INSTITUTE OF PLASMA PHYSICS
NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Application of Linear Response Theory to Dielectric and Conduction Problems in a Turbulent Plasma

Mitsuo Kono and Nobuo Yajima

IPPJ-325 March 1978

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

Permanent address : Research Institute for Applied Mechnics,
Kyushu University, Fukuoka, 812

Abstract

The dielectric function in a turbulent plasma is derived based on the linear response theory, in which a summation of the infinite series is made in order to get the pair-correlational effects exactly. This infinite sum can be performed with the aid of diagram techniques and reproduces the dielectric function obtained before by Kono and Yajima. The complex conductivity is also derived by using a similar diagram technique and rightly agrees with that obtained kinetic-theoretically by Nishikawa and Ichikawa.

This work is motivated by Ichimaru's comment in the January issue (1978) of J. Phys. Soc. Japan.

§1. Introduction

In a previous paper[1], we have investigated the propagation of a test wave in a current-driven ion-acoustic wave turbulence, showing that the phase velocity of a test wave decreases from the value in a quiescent plasma and there appears the secondary instability associated with the decrease of the phase velocity. This result is different from that of Ichimaru and Tange[2]. Ichimaru asserts in his comment[3] that the origin of the discrepancy is traced to ad hoc and unjustified assumptions and to neglect of the vertex corrections in our analysis. In opposition to Ichimaru's assertion, however, the discrepacy is originated from Ichimaru and Tange's incomplete calculations of the pair-correlational effects. Ichimaru and Tange carried out a high-frequency asymptotic expansion of the susceptibility

$$\chi(\mathbf{q},\omega) = \sum_{\ell=1}^{\infty} < \omega^{\ell-1} > /\omega^{\ell},$$

where

$$< \omega^{\ell-1}> = -\int_{-\infty}^{\infty} d\omega \ \omega^{\ell} \mathrm{Im}\chi(\mathbf{q},\omega),$$

and retained terms up to $\ell = 4$. However, the ℓ-th frequency moment is not equivalent to the $(\ell/2)$-th cumulant, but involves the contributions from all the correlations up to $(\ell/2)$-th order. Thus, in applying the sum-rule analysis to the dispersion relations in turbulent plasmas, one should make a summation of the infinite series to get the pair-correlational effects exactly. This infinite sum can be performed with the aid of diagram techniques and reproduces the dielectric function obtained before by Kono and Yajima(§2). Such a work of summing up an infinite series is overcome by regarding waves as dynamical variables. In §3, the complex conductivity is derived with the linear response analysis by using a similar diagram technique, leading to the result obtained kinetic=theoretically by Nishikawa and Ichikawa[4].

§2. Dielectric function in a turbulent plasma

The dielectric function $\varepsilon(\mathbf{q},\omega)$ of the plasma is defined in terms of the retarded density-density response function

$$\chi_{\alpha\beta}(\mathbf{q},\omega) = \frac{1}{i\hbar}\Phi_{\alpha\beta}(\mathbf{q})\int_0^\infty d\tau\ e^{i\omega\tau} < [\rho_\alpha(\mathbf{q},\tau),\rho_\beta^+(\mathbf{q})] >_0, \qquad (1)$$

as

$$\varepsilon^{-1}(\mathbf{q},\omega) = 1 + \sum_{\alpha}\sum_{\beta} \chi_{\alpha\beta}(\mathbf{q},\omega), \qquad (2)$$

where α and β denote the species of particles, and

$$\rho_\alpha(\mathbf{q}) = \sum_j e^{-i\mathbf{q}\mathbf{r}_{\alpha j}}, \qquad \rho_\alpha(\mathbf{q},\tau) = e^{i\mathcal{H}\tau/\hbar}\rho_\alpha(\mathbf{q})e^{-i\mathcal{H}\tau/\hbar},$$

$$\mathcal{H} = H_0 + H_I, \qquad H_0 = -\sum_{\alpha}\sum_{j} \frac{\hbar^2}{2m_\alpha}\nabla_{\alpha j}^2,$$

$$H_I = (1/2)\sum_{\alpha}\sum_{\beta}\sum_{\mathbf{k}} \Phi_{\alpha\beta}(\mathbf{k})[\ \rho_\alpha(\mathbf{k})\ \rho_\beta^+(\mathbf{k}) - N\delta_{\alpha\beta}\ ],$$

$$\Phi_{\alpha\beta}(\mathbf{k}) = 4\pi e_\alpha e_\beta/k^2, \qquad [\ A\ ,\ B\ ] = AB - BA.$$

The braket $\langle\cdots\rangle_0$ denotes an ensemble average in a sense of Klimontvich.

The main task is to calculate the r.h.s. of eq.(1) with the aid of the perturbation expansion. By using the following relations,

$$e^{\lambda H_0}\rho_\alpha(\mathbf{q})e^{-\lambda H_0} = \sum_j e^{-i\mathbf{q}\cdot(\mathbf{r}_{\alpha j} - \lambda\frac{\hbar^2}{m_\alpha}\nabla_{\alpha j})} = \hat{\rho}_\alpha(\mathbf{q},\lambda\mathbf{q}),$$

$$e^{\lambda\mathcal{H}} = e^{\lambda H_0}\ \mathrm{Pexp}[\ -\int_0^\lambda d\lambda_1\ H_I(\lambda_1)],$$

where $\lambda = (i/\hbar)t$, $H_I(\lambda) = e^{\lambda H_0}H_I e^{-\lambda H_0}$ and P indicates the chronological operator, the density fluctuation in the Heisenberg representation reduces to the form

$$\rho_\alpha(\mathbf{q},\lambda) = \sum_{n=0}^{\infty}\rho_\alpha^{(n)}(\mathbf{q},\lambda), \tag{3.1}$$

$$\rho_\alpha^{(n)}(\mathbf{q},\lambda) = \int_0^\lambda d\lambda_1\int_0^{\lambda_1}d\lambda_2\cdots\int_0^{\lambda_{n-1}}d\lambda_n[H_I(\lambda_n),[H_I(\lambda_{n-1}),[\cdots$$
$$[H_I(\lambda_1),\hat{\rho}_\alpha(\mathbf{q},\lambda\mathbf{q})]\cdots]. \tag{3.2}$$

Corresponding to the expansion(3), we put

$$\chi(\mathbf{q},\omega) = \sum_{n=0}^{\infty}\chi^{(n)}(\mathbf{q},\omega) = \sum_n^{\infty}\sum_\alpha\sum_\beta\chi_{\alpha\beta}^{(n)}(\mathbf{q},\omega), \tag{4}$$

$$\chi_{\alpha\beta}^{(n)}(\mathbf{q},\omega) = (1/i\hbar)\Phi_{\alpha\beta}(\mathbf{q})\int_0^\infty d\tau e^{i\omega\tau}<[\rho_\alpha^{(n)}(\mathbf{q},\lambda) ,\rho_\beta^{+}(\mathbf{q})]>_0.$$

