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Anomalous Plasma Transport

due to

Electromagnetic Drift Wave Fluctuations

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Abstract

The anomalous particle and heat fluxes in a finite β inhomogeneous plasma due to electromagnetic drift wave fluctuations in a strongly-sheared magnetic field are obtained correct to the first order with respect to β(plasma pressure / magnetic pressure). Using the proper eigenfunctions of perturbed field, we show that the transport is reduced by the finite β effect regardless of the formation of fluctuating magnetic islands. Broken ambipolarity is also found.

Effects of drift wave fluctuations on the plasma transport have been studied extensively. Recent experimental progress has achieved the confinement of a plasma of medium β value (the ratio of the plasma pressure to the magnetic pressure). When β becomes greater than m_e/m_i, the electron-to-ion mass ratio, drift waves are accompanied by magnetic perturbations[1] which modify the anomalous plasma transport by two possible mechanisms. On one hand, the excited magnetic perturbation in a sheared magnetic field forms fluctuating magnetic islands near the rational surface, and these magnetic islands give rise to a new source of anomalous transport. Some models are proposed to show this effect which indicate an anomalous enhancement of the electron heat flux, although the particle transport is left unaltered.[2] On the other hand, it has been shown that the convective damping due to the magnetic shear persists in a finite β plasma and that the growth rate is reduced by the electromagnetic (EM) correction.[1] Therefore the anomalous transport may be reduced by the finete- β effect.

In this letter, we show that in the presence of a strong magnetic shear with $\beta<1$, the effect of the latter mechanism dominates over the former, so that the finite β effect reduces the anomalous transport, either particle or heat, regardless of the formation of fluctuating magnetic islands. We first obtain the linear structures of the magnetic perturbation and the associated magnetic islands. We then calculate the particle and heat fluxes across the equilibrium magnetic surface in the consistent ordering of β. To calculate these fluxes, we use the general formula for the particle and heat fluxes due to EM fluctuations. We find that the fluxes are always reduced by the finite β effect, provided that the magnetic shear is fairly strong. In addition, an ambipolarity of the particle flux, which is maintained in the electrostatic (ES) fluctuations, is found to be broken in the order of β.

We start by solving the linear eigenmode structure of the EM drift wave for a collisionless, finite β plasma slab having a density inhomogeneity in the x-direction, $\nabla n = -\kappa n x$, perpendicular to the magnetic field, $\vec{B} = [o,\ x/L_s,\ (1+\beta x/L_s)]B_o$. We seek a perturbation of the form $\tilde{E}_y = E_y(x)\exp(-i\omega t + iky)$, where the $x=0$ plane corresponds to the rational surface, $\vec{k}\cdot\vec{B}(x=0)=0$, and the parallel wave vector $\vec{k}_{/\!/}$ is defined as $\vec{k}_{/\!/} = \vec{k}x/L_s$. The magnetic shear is generated by the electron current density, -neu.

The basic equations in the small $b(\equiv k^2\rho_i^2)$ limit, where ρ_i is the ion gyroradius, are given in Ref.1. We extend these equations to the mode with arbitrary b, assuming $\rho_i^2\partial^2/\partial x^2 < 1$ to obtain,[3]

$$\frac{\partial^2 E_y}{\partial\zeta^2} + [\,\lambda_E + \mu^2\zeta^2 + i\sigma_E(\zeta)\,]E_y = \frac{\nu}{\alpha}\frac{1}{\zeta}[\,\lambda_B - \mu^2\zeta^2 + i\sigma_B(\zeta)\,]B_x\ , \tag{1}$$

$$\frac{\partial^2 B_x}{\partial\zeta^2} + (\,b\Lambda' - \frac{\sigma_c}{\zeta}\,)B_x = \frac{\alpha}{\zeta}(\,-\frac{\partial^2}{\partial\zeta^2} + 1 - \Lambda\,)E_y\ , \tag{2}$$

where we used the following notation:

