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Modulated Langmuir Waves and Nonlinear Landau Damping

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Abstract

The nonlinear Schrödinger equation with a nonlocal-nonlinear term, which describes modulated Langmuir waves with the nonlinear Landau damping effect, is solved by numerical calculations. The effects of nonlinear Landau damping on solitary wave solutions are studied. The results show that the solitary waves deform in an asymmetric way changing its velocity. The time evolutions of a periodic modulation are also studied.

§1 Introduction

It has been shown by Taniuti and one of the authors (N.Y.) that one-dimensional nonlinear modulation of plane waves in dispersive systems can be described by the nonlinear schrödinger equation[1):

$$i\frac{\partial u}{\partial t} + (P/2)\frac{\partial^2 u}{\partial x^2} + Q|u|^2 u = 0, \tag{1}$$

where u is the complex amplitude of a plane wave varying slowly due to modulation and P and Q are parameters which represent the strength of dispersion and nonlinearity. This equation describes a wide class of physical phenomena which involve modulational instability of water waves[2)], propagation of heat pulses in anharmonic crystals[3)], helical motion of a very thin vortex filament[4)], nonlinear modulation of collisionless plasma waves[5),6)] and self-trapping of a light beam in colour-dispersive systems[7)].

In the modulationally unstable case (PQ>0), the initial value problem of the nonlinear Schrödinger equation was investigated numerically by Karpman and Krushkal'[8)], Yajima and Outi[9)], and Satsuma and Yajima[10)]. Zakharov and Shabat[11)] first obtained analytical solutions of eq.(1) by applying Lax's method[12)] to solve nonlinear evolution equations. A given initial disturbance breaks up to a train of solitons. The solitons work as stable entities through the time evolution of solution.

It is well known that in real plasma systems a wave interacts strongly with the resonant particles, for example, a nonlinear modulated wave is scattered by the particles moving with the velocity equal to its group velocity. If the velocity distribution of the particles is Maxwellian, this scattering leads to the nonlinear Landau damping. Ichikawa and Taniuti[13] studied this phenomenon and modified eq.(1) to :

$$i\frac{\partial u}{\partial t} + (P/2)\frac{\partial^2 u}{\partial x^2} + Q|u|^2 u + R\mathcal{P}\int_{-\infty}^{\infty} \frac{|u(x',t)|^2}{x-x'}dx'u = 0, \quad (2)$$

where $\mathcal{P}$ denotes the Cauchy principal value. The nonlocal-nonlinear integral term with coefficient R represents the resonant effect of nonlinaer Landau damping. It is noted that for the modulation of ion waves Q changes the sign depending on the ratio of the ion temperature to the electron temperature.

We now consider the effect of nonlinear Landau damping on nonlinear wave modulations. We substitute

$$u = [\phi_0+\phi_1 e^{i(qx-\omega t)} + \phi_2 e^{-i(qx-\omega^* t)}]e^{-i\omega_0 t} \quad (3)$$

into eq.(2) and linearize with respect to the perturbed amplitudes, ϕ_1 and ϕ_2, where asterisk denotes the complex conjugation. It can be readily shown that the dispersion equaiton is

$$\omega_0 = -Q\ |\phi_0|^2, \quad (4)$$

$$\omega^2 = (P^2 q^4/4)[1-(4Q/Pq^2)|\phi_0|^2+i(4\pi R/Pq|q|)|\phi_0|^2]. \quad (5)$$

We put $\omega = \Omega + i\Gamma$, where Ω and Γ are real, to obtain

$$\Omega^2 - \Gamma^2 = (P^2 q^4/4)[1 - (4Q/Pq^2)|\phi_0|^2], \tag{6a}$$

$$\Omega\Gamma = (\pi PRq|q|/2)|\phi_0|^2 . \tag{6b}$$

Without the nonlinear Landau damping, R = 0, eqs.(6a) and (6b) give the usual stability condition; the system is unstable if $(4Q/Pq^2)|\phi_0|^2 > 1$ and otherwise stable. Whilst, with $R \neq 0$, the growth rate Γ never vanishes and plasne waves become unstable. If $PR>0$, we find that $\Gamma>0$ for the disturbance with $\Omega q>0$ and $\Gamma<0$ for $\Omega q<0$. The amplitude of modulated wave, thereby, grows when it propagates in the positive x-direction and damps in the opposite direction. If $PR<0$, the situation is reversed.

In this paper† eq.(2) is solved by numerical computations in order to explore how the nonlinear modulated waves, particularly the solitary waves, evolve under the influence of the nonlocal-nonlinear integral term. In §2, modulationally unstable case ($PQ>0$) is studied. The modulationally stable case ($PQ<0$) is dealt with in §3. In both cases R is taken to be positive. The reductive perturbation method applies to investigate the behaviour of a slightly modulated plane wave with small nonlinear Landau damping for the case ($PQ<0$). Evolutions of a periodic modulation are studied in §4. The final section is devoted to the summary.

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† A part of this work (§§2 and 3) has been published preliminarily in Reports of Research Institute for Applied Mechanics 22 (1975) 89.

The difference scheme used to solve eq.(2) is presented in Appendix. In numerical computations, the runs were inspected at every step by using the conserved quantity, $I = \int |u(x,t)|^2 dx$. The relative error, $|\Delta I/I|$, was within 0.71% through our calculations.

§2 Numerical Solutions 1

—— Behaviour of Envelope Solitons (PQ>0) ——

In the present case (PQ>0), eq.(1) has an envelope-soliton solution, which satisfies the boundary condition that u(x,t) and its derivatives vanish at $x = \pm\infty$,

$$S(x,t) = \exp[i\{(V/P)x-(V^2/2P)t+(QA^2/2)t\}] \times A\mathrm{sech}[\sqrt{Q/P}A(x-Vt)]. \qquad (7)$$

Putting V = 0, we have a soliton at rest,

$$S_0(x,t) = \exp[i(QA^2/2)t]A\mathrm{sech}[\sqrt{Q/P}Ax]. \qquad (7')$$

In phrase of the Schrödinger equation, the nonlinear term of eq.(1) works as an attractive potential if Q>0 and prevents diffusion of wave packet due to the dispersion term, so that the stationary soliton solution (7) can exist.

