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Thermal-flux reduction by electromagnetic instabilities

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Abstract

From a dispersion relation, linear growth rates of electromagnetic instabilities are obtained in an electron plasma whose velocity distribution function has a high-energy-tail part. Theory is developed to derive the reduction in the thermal flux caused by these electromagnetic instabilities. Nonlinear theory leads to the saturation level of instabilities. Numerical simulations are carried out by using PIC method. Reduction rates in flux limited theory of thermal conduction predicted by the theory are found to be in good agreement with computer simulations.

§1. Introduction

In laser-irradiated target plasmas, the thermal conduction plays an important role in the implosion of targets and hence in thermonuclear-fusion reactions in plasmas. Recent several experiments have shown that the electron thermal conductivity in the target-plasma was much reduced from the classical value [1].

It is well known in the research field of laser fusion that the light energy absorbed in the target contributes mainly to increasing high energy electrons. Morse and Nielson [2] have attributed the reduction in the transport coefficients to these high energy electrons. Quite recently, Manheimer [3] proposed a theory that ion acoustic turbulence in laser-fusion schemes was a possible cause for energy-flux limitation.

On the other hand, purely growing electromagnetic waves were shown to be excited by electrons in a velocity distribution with an anisotropic temperature. The preceding paper written by the same authors [4] pointed out that propagating electromagnetic waves were excited by a microinstability induced in an electron plasma with nonvanishing thermal flux, vanishing current and vanishing temperature anisotropy. The purpose of this paper is to predict that electromagnetic waves induced by these microinstabilities reduce effectively the electron thermal conductivity in plasmas.

In §2, the dispersion relation of electromagnetic instability has been investigated by electrons in a velocity distribution with the high-energy-tail part. In §3, quasilinear theory is used to study the reduction in the thermal flux by electromagnetic instability. The saturation mechanism of the instability is presented in §4. Comparison between the theory and the computer simulation in made in §5, and finally in §6 we present summary and conclusions.

§2. Dispersion relation

The dynamics of the system under consideration are described by the Vlasov-Maxwell equations :

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} + \frac{q}{m}\left(\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B} \right)\cdot\frac{\partial f}{\partial \mathbf{v}} = 0 \quad , \tag{1}$$

$$\mathrm{rot}\ \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \ , \tag{2}$$

$$\mathrm{rot}\ \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi \mathbf{J}}{c} \ , \tag{3}$$

where c.g.s. gaussian units are used, f is the electron distribution function at position $\mathbf{r}$ and velocity $\mathbf{v}$ at time t, q and m, respectively, denote the charge (including sign) and mass of the electron, c is the light velocity, $\mathbf{J}$ the current, while $\mathbf{E}$ and $\mathbf{B}$ represent the electromagnetic field. Denoting the average of a quantity over the ensemble by $< >$ and its deviation from the average by δ for an unmagnetized plasma, we obtain

$$\frac{\partial \langle f \rangle}{\partial t} + \mathbf{v}\cdot\frac{\partial \langle f \rangle}{\partial \mathbf{r}} + \frac{q}{m}\left\langle \left(\delta\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\delta\mathbf{B} \right)\cdot\frac{\partial \delta f}{\partial \mathbf{v}} \right\rangle = 0 \ . \tag{4}$$

By subtracting (4) from (1), the equation for δf is obtained as

$$\frac{\partial \delta f}{\partial t} + \mathbf{v}\cdot\frac{\partial \delta f}{\partial \mathbf{r}} + \frac{q}{m}\left(\delta\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\delta\mathbf{B} \right)\cdot\frac{\partial (\langle f \rangle + \delta f\ )}{\partial \mathbf{v}} = 0 \ . \tag{5}$$

In obtaining (5), we have neglected term $\left\langle \left(\delta\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\delta\mathbf{B}\right)\cdot\frac{\partial \delta f}{\partial \mathbf{v}} \right\rangle$. To solve (5), we make use of a propagator $U(t-t')$ which is defined as the solution of

$$\frac{\partial U}{\partial t} + \mathbf{v}\cdot\frac{\partial U}{\partial \mathbf{r}} + \frac{q}{m}\left(\delta\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\delta\mathbf{B} \right)\cdot\frac{\partial U}{\partial \mathbf{v}} = 0, \tag{6}$$

$$U(t'-t') = 1 \ . \tag{7}$$

The solution of (5) can be written as

$$\delta f(t) = - \int_{-\infty}^{t} dt' \ U(t-t') \frac{q}{m} (\delta E(t') + \frac{1}{c} \mathbf{v}' \times \delta B(t')) \cdot \frac{\partial \langle f(\mathbf{r}', \mathbf{v}', t') \rangle}{\partial \mathbf{v}'} \ . \tag{8}$$