Noting the commutation relation

$$[\hat{\rho}_\alpha(\mathbf{k},\lambda\mathbf{k}),\hat{\rho}_\beta(\mathbf{k}_1,\lambda_1\mathbf{k}_1)] = = \delta_{\alpha\beta}2\sinh[(\lambda-\lambda_1)\hbar^2\mathbf{k}\cdot\mathbf{k}_1/2m_\alpha]\hat{\rho}_\alpha(\mathbf{k}+\mathbf{k}_1,\lambda\mathbf{k}+\lambda_1\mathbf{k}_1), \quad (5)$$

which is easily examined by direct calculations, we obtain

$$\chi_{\alpha\beta}^{(0)}(\mathbf{q},\omega) = (1/i\hbar)\Phi_{\alpha\beta}(\mathbf{q})\int_0^\infty d\tau\ e^{i\omega\tau}\delta_{\alpha\beta}2\sinh(-\lambda\frac{\hbar^2q^2}{2m_\alpha})<\hat{\rho}_\alpha(0,\lambda\mathbf{q})>_0$$

$$= (1/i\hbar)\Phi_{\alpha\beta}(\mathbf{q})\delta_{\alpha\beta}\int_0^\infty d\tau\ e^{i\omega\tau}<\sum_j 2\sinh(\frac{\hbar}{2m_\alpha}\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{v}_{\alpha j}})e^{-i\mathbf{q}\cdot\mathbf{v}_{\alpha j}\tau}>_0.$$

In the classical limit $\hbar \to 0$, we have

$$\chi_{\alpha\beta}^{(0)}(\mathbf{q},\omega) = \delta_{\alpha\beta}\Phi_{\alpha\beta}(\mathbf{q})<\sum_j\frac{\mathbf{q}}{m_\alpha}\cdot\frac{\partial}{\partial\mathbf{v}_{\alpha j}}\frac{1}{\omega-\mathbf{q}.\mathbf{v}_{\alpha j}}>_0.$$

Introducing a distribution function

$$F_\alpha(\mathbf{r},\mathbf{v}) = \sum_j \delta(\mathbf{r}-\mathbf{r}_{\alpha j})\delta(\mathbf{v}-\mathbf{v}_{\alpha j}), \quad (6)$$

gives

$$\chi_{\alpha\beta}^{(0)} = -\delta_{\alpha\beta}\Phi_{\alpha\beta}(\mathbf{q})\int_0^\infty d\mathbf{v}\ \frac{1}{\omega-\mathbf{q}\mathbf{v}}\ \frac{\mathbf{q}}{m_\alpha}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial\mathbf{v}}, \quad (7)$$

where $<F_\alpha(\mathbf{r},\mathbf{v})>_0 = F_\alpha(\mathbf{v})$.

Here, we define the linear dielectric function as

$$\varepsilon^{(0)}(\mathbf{q},\omega) = 1 - \sum_{\alpha}\sum_{\beta}\chi_{\alpha\beta}^{(0)}(\mathbf{q},\omega). \quad (8)$$

Proceeding to the next step, we find

$$[H_I(\lambda_1), \hat{\rho}_\alpha(\mathbf{q},\lambda\mathbf{q})] = \sum_{\gamma}\sum_{\mathbf{k}}\Phi_{\alpha\gamma}(\mathbf{k})\, 2\sinh[(\lambda-\lambda_1)\hbar^2\mathbf{q}\cdot\mathbf{k}/2m_\alpha]\times$$

$$\times\{\hat{\rho}_\alpha(\mathbf{q}-\mathbf{k},(\lambda-\lambda_1)\mathbf{q}+\lambda_1(\mathbf{q}-\mathbf{k}))\hat{\rho}_\gamma(\mathbf{k},\lambda_1\mathbf{k}) +$$

$$+ \delta_{\alpha\gamma}\sinh[-(\lambda-\lambda_1)\hbar^2\mathbf{q}\cdot\mathbf{k}/2m_\alpha]\hat{\rho}_\alpha(\mathbf{q},\lambda\mathbf{q})\}. \qquad (9)$$

The second term in a brace of the r.h.s. of eq.(9) has no contribution to $\chi^{(1)}_{\alpha\beta}$ for the classical limit. From eq.(9), $\chi^{(1)}$ is obtained as

$$\chi^{(1)}(\mathbf{q},\omega) = (1/i\hbar)\sum_{\alpha}\sum_{\beta}\Phi_{\alpha\beta}(\mathbf{q})\int_0^\infty d\tau\; e^{i\omega\tau}\sum_{\gamma}\sum_{\mathbf{k}}\Phi_{\alpha\gamma}(\mathbf{k})\;\times$$

$$\times\int_0^\lambda d\lambda_1\;\; 2\sinh[(\lambda-\lambda_1)\hbar^2\mathbf{q}\cdot\mathbf{k}/2m_\alpha]\;\times$$

$$\times\;\langle\sum_{i}\sum_{j}\{\delta_{\alpha\beta}\; e^{i\mathbf{k}\cdot(\mathbf{r}_{\alpha i}-\mathbf{r}_{\gamma j})}2\sinh[(\hbar/2m_\alpha)\mathbf{q}\cdot\partial/\partial\mathbf{v}_{\alpha i}] +$$

$$+\;\delta_{\gamma\beta}e^{i(\mathbf{k}-\mathbf{q})\cdot(\mathbf{r}_{\alpha i}-\mathbf{r}_{\gamma j})}2\sinh[(\hbar/2m_\gamma)\mathbf{q}.\partial/\partial\mathbf{v}_{\gamma j}]\}\;\times$$

$$\times\;\exp[-\hbar\lambda\mathbf{q}\cdot\mathbf{v}_{\alpha i} + \hbar\lambda_1\mathbf{k}\cdot(\mathbf{v}_{\alpha i}-\mathbf{v}_{\gamma j})]\rangle_0$$

$$\xrightarrow[\hbar\to 0]{}\;\sum_{\alpha}\sum_{\beta}\Phi^2_{\alpha\beta}(\mathbf{q})\int d\mathbf{v}\;\frac{-1}{\omega-\mathbf{q}\cdot\mathbf{v}}\;\frac{\mathbf{q}}{m_\alpha}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial\mathbf{v}}\int d\mathbf{v}_1\frac{-1}{\omega-\mathbf{q}\cdot\mathbf{v}_1}\;\frac{\mathbf{q}}{m_\beta}\cdot\frac{\partial F_\beta(\mathbf{v}_1)}{\partial\mathbf{v}_1}\;+$$

$$+\;\sum_{\alpha}\sum_{\beta}\sum_{\gamma}\sum_{\mathbf{k}}\Phi_{\alpha\beta}(\mathbf{k})\Phi_{\alpha\gamma}(\mathbf{k})\int d\mathbf{v}\int d\mathbf{v}_1\frac{-1}{\omega-\mathbf{q}\cdot\mathbf{v}}\;\frac{\mathbf{k}}{m_\alpha}\cdot\frac{-1}{\omega-(\mathbf{q}-\mathbf{k})\cdot\mathbf{v}-\mathbf{k}\cdot\mathbf{v}_1}\times$$

$$\times\;\{\delta_{\alpha\beta}(\mathbf{q}/m_\alpha)\cdot\frac{\partial}{\partial\mathbf{v}}G_{\alpha\gamma}(-\mathbf{k};\mathbf{v},\mathbf{v}_1) + \delta_{\gamma\beta}(\mathbf{q}/m_\beta)\cdot\frac{\partial}{\partial\mathbf{v}_1}G_{\alpha\gamma}(\mathbf{q}-\mathbf{k};\mathbf{v},\mathbf{v}_1)\}, \qquad (10)$$

where we have introduced the correlation function

$$G_{\alpha\beta}(\mathbf{r}-\mathbf{r}_1;\mathbf{v},\mathbf{v}_1) = \langle F_\alpha(\mathbf{r},\mathbf{v})F_\beta(\mathbf{r}_1,\mathbf{v}_1)\rangle_0 -$$

$$-\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}_1)\delta(\mathbf{v}-\mathbf{v}_1)F_\alpha(\mathbf{v}) - F_\alpha(\mathbf{v})F_\gamma(\mathbf{v}_1). \qquad (11)$$

It should be noted that eqs.(7) and (10) are all calculated by Ichimaru and Tange.

