\zeta^2/2}/\sqrt{n!}\ \}$. Expressing $\phi = \Sigma a_n\phi_n(\zeta)$, we obtain

$$\det\begin{vmatrix} \lambda-\lambda_0+\langle 0|\eta|0\rangle, & \langle 0|\eta|1\rangle, & \cdots \\ \langle 1|\eta|0\rangle, & \lambda-\lambda_1+\langle 1|\eta|1\rangle, & \cdots \\ \cdots & \cdots & \end{vmatrix} = 0 \qquad (5)$$

where $\langle i|F|j\rangle = \int\phi_i F\phi_j d\zeta$, $\langle i|1|j\rangle = \delta_{ij}$ and $\lambda_n = i(2n+1)\mu$. Since the large n mode ϕ_n is strongly stabilized ($\lambda_n \propto 2n+1$), the contributions of the couplings between higher n modes are unimportant. Retaining (a_0, a_1), we truncate this infinite matrix into 2 x 2. Because of the parity of ϕ_n and η, $\langle 1|\eta|0\rangle = \langle 0|\eta|1\rangle = 0$ holds having

$$\lambda = \lambda_0 - \langle 0|\eta|0\rangle \qquad (6)$$

for the least stable mode. Let be $\omega = \omega_0 + \delta\omega + i\gamma$, ω_0 is the solution of $\lambda = 0$, i.e., $\omega_0 = \omega_* \Lambda/(1+\tau-\tau\Lambda)$. Linearizing λ with respect to γ/ω_*, we obtain from Eq.(6)

$$\gamma/\omega_* = \frac{(\omega_0\tau+\omega_*)^2}{(\tau+1)\omega_*^2}\left[\ \mathrm{Im}\langle 0|\eta|0\rangle - \mu\ \right] . \qquad (7)$$

Now we evaluate the integral $\langle 0|\eta|0\rangle$. We take two expansion parameters ω/ν and $\varepsilon \equiv \sqrt{\mu}\zeta_e$ ($\zeta_e = L_s\omega/kv_e\tilde{\rho}_i$) for $\omega \ll \nu$, then η is approximately given by

$$\begin{aligned} &\frac{\omega_*-\omega}{\omega\tau+\omega_*}\left[\left(\frac{4\omega^2\zeta_e^4}{\nu^2\zeta^4} + \frac{2i\omega\zeta_e^2}{\nu\zeta^2}\right)/\left(1 + \frac{4\omega^2\zeta_e^4}{\nu^2\zeta^4}\right)\right] && \text{for } \frac{\zeta}{\zeta_e} \ll \frac{\nu}{\omega} , \\ &\frac{\omega_*-\omega}{\omega\tau+\omega_*}\left[\ \sqrt{\pi}\zeta_e/|\zeta|\ \right] && \text{for } \frac{\zeta}{\zeta_e} \ll \frac{\nu}{\omega} . \end{aligned} \qquad (8)$$

Executing the integration, to the lowest order of ω/ν and ε we have

$$\langle 0|\eta|0\rangle \simeq \left\{\frac{\omega_*-\omega}{\omega\tau+\omega_*}\right\} \times \begin{cases} \dfrac{\omega}{2\sqrt{\pi}\nu\varepsilon^2} F & \dfrac{\nu}{\omega} > \dfrac{1}{\varepsilon^2} \\ \varepsilon\sqrt{\nu/2\pi\omega} - \dfrac{\varepsilon}{2}\ln\varepsilon & \dfrac{1}{\varepsilon^2} > \dfrac{\nu}{\omega} > 1 \\ -\dfrac{\varepsilon}{2}\ln\varepsilon & 1 > \dfrac{\nu}{\omega} \end{cases} \qquad (9)$$

where $F(\varepsilon\sqrt{\nu/\omega})$ is the numerical coefficient of order of unity and goes to zero as $\varepsilon^2\nu/\omega \to \infty$. From Eq.(9) we see that the growth rate smoothly connects to those of hydrodynamic ($\gamma \propto \nu^{-1}$) and universal modes both in high and low collisional limits. In the intermediate (collisional) regime, $1 < \nu/\omega < 1/\varepsilon^2$, the growth rate is proportional to $\sqrt{\nu}$ not to ν. Equations (7) and (9) gives the critical shear parameter for the stability

$$\kappa L_s \leq \sqrt{-\Lambda'}/\Lambda \left[\frac{1+\tau-\tau\Lambda}{1-\Lambda} \right]^{2/3} (2\pi\tau m_i/m_e)^{1/3} (\omega_*/\nu)^{1/3} . \qquad (10)$$