To lowest order in perturbed field, (8) can be expressed in the following form,

$$\delta f(t) = - \int_{0}^{\infty} d\tau \sum_{\mathbf{k}} \frac{q}{m} \exp(-i\omega t + i\omega\tau + i\mathbf{k}\cdot\mathbf{r})$$

$$\langle [\ \delta\mathbf{E}_{\mathbf{k},\omega_{\mathbf{k}}} + \frac{\mathbf{v} + \Delta\mathbf{v}(\tau)}{\omega_{\mathbf{k}}} \times \mathbf{k} \times \delta\mathbf{E}_{\mathbf{k},\omega_{\mathbf{k}}}] \exp(i\mathbf{k}\cdot\Delta\mathbf{r}(\tau)) \rangle$$

$$\langle \ U(\tau) \ \rangle \ \frac{\partial \langle f(\mathbf{r},\mathbf{v},t) \rangle}{\partial \mathbf{v}'} \ , \tag{9}$$

where we have used $U(t-t')\mathbf{r}(t') = \mathbf{r}(t) + \Delta\mathbf{r}(t-t')$ and $U(t-t')\mathbf{v}(t') = \mathbf{v}(t) + \Delta\mathbf{v}(t-t')$. Here also the approximations $\langle f(t-\tau) \rangle \simeq \langle f(t) \rangle$ and $U(\tau) \simeq \langle U(\tau) \rangle$ are used and the Fourier transformation

$$G(\mathbf{r},t) = \sum_{\mathbf{k}} G_{\mathbf{k},\omega_{\mathbf{k}}} \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r}) \tag{10}$$

is used for $\delta\mathbf{E}$ and δB combined with Maxwell's equations. If we define $\mathbf{I}$ as

$$\mathbf{I} = \langle [\delta\mathbf{E}_{\mathbf{k},\omega_{\mathbf{k}}} + \frac{\mathbf{v} + \Delta\mathbf{v}(\tau)}{\omega_{\mathbf{k}}} \times \mathbf{k} \times \delta\mathbf{E}_{\mathbf{k},\omega_{\mathbf{k}}}] \exp(i\mathbf{k}\cdot\Delta\mathbf{r}(\tau)) \rangle \tag{11}$$

and confine ourselves to the configuration

$$\mathbf{k} = (\ k,\ 0,\ 0\ ),\ \delta\mathbf{E} = (\ 0,\ \delta E,\ 0\ ) \text{ and } \delta B = (\ 0,\ 0,\ \delta B\ ), \tag{12}$$

the equation (11) reduces to

$$\mathbf{I} = \delta E_{k,\omega_k} \langle [\ \frac{v_y + \Delta v_y(\tau)}{\omega_k} \ k e^{ik\cdot\Delta x(\tau)} \ ] \rangle \mathbf{i}$$

$$+ \ \delta E_{k,\omega_k} \langle [\ 1 - \frac{v_x + \Delta v_x}{\omega_k} \ k \ ] e^{ik\cdot\Delta x(\tau)} \rangle \mathbf{j} \ , \tag{13}$$

where **i** and **j** are the unit vectors in the x and y directions, respectively. Using the following relation

$$\frac{\partial \Delta x}{\partial \tau} = - (v_x + \Delta v_x) , \tag{14}$$

we write

$$\mathbf{I} = \delta E_{k,\omega_k} < \frac{kv_y}{\omega_k} e^{ik\cdot\Delta x(\tau)} > \mathbf{i} + \delta E_{k,\omega_k} <[1 + \frac{1}{i\omega_k}\frac{\partial}{\partial\tau}] e^{ik\cdot\Delta x(\tau)} > \mathbf{j} , \tag{15}$$

where Δv_y has been neglected because of its independence on Δx. Defining J as

$$J = \int_0^\infty e^{i\omega\tau} < e^{ik\cdot\Delta x(\tau)} > d\tau , \tag{16}$$

we get

$$\delta f_{k,\omega_k} = - \sum_k \frac{q}{m} \delta E_{k,\omega_k} [\frac{kv_y}{\omega_k} J \frac{\partial}{\partial v_x} - \frac{1}{i\omega_k} \frac{\partial}{\partial v_y}] <f> . \tag{17}$$

Substituting (17) into the space and time Fourier transform of Maxwell's equations, we obtain a nonlinear dispersion relation for a purely transverse mode :

$$1 + \frac{\omega_p^2}{n\omega^2} \int dv [\frac{kv_y^2}{i} J \frac{\partial}{\partial v_x} + v_y \frac{\partial}{\partial v_y}] <f> - \frac{k^2c^2}{\omega^2} = 0 , \tag{18}$$

where n is the number density and $\omega_p = \sqrt{4\pi nq^2/m}$.