Here, we notice that if we restrict our interest to the pair-correlational effects of turbulent waves on the dielectric function, the polarizability can be systematically calculated with the aid of diagrams listed in Table 1.

Table 1

Equation(10) is represented by diagrams as shown in Fig.1.

Figure 1

The construction rules of diagrams for $\chi^{(n)}(\mathbf{q},\omega)$ are quite simple. First write down the schematic figures for $\rho_\alpha^{(n)}(\mathbf{q},\lambda)$ which are easily anticipated from eqs.(3.1) and (5). For example, we find Fig.2-(a) for $\rho_\alpha^{(1)}$ and Fig.2-(b),(c) for $\rho_\alpha^{(2)}$.

Figure 2

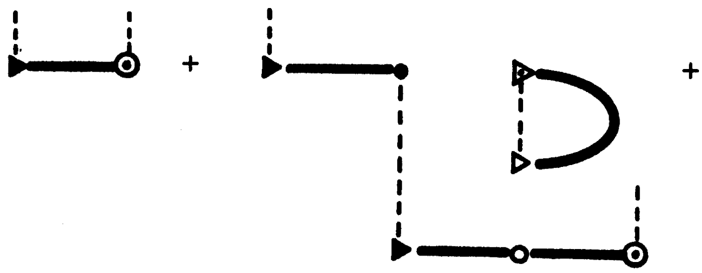
Next, as is clear from the correspondence of Fig.2-(a) to Fig.1, add external lines to the end of the solid line in the figure for $\rho_\alpha^{(n)}$, take an ensemble average and retain only contributions from the pair-correlations by using the Mayer cluster expansion. Diagrams for $\chi_\alpha^{(2)}$ corresponding to $\rho_\alpha^{(2)}$ are shown in Fig.3.

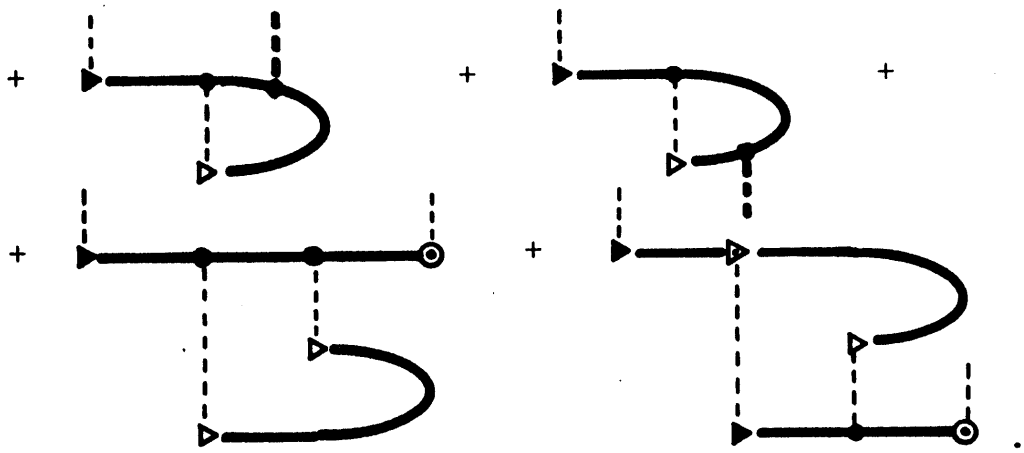
Figure 3

The disconnected diagram in Fig.3-(b) represents the renormalization of the pair-correlatinal effects to the "bare" distribution function(distribution function in a quiescent plasma). If $<F_\alpha(\mathbf{r},\mathbf{v})>_0$ is taken as the "dressed" distribution function (distribution function in the stationary turbulent state), this disconnected diagram must be discarded in order to avoid double counting. On the contrary, if $<F_\alpha(\mathbf{r},\mathbf{v})>_0$ is regarded as the "bare" distribution function, the disconnected diagram should be included. It is shortly examined that $\chi^{(3)}$ consists of forty-two topologically different diagrams.

Now, we are ready to make a summation of infinite series of the expansion(4). The diagramatic equation for $\chi(\mathbf{q},\omega)$ is written as

$\chi(q,\omega)$ =

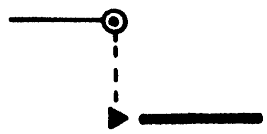




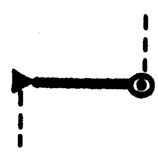
(12)

where the renormalized propagator is defined

 $= P_\alpha(\mathbf{k},\mathbf{v},\omega)$

$=$ $+$  . (13.1)

and the renormalized external line is defined

 $=$ $+$  . (14.1)

Equation (13.1) for the renormalized propagator $P_\alpha(\mathbf{q},\mathbf{v},\omega)$ is represented as

$$P_\alpha(\mathbf{k},\mathbf{v},\omega) = \frac{-1}{\omega-\mathbf{k}\cdot\mathbf{v}} + \frac{-1}{\omega-\mathbf{k}\cdot\mathbf{v}}\frac{e_\alpha}{m_\alpha}\frac{\mathbf{k}}{k^2}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial \mathbf{v}}\sum_\gamma 4\pi e_\gamma\int d\mathbf{v}_1 P_\gamma(\mathbf{k},\mathbf{v}_1,\omega). \quad (13.2)$$

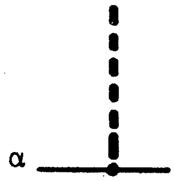
This can be solved as

$$P_\alpha(\mathbf{k},\mathbf{v},\omega) = \frac{-1}{\omega-\mathbf{k}\cdot\mathbf{v}}\{1 - \frac{1}{\varepsilon^{(0)}(\mathbf{k},\omega)}\frac{e_\alpha}{m_\alpha}\frac{\mathbf{k}}{k^2}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial \mathbf{v}}\sum_\alpha 4\pi e_\alpha\int d\mathbf{v}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}}\}, \quad (15)$$

where the expansion formula for operators

$$\frac{1}{A+B} = \frac{1}{A} + \frac{1}{A}B\,\frac{1}{A} + \cdots,$$

has been used. For the renormalized external line, we have from eqs.(14.1) and (15),



$$= \sum_\beta (4\pi)^{1/2}\frac{e_\beta}{q}\delta_{\alpha\beta}\frac{1}{\varepsilon^{(0)}(\mathbf{q},\omega)}\frac{\mathbf{q}}{m_\alpha}\cdot\frac{\partial}{\partial \mathbf{v}}. \quad (14.2)$$

As is noted before, since we restrict ourselves to the pair-correlational effects of turbulent waves on the dispersion relation, we may use the linear eigen-frequency for the turbulent waves in calculating the right hand side of eq.(12) except for the first term. Thus, we may assume for a stationary turbulence

$$G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}_1) = \frac{e_\alpha}{m_\alpha}\frac{-1}{\omega(\mathbf{k})-\mathbf{k}\cdot\mathbf{v}}\frac{\mathbf{k}}{k^2}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial \mathbf{v}}\sum_\gamma 4\pi e_\gamma\int d\mathbf{v}_2 G_{\gamma\beta}(\mathbf{k},\mathbf{v}_2,\mathbf{v}_1), \quad (16)$$

where $\omega(\mathbf{k})$ is determined by $\varepsilon^{(0)}(\mathbf{k},\omega(\mathbf{k})) = 0$.