For the unstable mode (Im $\langle 0|\eta|0\rangle > \mu$), the growth rate is

$$\frac{\gamma}{\omega_*} \simeq \frac{(\tau+1)(1-\Lambda)}{(\tau+1-\tau\Lambda)^2\sqrt{\tau}} [-\Lambda^2/\Lambda']^{1/4} \sqrt{\frac{m_e}{2\pi m_i}} \sqrt{\kappa L_s} \sqrt{\nu/\omega_*} \qquad (11)$$

both are in the collisional regime. In Fig.1, the stability criterion is shown on a ν/ω_{*max}- b plane for various values of $1/\kappa L_s$. The mass ratio m_i/m_e is taken 1836. As the shear increases the unstable region moves to higher ν/ω and b. In figures ν and γ are normalized by ω_{*max} ($= \kappa T/eB\rho_i$). In Fig. 2 and Fig.3 the contours of the growth rate are shown on b - ν/ω_{*max} and ν/ω_{*max} - $1/\kappa L_s$ planes.

Finally we briefly discuss the cross field diffusion due to the collisional drift mode. The diffusion coefficient D is given by $\Sigma|\bar{n}_k/n|^2\gamma_k/\kappa^2$ [7]. We assume the quasilinear saturation level $|\bar{n}_k/n|^2 \simeq \kappa^2/k_\perp^2 = \kappa^2/(k^2+k_x^2)$ [7] ($k_x^2 \simeq \mu/\tilde{\rho}_i^2$). Because the growth rate is an increasing function of b ($b \lesssim 1$), and for $b > 1$ modes the ion collisional damping may dominate[6], the main contribution comes from the short wave length mode with $b \sim 1$. D is given as

$$D \simeq \tilde{\rho}_i^2 \omega_{*max} \sqrt{\frac{\nu}{\omega_{*max}} \frac{m_e}{2\pi m_i} \kappa L_s} \; . \quad \text{(collisional regime)} \qquad (12)$$

The diffusion coefficient depends on $\sqrt{\nu}$ for the fixed κL_s. The analyses with a local approximation have shown $\gamma \propto \nu$, that is $D \propto \nu$. The collisional drift wave has been considered as the cause of the anomalous transport, the pseudo-classical law[7].

However, in the previous analyses, the parallel wave number $k_{\parallel}$ is ambiguously evaluated.

In the concept of the high density tokamak, the parallel electron velocity ($u = J/ne$) is inversely proportional to the density for the fixed q value, and is low enough for the current-driven mode to be stabilized, i.e., $u/v_e \lesssim 1/\kappa L_s$[4]. Assuming that the value of κL_s in Eq.(12) is evaluated by v_e/u, the diffusion coefficient D is proportional to the plasma density (i.e., $\tau_E \propto n^{-1}$). We may say that the collisional drift mode gives the density limit even in the sheared system.

Since the localization width of the wave ($\lesssim L_s\omega/v_i k$) is small compared with the density gradient length, above results are directly applicable to cylindrical plasmas. In addition, in high density tokamak plasmas, where $\nu R/a > \omega_b$ (R/a: the aspect ratio, ω_b: the bounce frequency of the trapped electrons[1]) holds, the effect of trapped particles are negligible. The typical features of collisional drift modes are essentially the same as that of the slab model discussed here. In applying the results to cylindrical and toroidal cases (for instance the tokamak simulation) the values (κ, L_s, ··) are to be evaluated by the local values.

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Figure Captions

Fig.1 The stability criterion on a ν/ω_{*max} - b plane. The parameter $1/\kappa L_s$ is indicated on each line. The upper part of the line is the unstable region. As $1/\kappa L_s$ increases the unstable region degenerates to higher b region.

Fig.2 The contour of γ/ω_{*max}. The shear parameter $1/\kappa L_s$ is taken as 0.034 (left) and 0.109 (right). The growth rate forms a " ridge " near $\nu/\omega_{*max} \sim 1/\varepsilon^2$.