$$\zeta = x/\tilde{\rho}_i,\quad \tilde{\rho}_i^2 = -\Lambda'\rho_i^2,\quad \Lambda = I_0(b)e^{-b},\quad \mu^2 = \frac{(\tau+1)k^2 v_i^2\tilde{\rho}_i^2}{\omega(\omega\tau+\omega_*)L_s^2}\ ,$$

$$\omega_* = \frac{\kappa T_e k}{eB_0}\ ,\quad v_j^2 = T_j/m_j,\quad \alpha = -\Lambda'\beta_i L_s\frac{(\omega\tau+\omega_*)c}{v_i^2 k\tilde{\rho}_i^2\tau}\ ,$$

$$\zeta_e = L_s\omega/kv_e\tilde{\rho}_i\ , \quad \nu/\alpha = \zeta_e v_e/c, \quad \beta_i = 8\pi n_i T_i/B_0^2, \quad \sigma_c = \frac{\alpha\omega_* u}{c(\omega\tau+\omega_*)}\ ,$$

$$\lambda_E = \Lambda - \frac{(1+\tau)\omega}{\tau\omega+\omega_*} + \beta_i\left[\frac{2\omega_*(1+\tau)(\Lambda+b\Lambda')}{\tau(\tau\omega+\omega_*)} + \frac{\tau\omega+\omega_*}{\tau\omega}\Lambda'^2\right]\ ,$$

$$\lambda_B = \frac{\omega-\omega_*}{\omega\tau+\omega_*}\ , \quad i\sigma_B = \eta Z(\eta)\frac{\omega-\omega_*-k_{/\!/}u}{\omega\tau+\omega_*}\ , \quad i\sigma_E = -i\sigma_B(1+\tau+\omega_*/\omega)\ ,$$

$$\eta = \omega/\sqrt{2}|k_{/\!/}|v_e \quad \text{and} \quad Z(\eta) = \frac{1}{\sqrt{\pi}}\int\frac{e^{-t^2}}{t-\eta}dt\ .$$

The components other than E_y and B_x are expressed in terms of E_y and B_x through Maxwell's equations.

The boundary conditions are : i) the wave is out-going and ii) the fields must be regular at $x = 0$. Using the complete ortho-normal functions $\{\phi_n|\phi_n(\zeta) = \sqrt[4]{i\mu/\pi}H_n(\sqrt{2i\mu}\zeta)\exp(-i\mu\zeta^2/2)/\sqrt{n!}\}$, E_y and B_x are expressed as $E_y = \Sigma A_n\phi_n$, $B_x = \Sigma B_n\phi_n$. From these basic equations we can derive a secular equation. Using a smallness parameter $\varepsilon = \sqrt{\mu}\zeta_e \sim \sqrt{\kappa L_s m_e/m_i}$ where we assume $1 < \kappa L_s << m_i/m_e$, we keep only the lowest order terms. For the least stable mode, we then obtain the eigen function as

$$E_y = A_0\phi_0\ , \qquad B_x = B_1\phi_1\ ,$$

$$\frac{B_1}{A_0} = -\alpha\sqrt{2i\mu}\ \frac{i\mu + 2 - 2\Lambda}{3i\mu - 2\Lambda'}\ . \tag{3}$$

In deriving Eq.(3), we neglected u/v_e for simplicity. For

$b \ll 1$, the growth rate is reduced to that obtained in Ref.1. More detailed arguments and calculations about the linear mode structure will be presented in a separate paper[3].

The resultant magnetic surface is illustrated in Fig.1. The normal mode given by Eq.(3) has nodes at $x_i = \tilde{\rho}_i\sqrt{(2m+1)\pi/\mu}$ (m : integer number) independent of the fluctuation amplitude. The stratification of magnetic islands appears for the perturbations of sufficiently large amplitude. The width of the stratified cells is of the order of $\rho_i/\sqrt{\mu}$. The consistent eigenmode of E_y and B_x (Eq.(3)) gives the full width δ of the central magnetic island, which is proportional to B_1/B_0, i.e.,

$$\delta = \frac{2L_s}{k\rho_i}\frac{B_1}{B_0} \sim \beta_i \kappa L_s \frac{cE_y}{\omega B_0} . \tag{4}$$

Note that the electron excurtion length due to this magnetic fluctuation across the magnetic surface has the same dependence on E_y/B_0 as that of the electric field fluctuation.