2.1 The soliton solution (7) does not satisfy eq.(2) and deforms under the effect of nonlocal-nonlinear integral term. We now study the initial value problem for eq.(2) with :

$$u(x,t=0) = A\mathrm{sech}[\sqrt{Q/P}Ax]. \qquad (8)$$

If $R = 0$, the solution is just the soliton at rest, (7'). For this u, the integral term in eq.(2) is positive for $x>0$ and negative for $x<0$. As we consider the case of $R>0$, this implies that the nonlinear attractive force is enhanced for $x>0$ and weakened for $x<0$. The wave form, therefore, rises more steeply from the right than from the left. The nonlinear Landau damping thus leads to an asymmetric deformation of wave form. The integral term produces another effect ; the amplitude A slowly increases with x and then the phase $(QA^2/2)t$ of soliton solution (see (7')) advances more rapidly in larger x. As can be seen from (7), this makes an effect of $V \neq 0$. Therefore, the soliton which is initially at rest comes to move.

The numerical solutuions to eq.(2) with initial value (8) are illustrated in Fig.1. There we take $P = Q = 1$ and $A = 1$. It is observed that the soliton deforms in an asymmetric way and comes to travel. This agrees with the feature shown in §1 by a linear analysis ; waves propagating in the positive x-direction grow, those in the negative x-direction damp and as a result the wave packet moves to the right.

2.2 Equation (1) has a solution[10),11)]

$$u(x,t)=4Ae^{iQA^2t/2}\,\frac{ch(3Bx)+3ch(Bx)e^{4iQA^2t}}{ch(4Bx)+4ch(2Bx)+3\cos(4QA^2t)}, \qquad (9)$$

which satisfies the initial condition

$$u(x,t=0) = 2A\,\mathrm{sech}(Bx), \qquad B = \sqrt{Q/P}\,A. \qquad (10)$$

This solution does not decay into a train of solitons and the envelope pulsates with a period $\pi/(2QA^2)$. It has been already shown that if a disturbance with an asymmetric imaginary part is inflicted on this bound state of solitons, it decays into constituent solitons[10).

We now solve eq.(2) with the initial condition (10). The symmetry of u breaks due to the nonlinear Landau damping. Owing to such an asymmetry in u, solitons which are bound in its initial state are made to be free. Each of solitons travels with changing its shape and velocity. Examples are illustrated in Fig.2, for P = Q = 1 and A = 1.

§3 Numerical Solutions 2

—— Deformation of Dark-Solitons (PQ<0) ——

We introduce the amplitude and the phase of u as

$$u(x,t) = \sqrt{n(x,t)}\,\exp[i\theta(x,t)/P], \tag{11}$$

and substitute this into eq.(2), to get

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \tag{12a}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} - PQ\frac{\partial n}{\partial x} - (P^2/4)\frac{\partial}{\partial x}[\{\frac{\partial}{\partial x}(\frac{\partial n}{\partial x}/\sqrt{n})\}/\sqrt{n}]$$

$$- PR\, P\!\!\int_{-\infty}^{\infty} \frac{dx'}{x-x'}\,\frac{\partial n(x',t)}{\partial x'} = 0, \tag{12b}$$

where $v = \partial\theta/\partial x$. If the higher derivative term and the non-local integral term are neglected, eqs.(12a) and (12b) are reduced to the hyperbolic system of equations (note that PQ<0), in which the nonlinear steepening occures.

The presence of the higher derivative term, which represents the dispersive effect, prevents the nonlinear steepening and leads to an emission of solitons.

Equations (12) without the intergral term have following soliton soltuion ;

$$n(x,t)=n_0\,[1-A\,\mathrm{sech}^2\{(c/P)\sqrt{A}(x-\lambda_\pm t)\}], \qquad (13a)$$

$$v(x,t)=\lambda_\pm \mp c\sqrt{1-A}/[1-A\,\mathrm{sech}^2\{(c/P)\sqrt{A}(x-\lambda_\pm t)\}], \qquad (13b)$$

$$\lambda_\pm = v_0 \pm c\sqrt{1-A}, \qquad c = \sqrt{-PQn_0}, \qquad (13c)$$

where n_0 and v_0 are the boundary values of n and v, respectively, at $x = \pm\infty$. The double sigh in eq.(13b) is chosen according to the propagation direction of soliton; i.e., -sign for λ_+ and +sign for λ_-. It is noted that the above soliton with λ_+ (or λ_-) represents the defect in the amplitude, propagating in the positive (or negative) x-direction. This soliton is called a dark-soliton.

3.1 The effect of the nonlinear Landau damping on such dark-soliton solutions is studied by numerical integration of eq.(2) with the initial value

$$u(x,t=0)=\sqrt{n_0}[1-A\,\mathrm{sech}^2\{(c/P)\sqrt{A}x\}]^{1/2}\exp[\mp i\{(c/P)\sqrt{1-A}x + \tan^{-1}(\sqrt{A/(1-A)}\tanh[(c/P)\sqrt{A}x])\}]. \qquad (14)$$

For R = 0, the solution evolving from this initial value corresponds to the dark-soliton solution (13), but is observed in the frame moving with the soliton velocity.

The numerical solutions to eq.(2) were obtained for $P = -1$, $Q = 1$, $R = 0.5$ and $A = 0.1$ and are illustrated in Figs.3 and 4. It is seen that the dark-soliton moving to the right damps and one to the left grows. The results in early stage are consistent with an anticipation by the linear analysis in §1 (note that $PR<0$). For large t (>2.56), however, nonlinear effect becomes dominant.

3.2 It is interesting to study the case that the nonlinear Lnadau damping is sufficiently small and is of the same order of magnitude as that of modulation. In this case, we can apply the reductive perturbation method[14]. Assuming the modulation to be small, we now expand n and v in powers of a small parameter ε,

$$n = n_0 + \varepsilon n_1 + \varepsilon^2 n_2 + \cdots, \tag{15a}$$

$$v = \varepsilon v_1 + \varepsilon^2 v_2 + \cdots. \tag{15b}$$

In order to take into account approximately the competition between two effects, nonlinear steepening and dispersion, we introduce the stretched coordinates :

$$\xi_1 = \sqrt{\varepsilon}\,(x - ct - \sqrt{\varepsilon}\psi_1), \tag{16a}$$

$$\xi_2 = \sqrt{\varepsilon}\,(x + ct - \sqrt{\varepsilon}\psi_2), \tag{16b}$$

$$\tau = \sqrt{\varepsilon^3}\,t, \tag{16c}$$

where c is given in eq.(13c). The stretched variable ξ_1 (or ξ_2) represents the phase of soliton which travels in the positive (or negative) x-direction. The phase shifts ψ_1 and ψ_2 are due to the mutual interaction between waves moving right and left and considered as functions of ξ_1, ξ_2 and τ. Suppose that

$$R = \varepsilon r, \tag{17}$$

where r is at most of the order of unity.