Fig.3 The growth rate contour on ν/ω_{*max} - $1/\kappa L_s$ plane. Both axis are in logalithmic scale. The finite Larmour radius effect, b = 0.1 (left) and b = 0.75 (right).

Fig.1

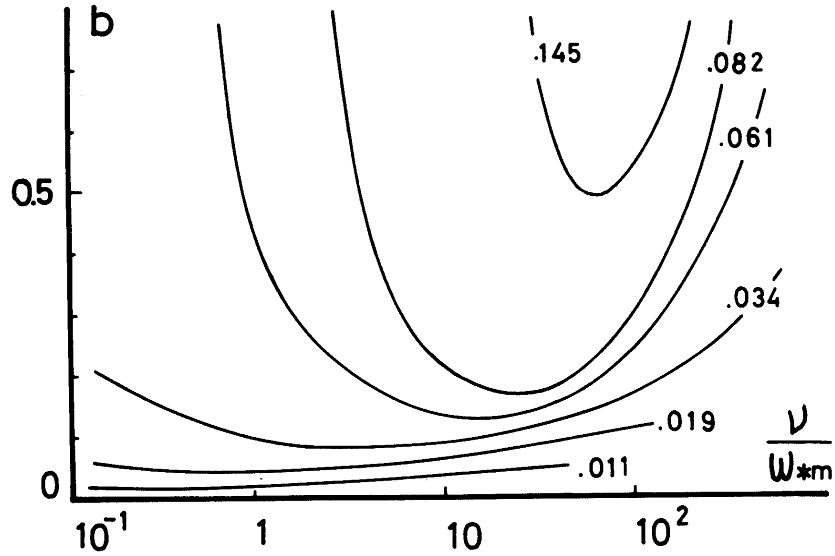
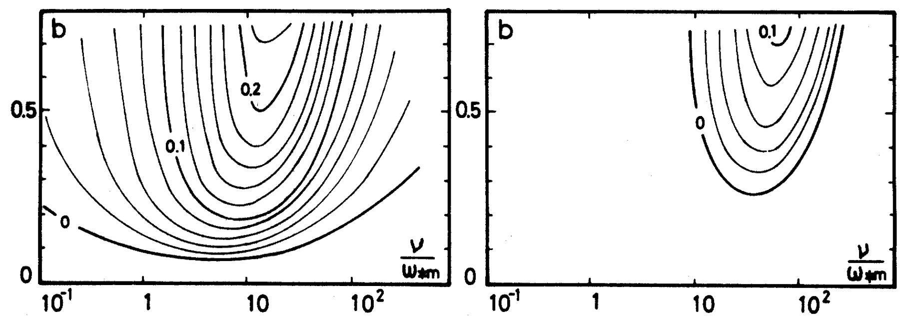
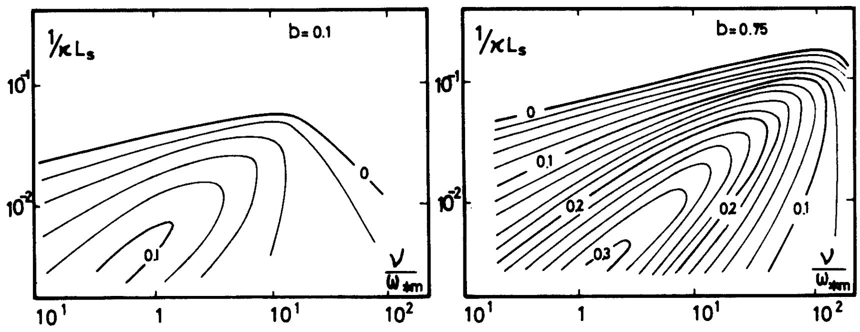


Fig.2



b
0.2
0.1
0
0.5
0
$\frac{\nu}{\omega_{*m}}$
10^{-1}
1
10
10^{2}
b
0.1
0
0.5
0
$\frac{\nu}{\omega_{*m}}$
10^{-1}
1
10
10^{2}

Fig.3


1/κLs
b=0.1
10⁻¹
10⁻²
0
0.1
ν/ω*m
10⁻¹
1
10¹
10²
1/κLs
b=0.75
0
0.1
0.2
0.3
0.2
0.1
ν/ω*m
10⁻¹
1
10¹
10²