Here, we summarize the identities proved straightfowardly for the convenience of following calculations.

$$+ \frac{\tilde{v}_1(\mathbf{q},\omega;\mathbf{k},\omega(\mathbf{k}))\tilde{v}_1(\mathbf{q}-\mathbf{k},\omega-\omega(\mathbf{k});\mathbf{q},\omega)}{\varepsilon^{(0)}(\mathbf{q}-\mathbf{k},\omega-\omega(\mathbf{k}))}\} \, U(\mathbf{k}), \qquad (18.2)$$

where we have used the abbreviations;

$$\tilde{v}_1(\mathbf{k},\omega;\mathbf{k}',\omega') = \sum_\alpha \frac{4\pi e_\alpha^2}{m_\alpha k^2} \frac{e_\alpha}{m_\alpha} \int d\mathbf{v} \, \frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}} \mathbf{k}'\cdot\frac{\partial}{\partial\mathbf{v}} \frac{1}{\omega-\omega'-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}} \times$$

$$\times \; (\mathbf{k}-\mathbf{k}')\cdot\frac{\partial}{\partial\mathbf{v}} F_\alpha(\mathbf{v}),$$

$$\tilde{v}_2(\mathbf{k},\omega;\mathbf{k}',\omega';\mathbf{k}'',\omega'') = -\sum_\alpha \frac{4\pi e_\alpha^2}{m_\alpha k^2}(e_\alpha/m_\alpha)^2 \int d\mathbf{v} \, \frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}} \mathbf{k}'\cdot\frac{\partial}{\partial\mathbf{v}} \times \qquad (19)$$

$$\times \; \mathbf{k}'\cdot\frac{\partial}{\partial\mathbf{v}} \frac{1}{\omega-\omega'-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}} \, \mathbf{k}''\cdot\frac{\partial}{\partial\mathbf{v}} \frac{1}{\omega-\omega'-\omega''-(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\cdot\mathbf{v}} \times$$

$$\times \; (\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\cdot\frac{\partial}{\partial\mathbf{v}} F_\alpha(\mathbf{v}),$$

$$U(\mathbf{k}) = \sum_\alpha\sum_\beta \frac{4\pi e_\alpha}{k^2} \frac{4\pi e_\beta}{k^2} \int d\mathbf{v} \int d\mathbf{v}' \; G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}').$$

The other terms of eq.(12) can be calculated in a similar way, and we put them in Appendix.

The result is now written down as

$$\chi(\mathbf{q},\omega) = \frac{1-\varepsilon^{(0)}(\mathbf{q},\omega)}{\varepsilon^{(0)}(\mathbf{q},\omega)} + \frac{1}{[\varepsilon^{(0)}(\mathbf{q},\omega)]^2} \sum_{\mathbf{k}} \{V_2(\mathbf{q},\omega;\mathbf{k},\omega(\mathbf{k});-\mathbf{k},-\omega(\mathbf{k})) +$$

$$+ \frac{V_1(\mathbf{q},\omega;\mathbf{k},\omega(\mathbf{k}))V_1(\mathbf{q}-\mathbf{k},\omega-\omega(\mathbf{k});\mathbf{q},\omega)}{\varepsilon^{(0)}(\mathbf{q}-\mathbf{k},\omega-\omega(\mathbf{k}))}\} \, U(\mathbf{k}), \qquad (20)$$

$$P_\alpha(\mathbf{k},\mathbf{v},\omega)G_{\alpha\beta}(\mathbf{k},\mathbf{v},\mathbf{v}_1) = \frac{-1}{\omega-\omega(\mathbf{k})}G_{\alpha\beta}(\mathbf{k},\mathbf{v},\mathbf{v}_1), \tag{17.1}$$

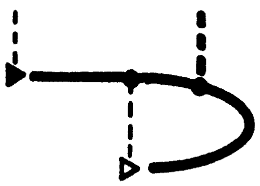
$$\sum_\alpha 4\pi e_\alpha\int d\mathbf{v}P_\alpha(\mathbf{k},\mathbf{v},\omega)A_\alpha(\mathbf{v}) = \frac{1}{\varepsilon^{(0)}(\mathbf{k},\omega)}\sum_\alpha 4\pi e_\alpha\int d\mathbf{v}\,\frac{-1}{\omega-\mathbf{k}\cdot\mathbf{v}}A_\alpha(\mathbf{v}), \tag{17.2}$$

$$P_\alpha(\mathbf{k},\mathbf{v},\omega)\cdot\frac{\mathbf{k}}{m_\alpha}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial\mathbf{v}}e_\alpha\delta_{\alpha\beta} = \frac{1}{\varepsilon^{(0)}(\mathbf{k},\omega)}\,\frac{-1}{\omega-\mathbf{k}\cdot\mathbf{v}}\,\frac{\mathbf{k}}{m_\alpha}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial\mathbf{v}}e_\beta\delta_{\alpha\beta}. \tag{17.3}$$

Then the r.h.s.of eq.(12) is calculated as follows;

$$= \sum_{\alpha\beta}(4\pi)^{1/2}\frac{e_\alpha}{q}\int d\mathbf{v}\;P_\alpha(\mathbf{q},\mathbf{v},\omega)\frac{\mathbf{q}}{m_\alpha}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial\mathbf{v}}\delta_{\alpha\beta}(4\pi)^{1/2}\frac{e_\beta}{q}$$

$$= \frac{1-\varepsilon^{(0)}(\mathbf{q},\omega)}{\varepsilon^{(0)}(\mathbf{q},\omega)}, \tag{18.1}$$



$$= \sum_{\alpha\beta}(4\pi)^{1/2}\frac{e_\alpha}{q}\int d\mathbf{v}\;P_\alpha(\mathbf{q},\mathbf{v},\omega)\sum_{\mathbf{k}}\frac{\mathbf{k}}{m_\alpha}\cdot\frac{\partial}{\partial\mathbf{v}}\sum_\gamma\Phi_{\alpha\gamma}(\mathbf{k})\int d\mathbf{v}_1\;\times$$

$$\times\int\frac{d\omega'}{2\pi i}P_\alpha(\mathbf{q}-\mathbf{k},\mathbf{v},\omega-\omega')P_\gamma(\mathbf{k},\mathbf{v}_1,\omega')(4\pi)^{1/2}\frac{e_\alpha}{q}\delta_{\alpha\beta}\frac{1}{\varepsilon^{(0)}(\mathbf{q},\omega)}\;\times$$

$$\times\;\frac{\mathbf{q}}{m_\alpha}\cdot\frac{\partial}{\partial\mathbf{v}}G_{\alpha\gamma}(-\mathbf{k};\mathbf{v},\mathbf{v}_1)$$

$$= \frac{1}{[\varepsilon^{(0)}(\mathbf{q},\omega)]^2}\sum_{\mathbf{k}}\sum_\alpha\frac{4\pi e_\alpha}{q^2}\int d\mathbf{v}\;\frac{-1}{\omega-\mathbf{q}\cdot\mathbf{v}}\;\frac{\mathbf{k}}{m_\alpha}\cdot\frac{\partial}{\partial\mathbf{v}}\sum_\gamma\Phi_{\alpha\gamma}(\mathbf{k})P_\alpha(\mathbf{q}-\mathbf{k},\mathbf{v},\omega-\omega(\mathbf{k}))\times$$