Substituting eqs.(15) - (17) into eqs.(12), we obtain a set of equations to be solved for the successive powers of ε. In the lowest order, we get

$$v_1 + (c/n_0)n = f(\xi_1, \tau), \tag{18a}$$

$$v_1 - (c/n_0)n_1 = g(\xi_2, \tau). \tag{18b}$$

The functions f and g can be obtained from the non-secularity condition of the next order equations.

In the order $\varepsilon^{5/2}$, we have

$$\frac{\partial}{\partial \xi_2}[2cF+\{\frac{fg}{4} - \frac{g^2}{8} + \frac{P^2}{8c}\frac{\partial^2 g}{\partial \xi_2^2} - \frac{rc}{2Q}\mathcal{P}\int_{-\infty}^{\infty}\frac{g(\xi')}{\xi_2-\xi'}d\xi'\}]$$
$$+[\frac{\partial f}{\partial \tau} + \frac{3}{4}f\frac{\partial f}{\partial \xi_1} - \frac{P^2}{8c}\frac{\partial^3 f}{\partial \xi_1^3} + \frac{rc}{2Q}\mathcal{P}\int_{-\infty}^{\infty}\frac{d\xi'}{\xi_1-\xi'}\frac{\partial f}{\partial \xi'}]$$
$$+[\frac{1}{4}g - 2c\frac{\partial \psi_1}{\partial \xi_2}]\frac{\partial f}{\partial \xi_1} = 0, \tag{19a}$$

$$\frac{\partial}{\partial \xi_1}[2cG+\{-\frac{fg}{4} + \frac{f^2}{8} + \frac{P^2}{8c}\frac{\partial^2 f}{\partial \xi_1^2} - \frac{rc}{2Q}\mathcal{P}\int_{-\infty}^{\infty}\frac{f(\xi')}{\xi_1-\xi'}d\xi'\}]$$
$$+[\frac{\partial g}{\partial \tau} + \frac{3}{4}g\frac{\partial g}{\partial \xi_2} + \frac{P^2}{8c}\frac{\partial^3 g}{\partial \xi_2^3} - \frac{rc}{2Q}\mathcal{P}\int_{-\infty}^{\infty}\frac{d\xi'}{\xi_2-\xi'}\frac{\partial g}{\partial \xi'}]$$

$$-[\frac{1}{4}f + 2c\frac{\partial\psi_2}{\partial\xi_1}]\frac{\partial g}{\partial\xi_2} = 0, \tag{19b}$$

where

$$F = v_2 + (c/n_0)n_2, \tag{20a}$$

$$G = v_2 - (c/n_0)n_2. \tag{20b}$$

The functions ψ_1 and ψ_2 are chosen such that

$$\frac{\partial\psi_1}{\partial\xi_2} = \frac{g}{8c}, \qquad \frac{\partial\psi_2}{\partial\xi_1} = -\frac{f}{8c}. \tag{21}$$

Substituting eqs.(21) into eqs.(19a) and (19b) and imposing the condition that F and G are bounded, the non-securality condition, yields

$$\frac{\partial f}{\partial\tau} + \frac{3}{4}f\frac{\partial f}{\partial\xi_1} - \frac{P^2}{8c}\frac{\partial^3 f}{\partial\xi_1^{\ 3}} + \frac{rc}{2Q}\,\mathcal{P}\int_{-\infty}^{\infty}\frac{d\xi'}{\xi_1-\xi'}\frac{\partial f}{\partial\xi'} = 0, \tag{22a}$$

$$\frac{\partial g}{\partial\tau} + \frac{3}{4}g\frac{\partial g}{\partial\xi_2} + \frac{P^2}{8c}\frac{\partial^3 g}{\partial\xi_2^{\ 3}} - \frac{rc}{2Q}\,\mathcal{P}\int_{-\infty}^{\infty}\frac{d\xi'}{\xi_2-\xi'}\frac{\partial g}{\partial\xi'} = 0. \tag{22b}$$

These equations are the same as that obtained by Ott and Sudan[15)] in studying ion acoustic waves of finite amplitude with the linear Landau damping by electrons.

We note here that eq.(22b) reduces to eq.(22a) by the transformation $g \to -f$, $\tau \to -\tau$. If $r = 0$, eqs.(22a) and (22b) have soliton solutions with negative and positive amplitudes, respectively. For the case $r \neq 0$, the soliton moving to the right, f, damps as time increases and the soliton moving to the left, g, grows. This tendency agrees with the numerical solutions with the initial condition (14) (see Figs.3 and 4).

§ 4 Numerical Solutions 3

——— Evolution of a Periodic Modulation ———

Here we study the numerical solution to eq.(2) obtained with initial value

$$u(x,t=0) = 1 + B\cos(\pi x). \tag{23}$$

If the nonlinear Landau damping is disregarded, i.e., if $R = 0$, modulation goes as follows : For $PQ>0$, the modulation once grows, then damps, and the initial state is almost recovered. This process is repeated. When $PQ<0$, the modulation separates into two series of dark-solitons which propagate in the both directions, right and left, and after a recurrence time they focus at a common spatial point and almost reconstructed the initial state.

Under the effect of nonlinear Landau damping, the modulation behaves in an asymmetric way and the recurrence property is broken. In course of process, the envelope soliton($PQ>0$) or the dark-soliton ($PQ<0$) should play an important role. Especially in the case $PQ<0$, because the dark-solitons grow or damp depending on their direction of propagation, soliton formation can be seen more conspicuously.

4.1 For the case $PQ>0$, the numerical results are shown in Figs.5 and 6, in which the calculations are made for $B = 0.1$ and 0.2, respectively, with the parameters $P = 0.2$, $Q = 1$ and $R = 0$, 0.05, 0.1. Under the effect of nonlinear Landau damping, the periodic modulation grwos in an asymmetric way into a soliton-like pulse, which damps not as simply as shown in Fig.1 but pulsating because of periodic boundary conditions.