We now consider the particle and heat fluxes. From the Klimontovich equation, we can derive the following general expressions for the particle flux across the magnetic surface[3]:

$$\Gamma_x = \frac{1}{B_0}\left[\, c\langle \tilde{E}_y \tilde{n}\rangle + \langle (\, \tilde{\vec{\Gamma}} \times \tilde{\vec{B}}\,)_y \rangle \,\right] , \tag{5}$$

where $\tilde{n}$ and $\tilde{\vec{\Gamma}}$ are expressed by the fluctuating part of the distribution function $\tilde{f}$ [4] as $\tilde{n} = \int d\vec{v}\tilde{f}$, $\tilde{\vec{\Gamma}} = \int \vec{v}d\vec{v}\tilde{f}$. The bracket $\langle\ \rangle$ denotes the time average over the time scale T_a which satisfies the inequalities, $\omega^{-1} \ll T_a \ll$ (diffusion time). In Eq.(5), the terms of order $(\kappa\rho_i)^2$ are ignored. We calculate the electron flux retaining the terms up to the 1st order with respect to β, obtaining,

$$\Gamma_x{}^e = \Gamma_{ES}\left[1 + \frac{2\beta\Lambda'}{\sqrt{\pi\mu}} \frac{b\Lambda' + 3(1-\Lambda)}{(b\Lambda')^2+9\mu^2/4} \right], \tag{6}$$

where

$$\Gamma_{ES} \equiv \frac{ce|E_y|^2}{kB_oT_e} \frac{\omega_*-\omega}{\omega} \mathrm{Im}[\eta Z(\eta)]$$

is the particle flux obtained by the ES approximation. Equation (6) shows that only the resonant electrons contribute to the flux. They do not circumnavigate the magnetic island, because their parallel velocity is close to the phase velocity of the wave. The important feature is that the flux is reduced (note $\Lambda' < 0$) by the finite β effect independently of the presence of the magnetic island.

For the ion flux we use Eq. (5) with Maxwell's equations assuming $k^2\lambda_D{}^2 << 1$ (that is $\tilde{n}_e = \tilde{n}_i$), and obtain

$$\Gamma_x{}^i - \Gamma_x{}^e = \frac{c}{4\pi eB_0} <[(\nabla \times \tilde{\vec{B}}) \times \tilde{\vec{B}}]_y>. \tag{7}$$

Substituting Eq. (3) into Eq. (7) we get the ion flux as

$$\Gamma_x{}^i = \Gamma_x{}^e + \frac{cne}{kB_0T}|E_y|^2\zeta^2\frac{(\omega_*+\omega\tau)}{\omega\tau}\beta(-\Lambda')\frac{(1-\Lambda)^2+\mu^2/4}{(b\Lambda')^2+9\mu^2/4} \tag{8}$$

We can see from this formula the broken ambipolarity of the order of β. The magnetic shear causes convective damping in the x-direction, and ions feel this damping during their gyromotions, thereby diffuse faster than electrons. Due to this broken ambipolarity a static electric field starts piling up as

$$\frac{\partial}{\partial x} E_x = 4\pi\sigma = -4\pi\int\frac{\partial}{\partial x}(\Gamma_x{}^i - \Gamma_x{}^e)dt \ .$$

This field has negative curvature, $d^2E/dx^2 < 0$, and therefore reduces the instability. Thus the EM correction gives a new aspect of the nonlinear stabilization. Note that the ambipolarity is maintained when the magnetic field is shearless and/or the fluctuation is electrostatic.