To treat the problem of electromagnetic instability, we model the following electron distribution function with high energy part as the ensemble-averaged distribution function

$$\langle f \rangle = \frac{n(1-\alpha)}{2\, v_{Ti}^{c} v_{Tj}^{c}} \exp\left[- \frac{(v_i + v_d^c)^2}{2 v_{Ti}^{c2}} - \frac{v_j^2}{2 v_{Tj}^{c2}} \right]$$

$$+ \frac{n\alpha}{2\pi v_{Ti}^{h} v_{Tj}^{h}} \exp\left[- \frac{(v_i - v_d^h)^2}{2 v_{Ti}^{h2}} - \frac{v_j^2}{2 v_{Tj}^{h2}} \right] , \qquad (19)$$

where v_i, v_j are the velocity components in the i and j directions, v_{Ti}, v_{Tj} are the thermal-velocity components, v_d is the drift velocity and α is the fraction of high-energy electrons. Suffices c and h refer to cold and high-energy electrons, respectively. Using this model distribution function, we can evaluate the dispersion relation in the linear theory (by putting $J = i/(\omega_k - kv_x)$ in (18)). By taking i=x, j=y or i=y, j=x in equation (19), we expect to have two modes of instability as follows :

1) If we take i=x and j=y in equation (19) (mode 1), the dispersion relation can be obtained as follows :

$$\omega^2 - (\omega_p^2 + k^2 c^2) + \frac{(1-\alpha)\,\omega_p^2 v_{Ty}^{c2}}{v_{Tx}^{c2}} W\left(\frac{\omega + k v_d^c}{k v_{Tx}^c}\right) + \frac{\alpha \omega_p^2 v_{Ty}^{h2}}{v_{Tx}^{h2}} W\left(\frac{\omega - k v_d^h}{k v_{Tx}^h}\right) = 0 , \qquad (20)$$

where the function W is defined in the form

$$W(\xi) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \frac{\zeta}{\zeta - \xi} \exp(-\zeta^2/2)\, d\zeta \ . \qquad (21)$$

To determine growth rates of induced electromagnetic waves, we examine (20) in the following limits. In the limits $|(\omega + k v_d^c)/k v_{Tx}^c| \ll 1$ and $|(\omega - k v_d^h)/k v_{Tx}^h| \ll 1$, (20) can be approximately solved as

$$\omega_r = k\left[\alpha \frac{v_{Ty}^{h2} v_d^h}{v_{Tx}^{h3}} - (1-\alpha) \frac{v_{Ty}^{c2} v_d^c}{v_{Tx}^{c3}} \right]\left[\alpha \frac{v_{Ty}^{h2}}{v_{Tx}^{h3}} + (1-\alpha) \frac{v_{Ty}^{c2}}{v_{Tx}^{c3}} \right]^{-1}, \qquad (22)$$

$$\gamma = (2/\pi)^{1/2} k[\alpha \frac{v_{Ty}^{h2}}{v_{Tx}^{h2}} + (1-\alpha) \frac{v_{Ty}^{c2}}{v_{Tx}^{c2}} - \frac{k^2c^2}{\omega_p^2} - 1]$$

$$\times [\alpha \frac{v_{Ty}^{h2}}{v_{Tx}^{h3}} + (1-\alpha) \frac{v_{Ty}^{c2}}{v_{Tx}^{c3}}]^{-1} , \qquad (23)$$

where we write ω as $\omega = \omega_r + i\gamma$.

Instability occurs for wave numbers such that

$$k^2 < k_0^2 \equiv \frac{\omega_p^2}{c^2} [(1-\alpha) \frac{v_{Ty}^{c2}}{v_{Tx}^{c2}} + \alpha \frac{v_{Ty}^{h2}}{v_{Tx}^{h2}} - 1] . \qquad (24)$$

The growth rate vanishes for $k=0$ and $k=k_0$ and has a maximum

$$\gamma_{Max} = (8/27\pi)^{1/2} \frac{\omega_p}{c} [(1-\alpha) \frac{v_{Ty}^{c2}}{v_{Tx}^{c2}} + \alpha \frac{v_{Ty}^{h2}}{v_{Tx}^{h2}} - 1]^{3/2}$$

$$\times [(1-\alpha) \frac{v_{Ty}^{c2}}{v_{Tx}^{c3}} + \alpha \frac{v_{Ty}^{h2}}{v_{Tx}^{h3}}]^{-1}, \qquad (25)$$

at $k_{Max}^2 = k_0^2/3$.