4.2 When $PQ<0$, the situation is quite different form the case $PQ>0$. In Figs.7 and 8, the numerical results for $B = 0.1$ and 0.2 are illustrated, where the parameters are taken as $P = -0.01$, $Q = 1$, $R = 0$, 0.05 and 0.1. For $R = 0$, one can see a tendency of dark-soliton formation. When $R \neq 0$, the growth of dark-solitons travelling to the left is strongly in evidence. The tendency is more clear for larger R. One can say that the periodically modulated plane wave becomes broken in fragments under the nonlinear Landau damping although the case $PQ<0$ is modulationally stable for $R = 0$. The results are understood from the discussions in §3, that the dark-soliton damps or grows according as it moves to the right or the left (note that the present case is $PR<0$).

§5 Summary

We solved numerically eq.(2) under various initial conditions. The obtained results are as follows :

1. For $PQ>0$, the envelope soliton damps deforming asymmetrically and changing the velocity, due to the nonlinear Landau damping. The bound state of solitons decays into a series of solitons, which behave themselves in a similar way.
2. For $PQ<0$, the dark-soliton displays a different character from the above envelope soliton. The dark-soliton damps or grows depending on its direction of propagation.

3. The periodic modulation for $PQ>0$ deforms asymmetrically developing into an envelope soliton which damps due to the nonlinear Landau damping effect.

4. The periodically modulated plane wave, for $PQ<0$, decays into a series of growing dark-solitons and becomes broken in fragments. In early stage, the behaviours of these numerical solutions can be interpretted by a linear analysis of instability and the reductive perturbation method.

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Appendix

To solve eq.(2) numerically, we replace the partial derivatives by the central difference quotients,

$$\partial u/\partial y \rightarrow [u(x,t+\Delta t)-u(x,t-\Delta t)]/(2\Delta t),$$

$$\partial^2 u/\partial x^2 \rightarrow [u(x+\Delta x,t)-2u(x,t)+u(x-\Delta x,t)]/\Delta x^2 \quad ,$$

where Δt and Δx are mesh size in the x-t space.

The integral term in eq.(2) is approximated as

$$\mathcal{P}\int_{-\infty}^{\infty} dx'\frac{f(x')}{x-x'} = -6\Delta x\left[\frac{df}{dx} + \frac{\Delta x^2}{2}\frac{d^3 f}{dx^3} + \frac{27}{200}\Delta x^4\frac{d^5 f}{dx^5}\right] + \left[\int_{-\infty}^{x-3\Delta x} + \int_{x+3\Delta x}^{\infty}\right]dx'\frac{f(x')}{x-x'} \ .$$

In deriving the above expression f(x') is expanded in Taylor series of (x'-x) for the region $x-3\Delta x \leqq x' \leqq x+3\Delta x$. The differential coefficients are approximated by the seven-points difference quotients and the integrals over the residual intervals are calculated by using Weddle's formula.

Figure Captions

Fig.1 Time development of solution for the initial condition (8) with A=1. (a) R=0.2 and (b) R=0.5. In both cases, P=Q=1.

Fig.2 Time development of solution for the initial condition (10) with A=1. (a) R=0, (b) R=0.2 and (c) R=0.5. In all cases, P=Q=1.

Fig.3 Time development of solution for the initial condition (14) with - sign. A=0.1, P=-1, Q=1 and R=0.5.

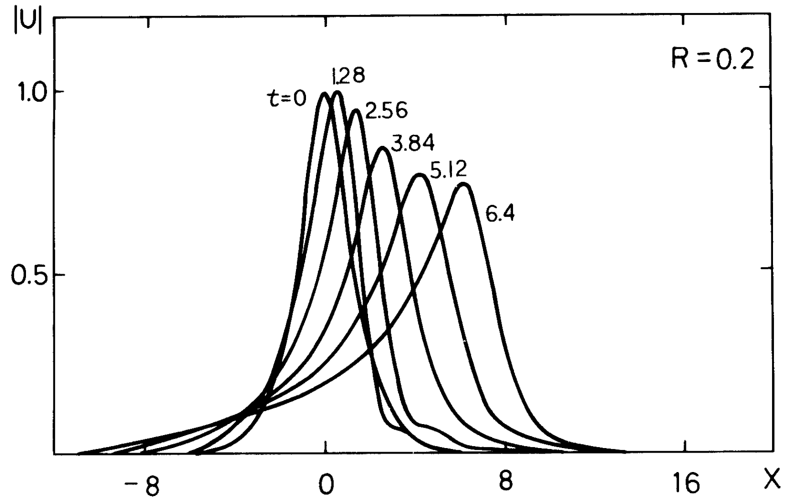
Fig.4 Time development of solution for the initial condition (14) with + sign. A=0,1, P=-1, Q=1 and R=0.5.

Fig.5 Evolution of periodic modulation (23) with B=0.1 (PQ>0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=0.2 and Q=1)

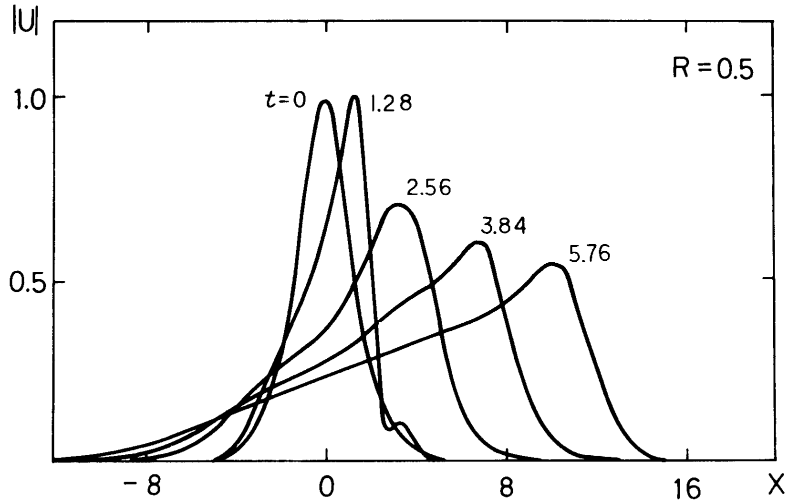
Fig.6 Evolution of periodic modulation (23) with B=0.2 (PQ>0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=0.2 and Q=1)

Fig.7 Evolution of periodic modulation (23) with B=0.1 (PQ<0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=-0.01 and Q=1)

Fig.8 Evolution of periodic modulation (23) with B=0.2 (PQ<0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=-0.01 and Q=1)