$$\times\;\frac{\mathbf{q}}{m}\cdot\frac{\partial}{\partial\mathbf{v}}\;\frac{-1}{\omega(\mathbf{k})-\mathbf{k}\cdot\mathbf{v}}\;\frac{e_\alpha}{m_\alpha}\;\frac{\mathbf{k}}{k^2}\cdot\frac{\partial F_\alpha(\mathbf{v})}{\partial\mathbf{v}}\sum_\eta 4\pi e_\eta\int d\mathbf{v}_1\int d\mathbf{v}_2 G_{\gamma\eta}(\mathbf{k},\mathbf{v}_1,\mathbf{v}_2).$$

$$= \frac{1}{[\varepsilon^{(0)}(\mathbf{q},\omega)]^2}\sum_{\mathbf{k}}\{\tilde{v}_2(\mathbf{q},\omega;\mathbf{k},\omega(\mathbf{k});\mathbf{q},\omega)\;+$$

where

$$V_1(y,y_1) = \tilde{v}_1(y,y_1) + \tilde{v}_1(y,y-y_1),$$

$$V_2(y,y_1,y_2) = \tilde{v}_2(y,y_1,y_2) + \tilde{v}_2(y,y-y_1-y_2,y_2) + \tilde{v}_2(y,y_1,y-y_1-y_2),$$

$$y \equiv (\mathbf{k},\omega).$$

From eqs.(2) and (20), we get the dispersion relation by retaining the effects of the turbulent waves in the lowest order;

$$\varepsilon(\mathbf{q},\omega) = \varepsilon^{(0)}(\mathbf{q},\omega) - \sum_{\mathbf{k}} \{ V_2(\mathbf{q},\omega;\mathbf{k},\omega(\mathbf{k});-\mathbf{k},-\omega(\mathbf{k})) + \\ + \frac{V_1(\mathbf{q},\omega;\mathbf{k},\omega(\mathbf{k}))V_1(\mathbf{q}-\mathbf{k},\omega-\omega(\mathbf{k});\mathbf{q},\omega)}{\varepsilon^{(0)}(\mathbf{q}-\mathbf{k},\omega-\omega(\mathbf{k}))} \} U(\mathbf{k}), \quad (21)$$

which exactly agrees with eq.(22) in Ref.1).(Note that the calculation of Ref.1) is based on the representation by the " bare" distribution function.)

Above calculations carried out for the system of many species of particles show that Ichimaru and Tange's analysis is incomplete for both the Langmuir turbulence and the ion acoustic wave turbulence.

As to Ichimaru's comment on the vertex correction by which he means[5] the orbit modification of plasma particles caused by scatterings with turbulent waves, our analysis takes acount of it in the lowest order with respect to the wave energy.

This is clear from Fig.4 which represents $\tilde{v}_2(\mathbf{q},\omega;\mathbf{k},\omega(\mathbf{k});\mathbf{q},\omega)$ in eq.(21) in terms of Ichimaru's diagrams[5].

Figure 4

At the end of this section, it should be remarked that Ref.5) by Ichimaru does not give a correct dispersion relation in a turbulent plasma, because he has not carried out the renormalization of the wave-wave interaction to the wave propagator. It was only the renormalization of the wave-particle interaction to the particle propagator that he made.

§3. Complex conductivity

In this section, we calculate the complex conductivity. The complex conductivity is expressed in terms of the current=current response function as

$$\sigma_{\mu\nu}(\omega) = \sigma_0(\omega)\delta_{\mu\nu} + (1/i\omega)(1/i\hbar)\int_0^\infty d\tau\ e^{i\omega\tau} <[J_\mu(\omega),J_\nu(0)]>_0, \quad (22)$$

where μ and ν indicate the component of the tensor, and $\sigma_0(\omega)$ is a reactive conductivity; $\sigma_0(\omega) = (1/i\omega)\sum_\alpha Ne_\alpha^2/m_\alpha$. Since the current operator can be expressed as

$$J_\mu = \sum_\alpha (e_\alpha/m_\alpha)[\frac{\partial}{\partial s_\mu}\hat{\rho}_\alpha(0,-\frac{m_\alpha}{\hbar}\mathbf{s})]_{\mathbf{s}=0}, \quad (23)$$

the second term of the r.h.s. of eq.(22) is easily calculated in a similar way as before. In this case, that J_μ does not depend on the wave-number gives particular simplicity which is anticipated from the relation

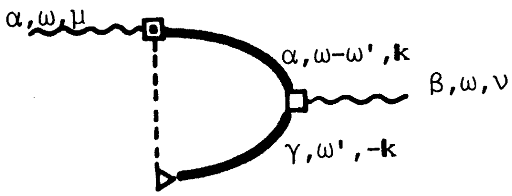
$$[H_I(\lambda), J_\mu] = \sum_{\alpha}\sum_{\beta}\sum_{\mathbf{k}} (e_\alpha/m_\alpha)\,\Phi_{\alpha\beta}(\mathbf{k})\,\hbar k_\mu\,\hat{\rho}_\alpha(\mathbf{k},\lambda\mathbf{k})\,\hat{\rho}^\dagger_\beta(\mathbf{k},\lambda\mathbf{k}). \tag{23}$$

Introducing diagrams listed in Table 2,

Table 2

the complex conductivity can be expressed as

$$i[\sigma_{\mu\nu}(\omega) - \sigma_0(\omega)\delta_{\mu\nu}] =$$



$$= (1/\omega^2)\sum_{\alpha}\sum_{\beta}\sum_{\mathbf{k}}(e_\alpha/m_\alpha)\,k_\mu\,\Phi_{\alpha\beta}(\mathbf{k})\int d\mathbf{v}\int d\mathbf{v}'\int\frac{d\omega'}{2\pi i}\,P_\alpha(\mathbf{k},\mathbf{v},\omega-\omega')\,P_\gamma(-\mathbf{k},\mathbf{v}',\omega')\times$$

$$\times\, G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}')\,(e_\alpha/m_\alpha - e_\beta/m_\beta)\,k_\nu$$

$$= (1/\omega^2)\sum_{\alpha}\sum_{\beta}\sum_{\mathbf{k}}(1/2)(e_\alpha/m_\alpha - e_\beta/m_\beta)\,k_\mu k_\nu\,\Phi_{\alpha\beta}(\mathbf{k})\int d\mathbf{v}\int d\mathbf{v}'\,\times$$

$$\times\,\{(e_\alpha/m_\alpha)P_\alpha(\mathbf{k},\mathbf{v},\omega+\omega(\mathbf{k})) - (e_\beta/m_\beta)P_\beta(-\mathbf{k},\mathbf{v}',\omega-\omega(\mathbf{k}))\}\,G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}'). \tag{24}$$

Note the relation

$$\int d\mathbf{v}\int d\mathbf{v}' \frac{1}{\omega-\omega(\mathbf{k})+\mathbf{k}\cdot\mathbf{v}'} G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}') =$$

$$= -(e_\beta/m_\beta)(1/\omega)\int d\mathbf{v}\int d\mathbf{v}' \left[\frac{1}{\omega(\mathbf{k})-\mathbf{k}\cdot\mathbf{v}'} + \frac{1}{\omega-\omega(\mathbf{k})+\mathbf{k}\cdot\mathbf{v}'}\right] \times$$

$$\times(\mathbf{k}/k^2)\cdot\frac{\partial}{\partial\mathbf{v}'}F_\beta(\mathbf{v}')\sum_\gamma 4\pi e_\gamma\int d\mathbf{v}'' G_{\alpha\gamma}(\mathbf{k};\mathbf{v},\mathbf{v}'')$$

$$= (1/\omega)S_{\alpha\beta}(\mathbf{k}) - (1/\omega)4\pi\chi_\beta^{(0)}(-\mathbf{k},\omega-\omega(\mathbf{k}))\sum_\gamma\frac{e_\gamma}{e_\beta} S_{\alpha\gamma}(\mathbf{k}), \tag{25}$$

where $S_{\alpha\beta}(\mathbf{k}) = \int d\mathbf{v}\int d\mathbf{v}' G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}')$ and $4\pi\chi_\alpha^{(0)}(\mathbf{k},\omega) = -\sum_\beta\chi_{\alpha\beta}^{(0)}(\mathbf{k},\omega)$.