2) If we take $i=y$ and $j=x$ in equation (19) (mode 2), the dispersion relation can be calculated as follows :

$$\omega^2 - (c^2k^2 + \omega_p^2) + \omega_p^2 (1-\alpha) \frac{v_{Ty}^{c2} + v_d^{c2}}{v_{Tx}^{c2}} W(\frac{\omega}{kv_{Tx}^c})$$

$$+ \omega_p^2 \alpha \frac{v_{Ty}^{h2} + v_d^{h2}}{v_{Tx}^{h2}} W(\frac{\omega}{kv_{Tx}^h}) = 0 . \qquad (26)$$

In the limits $|\omega/kv_{Tx}^c| \ll 1$ and $|\omega/kv_{Tx}^h| \ll 1$, (26) can be approximately solved as

$$\omega_r = 0 \ ,$$

$$\gamma = (2/\pi)^{1/2} k [(1-\alpha) \frac{v_{Ty}^{c2} + v_d^{c2}}{v_{Tx}^{c2}} + \alpha \frac{v_{Ty}^{h2} + v_d^{h2}}{v_{Tx}^{h2}} - 1 - \frac{c^2 k^2}{\omega_p^2}]$$

$$\times [(1-\alpha) \frac{v_{Ty}^{c2} + v_d^{c2}}{v_{Tx}^{c3}} + \alpha \frac{v_{Ty}^{h2} + v_d^{h2}}{v_{Tx}^{h3}}]^{-1} . \qquad (28)$$

Instability is found for wave numbers which satisfy

$$k^2 < \frac{\omega_p^2}{c^2} [(1-\alpha) \frac{v_{Ty}^{c2} + v_d^{c2}}{v_{Tx}^{c2}} + \alpha \frac{v_{Ty}^{h2} + v_d^{h2}}{v_{Tx}^{h2}} - 1] . \qquad (29)$$

The maximum growth rate is obtained as

$$\gamma_{Max} = (8/27\pi)^{1/2} \frac{\omega_p}{c} [(1-\alpha) \frac{v_{Ty}^{c2} + v_d^{c2}}{v_{Tx}^{c2}} + \alpha \frac{v_{Ty}^{h2} + v_d^{h2}}{v_{Tx}^{h2}} - 1]^{3/2}$$

$$\times [(1-\alpha) \frac{v_{Ty}^{c2} + v_d^{c2}}{v_{Tx}^{c3}} + \alpha \frac{v_{Ty}^{h2} + v_d^{h2}}{v_{Tx}^{h3}}]^{-1} , \qquad (30)$$

at

$$k_{Max}^2 = \frac{1}{3} \frac{\omega_p^2}{c^2} [(1-\alpha) \frac{v_{Ty}^{c2} + v_d^{c2}}{v_{Tx}^{c2}} + \alpha \frac{v_{Ty}^{h2} + v_d^{h2}}{v_{Tx}^{h2}} - 1] . \qquad (31)$$

§3. Anomalous thermal-flux reduction

To show the reduction of thermal conduction by the electromagnetic instability, we begin with the equation for $\langle U(t) \rangle$ [5],

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} \right) \langle U(t) \rangle = \left(\frac{q}{m} \right)^2 \sum_{\mathbf{k}} \frac{\partial}{\partial \mathbf{v}} \int_0^\infty \left[\delta \mathbf{E}_{-\mathbf{k},\omega_{-\mathbf{k}}} + \frac{\mathbf{v} \times \mathbf{k} \times \delta \mathbf{E}_{-\mathbf{k},\omega_{-\mathbf{k}}}}{\omega_{-\mathbf{k}}} \right]$$

$$\cdot e^{i\omega\tau} \mathbf{I} \cdot \frac{\partial}{\partial \mathbf{v}} \langle U(t-\tau) \rangle \, d\tau \; . \qquad (32)$$

We suppose that $\langle U(t-\tau)\rangle \simeq \langle U(t)\rangle$ and $J=i/(\omega_k - kv_x)$. In the configuration (12), (32) reduces to

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot \frac{\partial}{\partial \mathbf{r}} \right) \langle U(t) \rangle = i\omega_p^2 \sum_{\mathbf{k}} \frac{|\delta B_k|^2}{4\pi n m c^2 k^2} \left[kv_y \frac{\partial}{\partial v_x} - (\omega_{-k} + kv_x) \frac{\partial}{\partial v_y} \right]$$

$$\times \left[\frac{kv_y}{\omega_k - kv_x} \frac{\partial}{\partial v_x} + \frac{\partial}{\partial v_y} \right] \langle U(t) \rangle . \qquad (33)$$