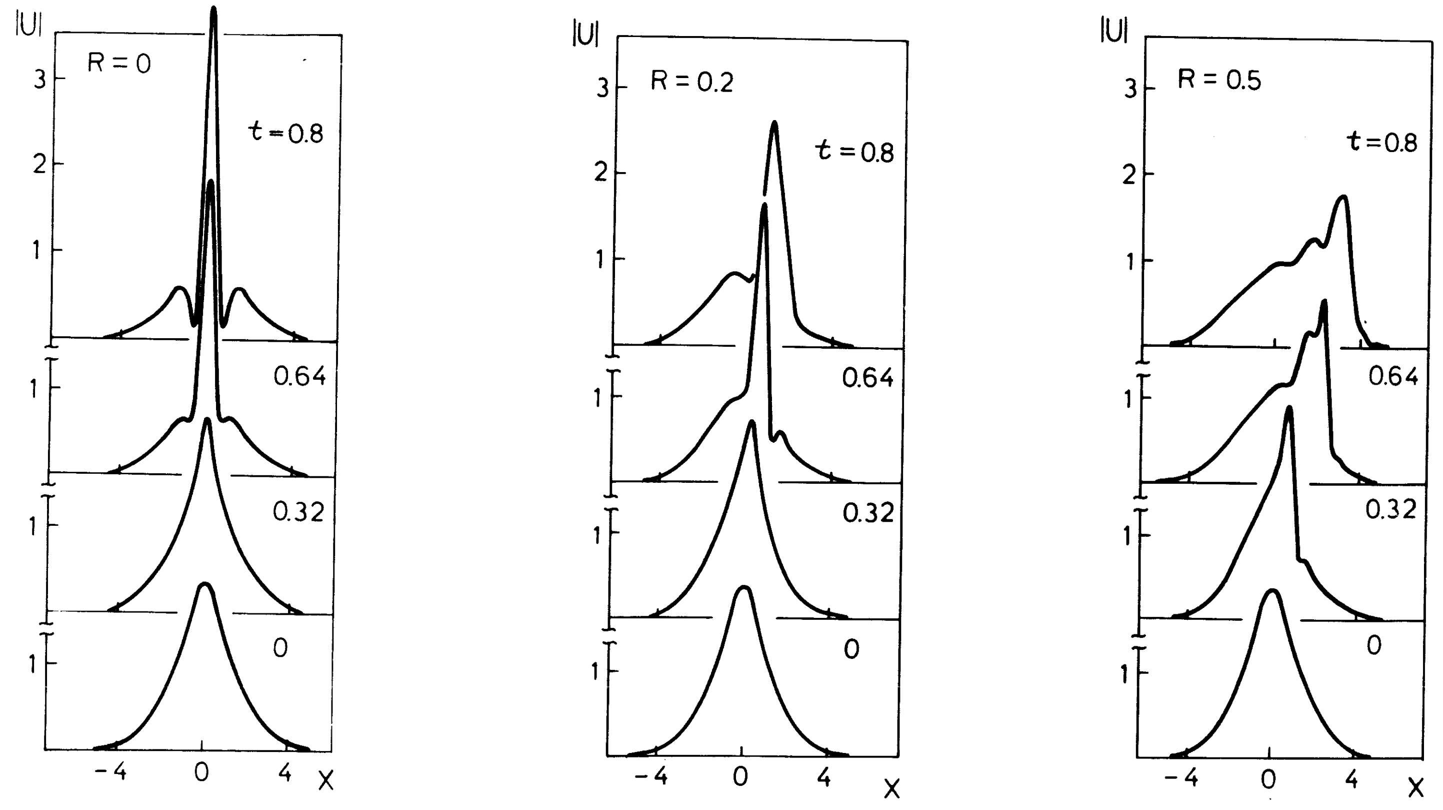


(a)



(b)

Fig.1 Time development of solution for the initial condition (8) with A=1. (a) R=0.2 and (b) R=0.5. In both cases, P=Q=1.



(a) (b) (c)

Fig.2 Time development of solution for the initial condition (10) with A=1. (a) R=0, (b) R=0.2 and (c) R=0.5. In all cases, P=Q=1.

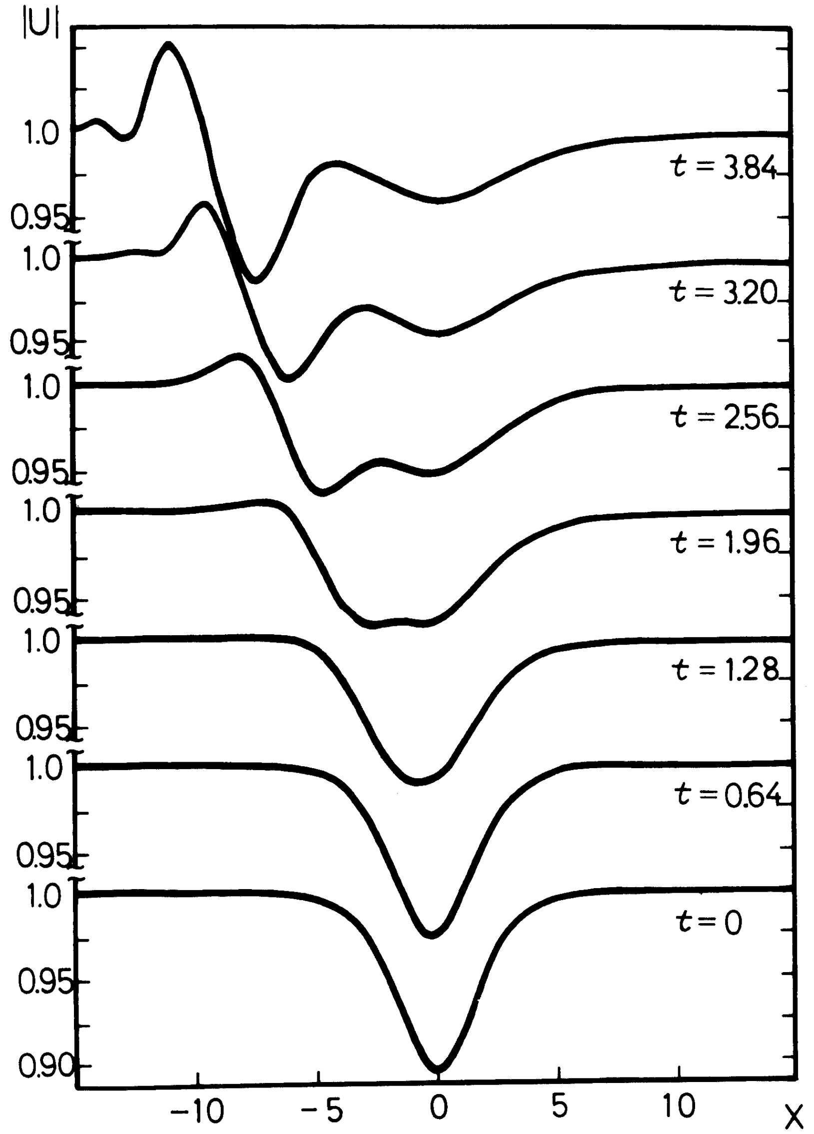


Fig.3 Time development of solution for the initial condition (14) with - sign. A=0.1, P=-1, Q=1 and R=0.5.