We next calculate the heat flux. We define the pressure and the heat flux tensors as

$$P_{ij} \equiv \frac{m}{2}\int(v_i-V_i)(v_j-V_j)f(\vec{v})d\vec{v} \quad ,$$

$$Q_{ijk} \equiv \frac{m}{2}\int(v_i-V_i)(v_j-V_j)(v_k-V_k)f(\vec{v})d\vec{v} \ ,$$

where f is the distribution function, $nV_j = \int v_j f(\vec{v})d\vec{v}$ and $i = x,y,z$. One can define $\tilde{P}_{ij}$ and $\tilde{Q}_{ijk}$ in the same way by replacing f by $\tilde{f}$. The heat conduction in the x-direction on the frame which moves with the fluid is $Q_{xxx} + Q_{xyy} + Q_{xzz}$. Including the convection term, the total heat flux in the x-direction is

$$Q_x \equiv \sum_i (Q_{xii} + \Gamma_x P_{ii}) \ .$$

The equation for $\sum_i Q_{xii}$ is obtained as

$$\frac{B_0}{c}\sum_i Q_{xii} = \sum_i \Big[\Big\langle \Big(\tilde{\vec{E}} + \frac{\tilde{\vec{V}} \times \tilde{\vec{B}}}{c} \Big)_y \tilde{P}_{ii} \Big\rangle + 2\Big\langle \Big(\tilde{\vec{E}} + \frac{\tilde{\vec{V}} \times \tilde{\vec{B}}}{c} \Big)_i \tilde{P}_{iy} \Big\rangle$$

$$- \frac{P_{ii}}{n}\Big\langle \Big(\tilde{n}\tilde{\vec{E}} + \frac{\tilde{\vec{\Gamma}} \times \tilde{\vec{B}}}{c} \Big)_y \Big\rangle - 2\frac{P_{iy}}{n}\Big\langle \Big(\tilde{n}\tilde{\vec{E}} + \frac{\tilde{\vec{\Gamma}} \times \tilde{\vec{B}}}{c} \Big)_i \Big\rangle$$

$$+ \frac{1}{c} \langle \tilde{B}_x \tilde{Q}_{zii} - \tilde{B}_z \tilde{Q}_{xii} \rangle \Big] , \qquad (9)$$

where the terms of order $(\kappa\rho_i)^2$ are neglected as before. We then obtain the electron heat flux in the sheared field as

$$Q_x = Q_{ES}\Big[1 - 4\beta_i \Lambda' (\tau + \frac{\omega_*}{\omega}) + \frac{\beta\Lambda'}{\sqrt{\mu\pi}} \frac{b\Lambda' + 3(1-\Lambda)}{(b\Lambda')^2 + 9\mu^2/4} \Big] , \qquad (10)$$

where

$$Q_{ES} = \frac{2ne|E_y|^2}{kB_0} \frac{\omega_* - 1}{\omega} \, \mathrm{Im}[\eta Z(\eta)] .$$

We again see a reduction due to the finite β effect. Comparing Eq. (10) with Eq. (6), we find that the finite β supression of the heat flux Q_x is smaller than that of Γ_x.

Finally, we discuss the effect of the fluctuating magnetic island formation. The EM perturbation causes a distortion of the lines of force, which gives rise to an additional excursion of electrons across the equilibrium magnetic surface. However, if we estimate the additional diffusion coefficient due to this effect by $\gamma\delta^2$, where δ is given by Eq.(4), and compare it with the diffusion coefficient due to the ES fluctuations, $\gamma\delta_E{}^2$, where $\delta_E = cE_y/\omega B_0$, we find that $\gamma\delta^2/\gamma\delta_E{}^2 \sim \beta^2(\kappa L_s)^2$, which is of order β^2 for a fairly strong magnetic shear $\beta\kappa L_s << 1$, and hence is negligible in our β-ordering.

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References

1] R.J.Catto. A.M.El-Nadi, C.S.Liu and M.N.Rosenbluth, Nucl. Fusion 14 405 (1974).

2] B.B.Kadomtsev, Sov. J. Plasma Phys. 1 512 (1975) ; J.D.Callen, Preprint, ORNL/TM-5974.