Then we get

$$\sigma_{\mu\nu}(\omega) - \sigma_0(\omega) = (i4\pi/\omega^3)\sum_\alpha\sum_\beta\sum_{\mathbf{k}}\frac{k_\mu k_\nu}{k^2}(e_\alpha/m_\alpha - e_\beta/m_\beta)\frac{1}{\varepsilon^{(0)}(-\mathbf{k},\omega-\omega(\mathbf{k}))} \times$$

$$\times\left\{\left(\frac{e_\alpha}{e_\beta}\frac{e_\alpha}{m_\alpha} - \frac{e_\alpha}{m_\beta}\right)\varepsilon_\alpha^{(0)}4\pi\chi_\beta^{(0)}S_{\alpha\alpha}(\mathbf{k}) +\right.$$

$$\left.+\left[(e_\alpha/m_\alpha)4\pi\chi_\alpha^{(0)}4\pi\chi_\beta^{(0)} - (e_\beta/m_\beta)\varepsilon_\alpha^{(0)}\varepsilon_\beta^{(0)}\right] S_{\alpha\beta}(\mathbf{k})\right\}, \tag{26}$$

where $\varepsilon_\alpha^{(0)} = 1 + 4\pi\chi_\alpha^{(0)}$ and we have suppressed the common argument $(-\mathbf{k},\omega-\omega(\mathbf{k}))$ in the functions $\varepsilon^{(0)}$ and $\chi^{(0)}$. Equation(26) is equivalent to eq.(32) in Ref.4) by Nishikawa and Ichikawa. This shows also the invalidity of Ichimaru and Tange's truncation.

§4. Discussions

In this paper, we have applied the linear response theory to dielectric and conduction problems in a turbulent plasma and obtained well-established results.

In the course of calculation, the ensemble has not been defined explicitly and there have been appeared two possibilities in handling the disconnected diagrams. As for the well=established weak turbulence theory based on the representation by the "dressed" distribution function, it is not simple to get the "dressed" distribution function explicitly. It is still an open problem to settle the "physical vacuum" in the stationary turbulent state.

References

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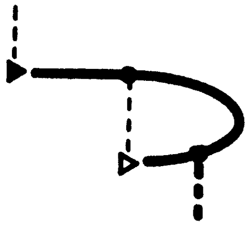
2) S.Ichimaru and T.Tange : Phys. Rev. Letters 32 (1974) 102.

3) S.Ichimaru: J. Phys. Soc. Japan 44 (1978)

4) K.Nishikawa and Y.H.Ichikawa: Phys. Fluids 12 (1969) 2563.

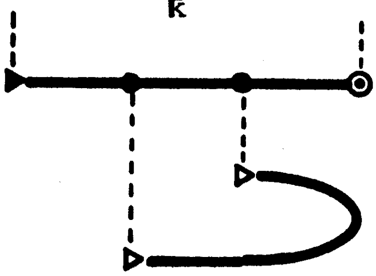
5) S.Ichimaru: Phys. Rev. A15 (1977) 744.

Appendix



$$= \frac{1}{[\varepsilon^{(0)}(q,\omega)]^2} \times$$

$$\times \sum_{\mathbf{k}} \frac{\tilde{v}_1(q,\omega;q-\mathbf{k},\omega-\omega(\mathbf{k}))\,\tilde{v}_1(q-\mathbf{k},\omega-\omega(\mathbf{k});q,\omega)}{\varepsilon^{(0)}(q-\mathbf{k},\omega-\omega(\mathbf{k}))}\; U(\mathbf{k}) . \qquad (A.1)$$



$$=\sum_{\alpha}\sum_{\beta}(4\pi e_\alpha/q^2)\int d\mathbf{v}\; P_\alpha(q,\mathbf{v},\omega)\sum_{\mathbf{k}}(\mathbf{k}/m_\alpha)\cdot\frac{\partial}{\partial \mathbf{v}}\sum_{\gamma}\Phi_{\alpha\gamma}(\mathbf{k})\times$$

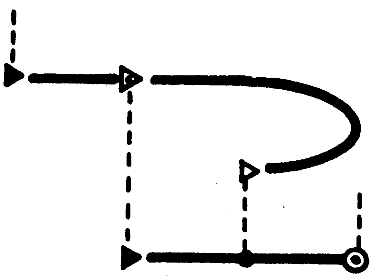
$$\times \int d\mathbf{v}'\int\frac{d\omega'}{2\pi i}\; P_\alpha(q-\mathbf{k},\mathbf{v},\omega-\omega')P_\gamma(\mathbf{k},\mathbf{v}',\omega')(-\mathbf{k}/m_\alpha)\cdot\frac{\partial}{\partial \mathbf{v}}\sum_{\eta}\Phi_{\alpha\eta}(-\mathbf{k}) \times$$

$$\times \int d\mathbf{v}''\int\frac{d\omega''}{2\pi i}\; P_\alpha(\mathbf{k},\mathbf{v},\omega-\omega'-\omega'')(q/m_\alpha)\cdot\frac{\partial}{\partial \mathbf{v}}F_\alpha(\mathbf{v})\quad e_\alpha\delta_{\alpha\beta}\times$$

$$\times P_\eta(-\mathbf{k},\mathbf{v}'',\omega'')G_{\eta\gamma}(-\mathbf{k},\mathbf{v}'',\mathbf{v}')$$

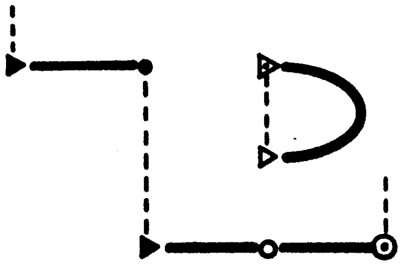
$$= \frac{1}{[\varepsilon^{(0)}(q,\omega)]^2}\sum_{\mathbf{k}}\{\tilde{v}_2(q,\omega;\mathbf{k},\omega(\mathbf{k});-\mathbf{k},-\omega(\mathbf{k})) \quad +$$

$$+\frac{v_1(q,\omega;\mathbf{k},\omega(\mathbf{k}))\,v_1(q-\mathbf{k},\omega-\omega(\mathbf{k});-\mathbf{k},-\omega(\mathbf{k}))}{\varepsilon^{(0)}(q-\mathbf{k},\omega-\omega(\mathbf{k}))}\}U(\mathbf{k}) . \qquad (A.2)$$



$$= \frac{1}{[\varepsilon^{(0)}(q,\omega)]^2}\sum_{\mathbf{k}}\frac{U(\mathbf{k})}{\varepsilon^{(0)}(q-\mathbf{k},\omega-\omega(\mathbf{k}))}\times$$

$$\times\; \tilde{v}_1(q,\omega;q-\mathbf{k},\omega-\omega(\mathbf{k}))\,\tilde{v}_1(q-\mathbf{k},\omega-\omega(\mathbf{k});-\mathbf{k},-\omega(\mathbf{k})) . \qquad A.3)$$