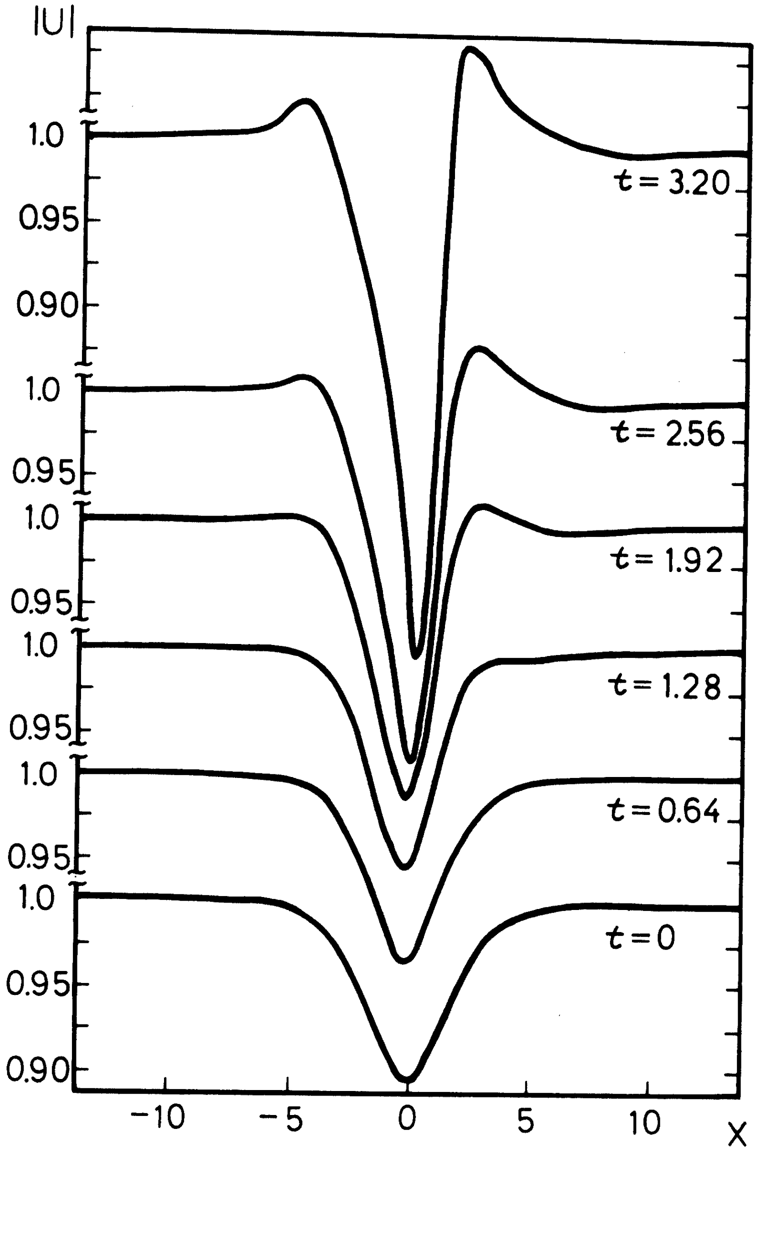


Fig.4 Time development of solution for the initial condition (14) with + sign. A=0,1, P=-1, Q=1 and R=0.5.