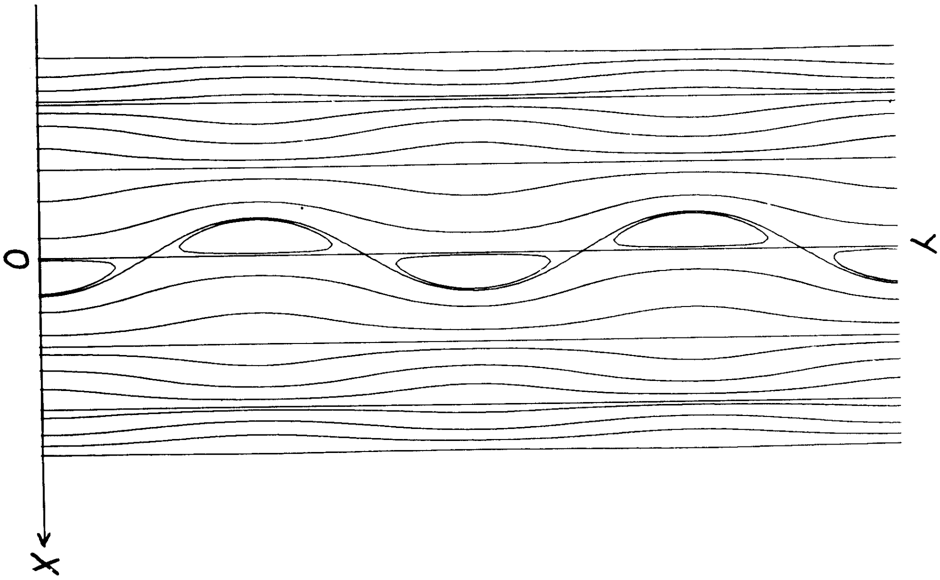
3] S.Inoue, K.Itoh and T.Tange, to be published.

4] The expression of $\tilde{f}$ is given in, for instance, Ref.1.

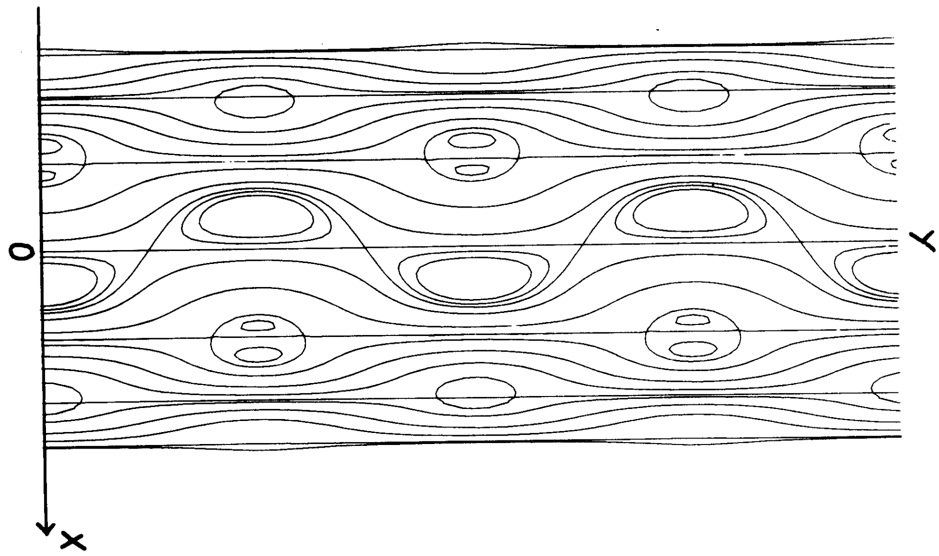
Figure Caption

Fig. 1. The resultant magnetic surfaces near the rational surface. The value γ/ω is taken .2 and $L_s B_1/B_0 \rho_i^2 k$ is taken a) .3 and b) .75.

Fig. 1



a)



b)