$$= \sum_{\alpha}\sum_{\beta}(4\pi e_\alpha/q^2)\int d\mathbf{v}\ P_\alpha(\mathbf{q},\mathbf{v},\omega)\,(\mathbf{q}/m_\alpha)\cdot\frac{\partial}{\partial\mathbf{v}}\sum_{\gamma}\Phi_{\alpha\gamma}(\mathbf{q})\int d\mathbf{v}'P_\gamma(\mathbf{q},\mathbf{v}',\omega)\ \times$$

$$\times\sum_{\mathbf{k}}(\mathbf{k}/m_\alpha)\cdot\frac{\partial}{\partial\mathbf{v}}\sum_{\eta}\Phi_{\alpha\eta}(\mathbf{k})\int d\mathbf{v}''\int\frac{d\omega'}{2\pi i}P_\alpha(\mathbf{k},\mathbf{v},\omega')\,P_\alpha(-\mathbf{k},\mathbf{v},\omega'')\,G_{\alpha\eta}(\mathbf{k};\mathbf{v},\mathbf{v}'')\times$$

$$\times P_\gamma(\mathbf{q},\mathbf{v}',\omega-\omega'-\omega'')\,(\mathbf{q}/m_\gamma)\cdot\frac{\partial}{\partial\mathbf{v}'}F_\gamma(\mathbf{v}')\,e_\beta\delta_{\gamma\beta}$$

$$= \frac{1}{[\varepsilon^{(0)}(\mathbf{q},\omega)]^2}\sum_{\mathbf{k}}\sum_{\mathbf{k}_1}\sum_{\alpha}(4\pi e_\alpha^2/m_\alpha q^2)\,(e_\alpha/m_\alpha)^2\int d\mathbf{v}\ \frac{-1}{\omega-\mathbf{q}\cdot\mathbf{v}}\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{v}}\ \times$$

$$\times\ \Big[\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{v}}\ \frac{-1}{\omega(\mathbf{k})-\mathbf{k}\cdot\mathbf{v}}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}F_\alpha(\mathbf{v})\ +\ \mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\ \frac{-1}{\omega(\mathbf{k}_1)-\mathbf{k}_1\cdot\mathbf{v}}\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{v}}F_\alpha(\mathbf{v})\Big]\,\delta_{\mathbf{k}+\mathbf{k}_1,0}U(\mathbf{k})$$

$$= \frac{1}{[\varepsilon^{(0)}(\mathbf{q},\omega)]^2}\sum_{\mathbf{k}}\sum_{\mathbf{k}_1}\ \delta_{\mathbf{k}+\mathbf{k}_1,0}\,[\tilde{v}_2(\mathbf{q},\omega;\mathbf{q}-\mathbf{k}-\mathbf{k}_1,\omega-\omega(\mathbf{k})-\omega(\mathbf{k}_1);\mathbf{k}_1,\omega(\mathbf{k}_1))+$$

$$+\ \tilde{v}_2(\mathbf{q},\omega;\mathbf{q}-\mathbf{k}-\mathbf{k}_1,\omega-\omega(\mathbf{k})-\omega(\mathbf{k}_1);\mathbf{k},\omega(\mathbf{k}))]\ \ U(\mathbf{k})\,. \qquad \text{(A.4)}$$

Figure Captions

Fig. 1 Diagramatic representations of eq.(10).

Fig. 2 Schematic figures for $\rho_\alpha^{(0)}$. (a) for $\rho_\alpha^{(1)}$ and (b),(c) for $\rho_\alpha^{(2)}$.

Fig. 3 Diagrams for $\chi^{(2)}$. (b) corresponds to Fig.2-(b) and (c) to Fig.2-(c).

Fig. 4 Figure for $\tilde{v}_2(q,\omega;k,\omega(k);q,\omega)$ represented by Ichimaru's diagrams[5)]

Table 1

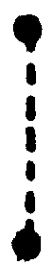
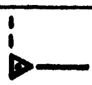
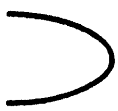
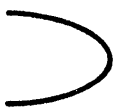
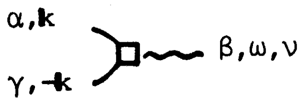
Propagator		$-1/(\omega - \mathbf{q}\cdot\mathbf{v})$
		$\frac{-1}{\omega - \mathbf{q}\cdot\mathbf{v}} \; \frac{-1}{\omega-\omega'-\mathbf{q}\cdot\mathbf{v}}$
Internal line		$\Phi_{\alpha\beta}(\mathbf{k})$
External line		$\sum_{\alpha} (4\pi/q^2)^{1/2} e_{\alpha} \delta_{\alpha\gamma}$
Vertex		$\sum_{\mathbf{k}} (\mathbf{k}/m_{\alpha})\cdot\frac{\partial}{\partial \mathbf{v}}$
		$(\mathbf{k}/m_{\alpha})\cdot\frac{\partial}{\partial \mathbf{v}} F_{\alpha}(\mathbf{v})$
		$\sum_{\gamma} \int d\mathbf{v} \int \frac{d\,\omega}{2\pi i}$
		$\sum_{\gamma} \int d\mathbf{v}$
		$(\mathbf{k}/m_{\alpha})\cdot\frac{\partial}{\partial \mathbf{v}} \int \frac{d\,\omega}{2\pi i}$
Correlation function		$\frac{-1}{\omega-\mathbf{k}\cdot\mathbf{v}} \; \frac{-1}{\omega'+\mathbf{k}\cdot\mathbf{v}'} G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}')$
		$\frac{-1}{\omega-\mathbf{k}\cdot\mathbf{v}} \; G_{\alpha\beta}(\mathbf{k};\mathbf{v},\mathbf{v}')$

Table 2

Propagator		$-1/\omega$
Vertex		$\sum_{\alpha} (e_\alpha/m_\alpha)\, k_\mu \int d\mathbf{v}$
		$(e_\alpha/m_\alpha - e_\gamma/m_\gamma)\, k_\nu$

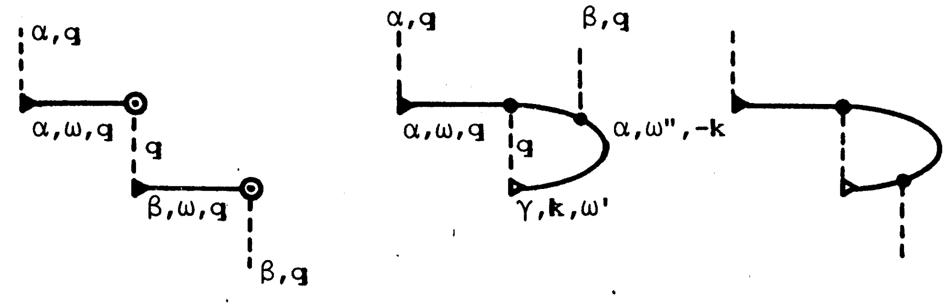

α,q
α,ω,q
q
β,ω,q
β,q
α,q
β,q
α,ω,q
q
α,ω",-k
γ,k,ω'