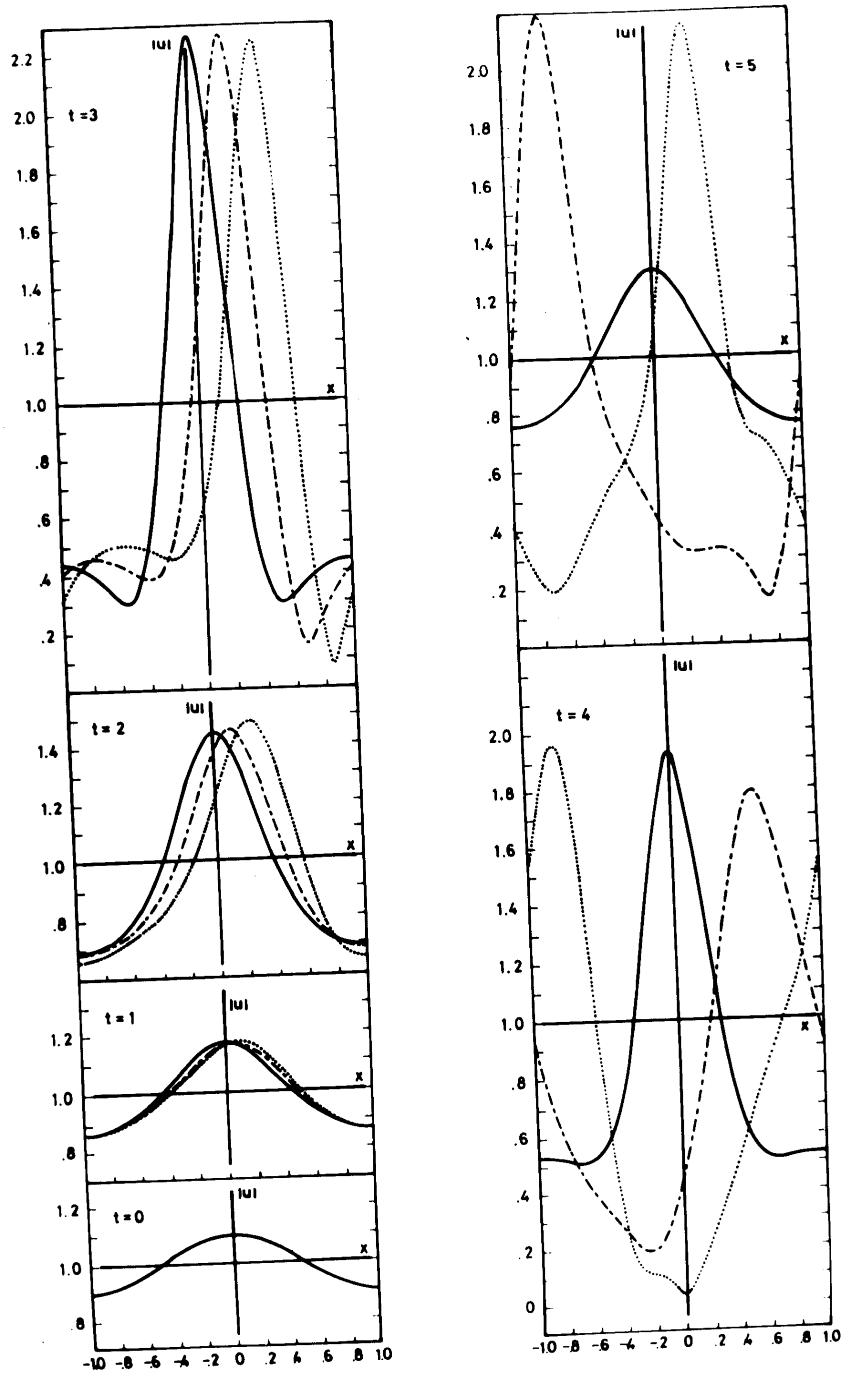


Fig.5 Evolution of periodic modulation (23) with B=0.1 (PQ>0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=0.2 and Q=1)

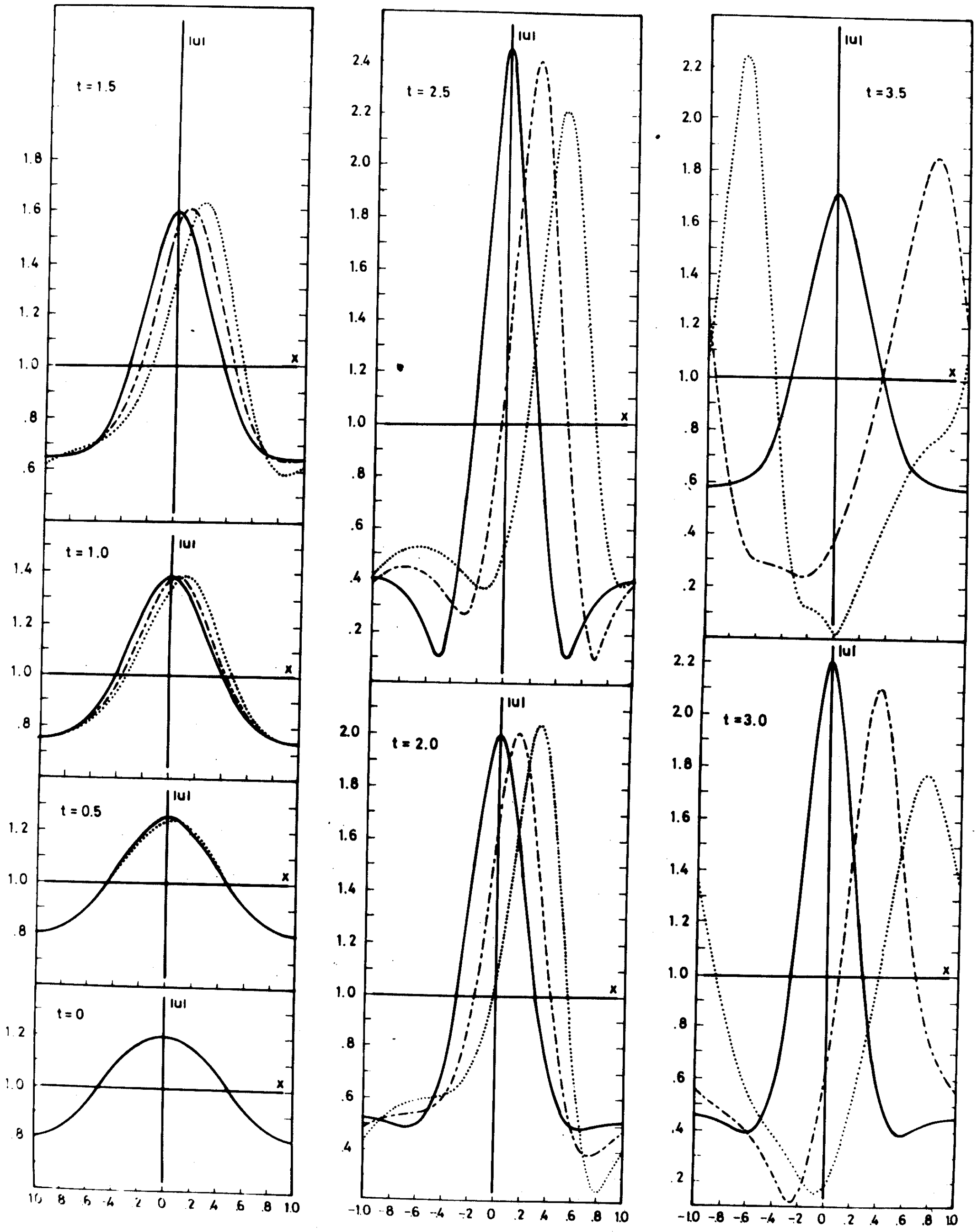


Fig.6 Evolution of periodic modulation (23) with B=0.2 (PQ>0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=0.2 and Q=1)

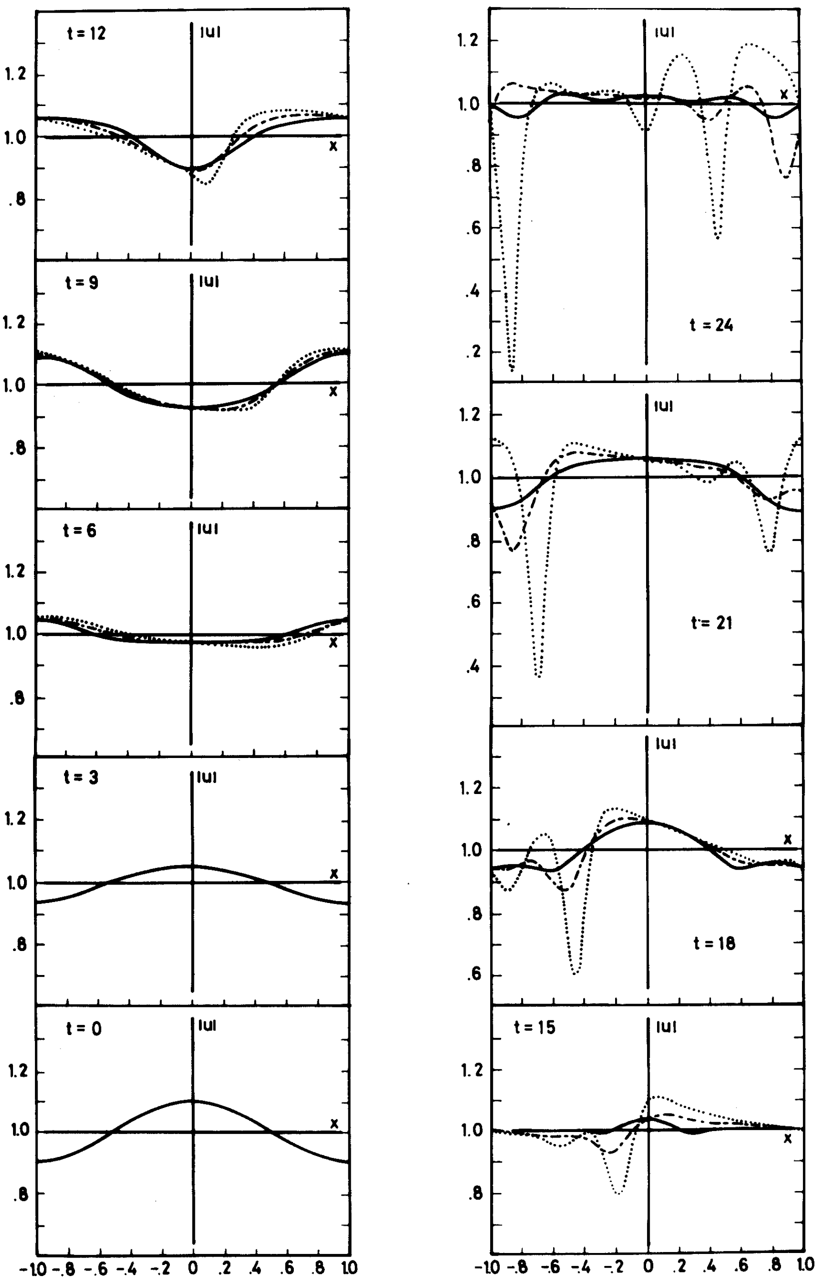


Fig.7 Evolution of periodic modulation (23) with B=0.1 (PQ<0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=-0.01 and Q=1)

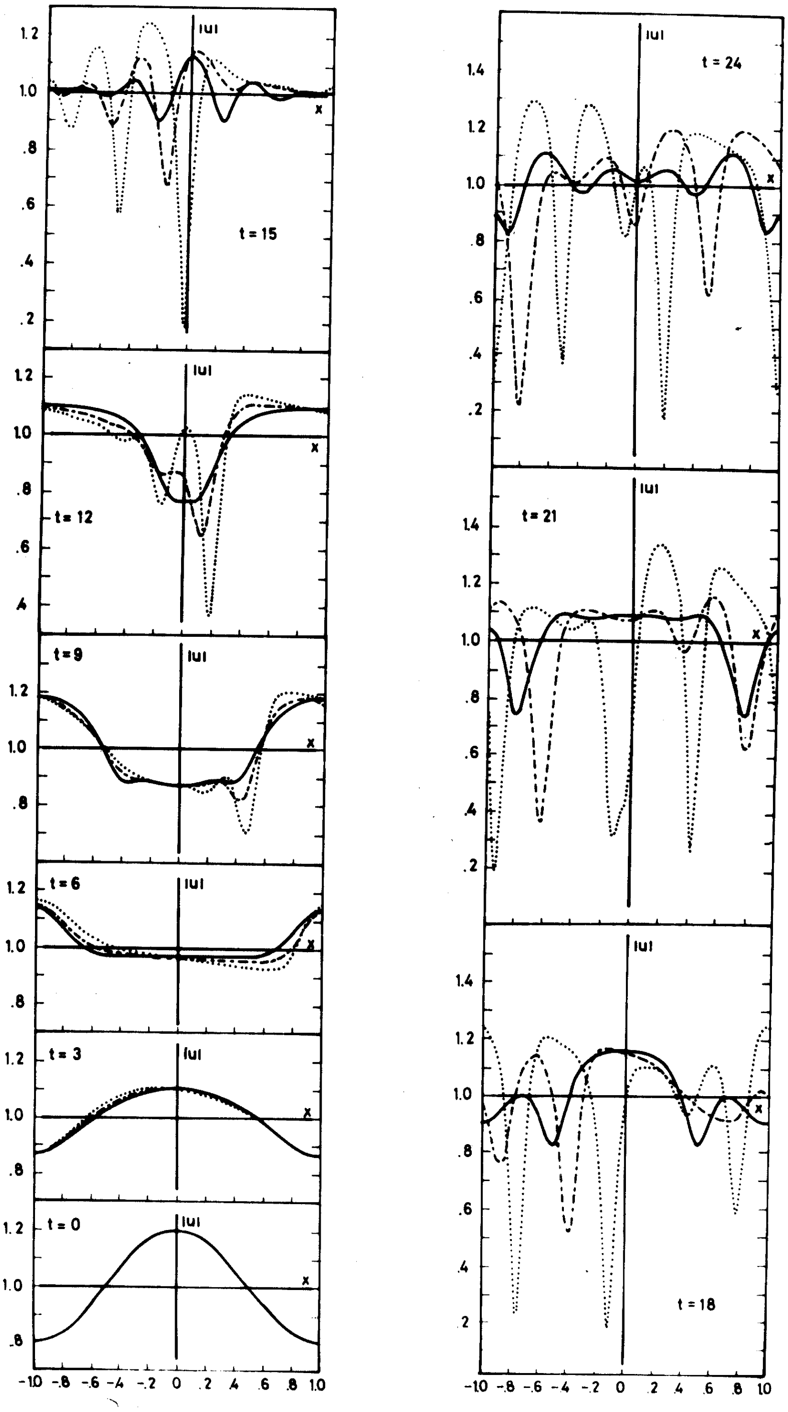


Fig.8 Evolution of periodic modulation (23) with B=0.2 (PQ< 0). The solid line corresponds to the case R=0, the broken line to R=0.05 and the dotted line to R=0.1. (P=-0.01 and Q=1)