Fig. 1

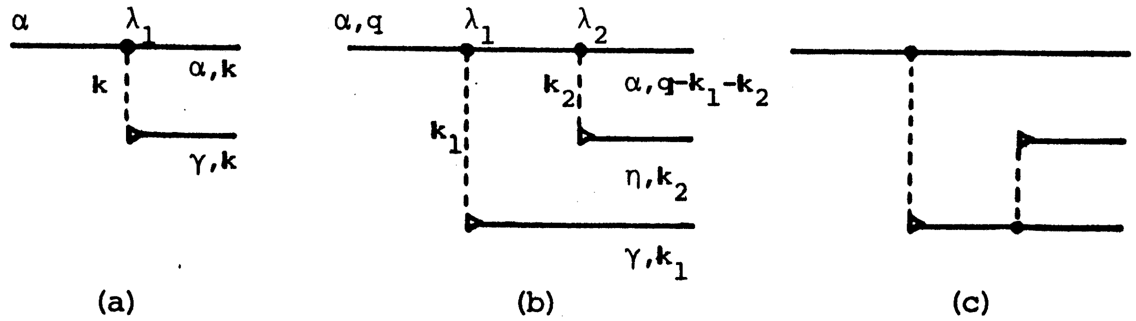

α
λ1
α,k
k
γ,k
(a)
α,q
λ1
λ2
k2
α,q-k1-k2
k1
η,k2
γ,k1
(b)
(c)

Fig. 2

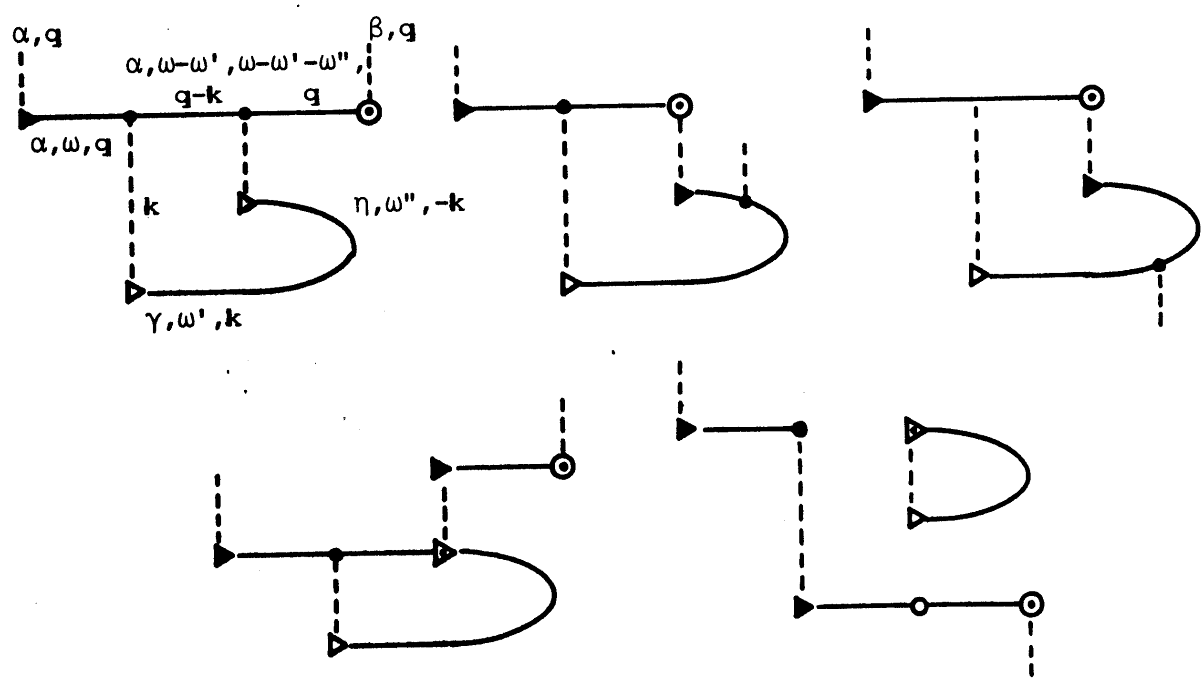


Fig. 3-(b)

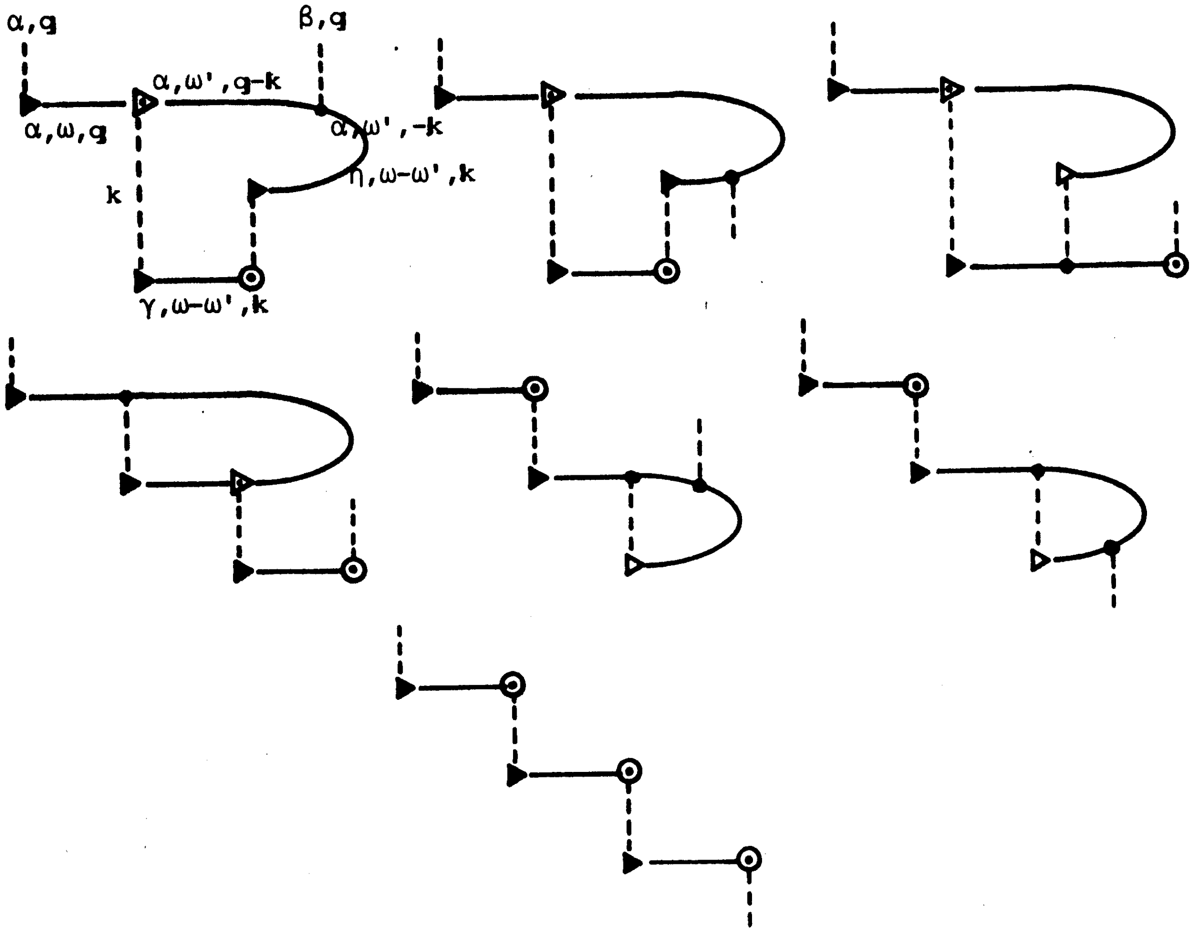


Fig. 3-(c)

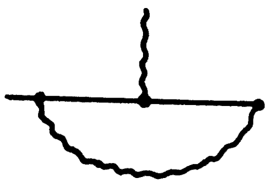


Fig. 4