This is also the equation for $\langle f(t) \rangle$:

$$\frac{\partial \langle f \rangle}{\partial t} + \mathbf{v}\cdot \frac{\partial \langle f \rangle}{\partial \mathbf{r}} = i\omega_p^2 \sum_{k} \frac{|\delta B_k|^2}{4\pi n m c^2 k^2} \left[kv_y \frac{\partial}{\partial v_x} - (\omega_{-k} + kv_x) \frac{\partial}{\partial v_y} \right]$$

$$\times \left[\frac{kv_y}{\omega_k - kv_x} \frac{\partial}{\partial v_x} + \frac{\partial}{\partial v_y} \right] \langle f \rangle \qquad (34)$$

For mode 1, we take $i=x$ and $j=y$ in (19). The third moment of (34) by use of (19) leads to the following approximate equation for the thermal flux :

$$\frac{\partial Q}{\partial t} + \frac{nT_a}{m} \frac{\partial T_a}{\partial x} \simeq - (18\pi)^{1/2} \sum_{k} \frac{|\delta B_k|^2}{4\pi n m c^2} \frac{\omega_p^2}{kv_d^h} \frac{kv_d^h - \omega_r}{kv_{Tx}^h} Q = -\nu_{eff} Q, \quad (35)$$

where Q is the thermal-flux defined by $Q = m/2 \int v_x (v_x^2 + v_y^2) \langle f \rangle d\mathbf{v}$ and subscript a denotes averaged values with respect to cold and high energy electrons. For the derivation of (35), we have assumed that $\langle f \rangle$ depends on x and t and velocities are in the following ranges :

$$v_{Ty}^h \; (\simeq v_{Tx}^h \simeq v_d^h) \gg v_{Ty}^c \; (\simeq v_{Tx}^c \simeq v_d^c) . \tag{36}$$

In the stationary state ($\partial/\partial t=0$), we have

$$Q = - \frac{nT_a}{m\nu_{eff}} \frac{\partial T_a}{\partial x} \equiv - K_{eff} \frac{\partial T_a}{\partial x} , \tag{37}$$

where K_{eff} denotes the effective thermal conductivity.

For mode 2, if we take i=y and j=x, we obtain the equation for the thermal flux as follows :

$$\frac{\partial Q'}{\partial t} + \frac{nT_a}{m} \frac{\partial T_a}{\partial y} \simeq -(\pi/2)^{1/2} \sum_k \frac{|\delta B_k|^2}{4\pi nmc^2} \frac{\omega_p^2}{kv_{Txa}} \left(\frac{R}{v_{Txa}} \right)^2 Q'$$

$$\equiv -\nu'_{eff} Q' , \tag{38}$$

where Q' is the thermal-flux defined by $Q' = m/2 \int v_y (v_x^2 + v_y^2) \langle f \rangle d\mathbf{v}$ and the resonance broadening effects are taken into account by use of the resonance width [6]

$$R \equiv \left[(\pi/18)^{1/2} \sum_k \frac{|\delta B_k|^2}{4\pi nmc^2} \left(\frac{v_{Txa}\omega_p}{k} \right)^2 \right]^{1/4} , \tag{39}$$

and relations $v_{Tx}^h = v_{Ty}^h$ and $v_{Tx}^c = v_{Ty}^c$ are assumed to be held. Then we similarly obtain the following effective thermal conductivity in the stationary state,

$$Q' = - \frac{nT_a}{m\nu'_{eff}} \frac{\partial T_a}{\partial y} \equiv -K'_{eff} \frac{\partial T_a}{\partial y} . \tag{40}$$

§4. Nonlinear saturation level

In order to obtain the saturation level of the instability, one has to consider nonlinear process. It is proposed that the nonlinear effect of the instability is a scattering of particle orbits by waves. Waves which are linearly unstable grow until the enhanced scattering causes their nonlinear growth rates to vanish. To evaluate the function J, we start with equation (32). Multiplying (32) from the right by $\exp(ik\cdot x(0))$, we get

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} \right) \langle U(t) \rangle e^{ik\cdot x(0)} = i\omega_p^2 \sum_k \frac{|\delta B_k|^2}{4\pi nmc^2 k^2} \left[kv_y \frac{\partial}{\partial v_x} - (\omega_{-k} + kv_x) \frac{\partial}{\partial v_y} \right] \left[\frac{\partial}{\partial v_y} - ikv_y J \frac{\partial}{\partial v_x} \right] \langle U(t) \rangle e^{ik\cdot x(0)} . \quad (41)$$

The last factor in the right hand side of (41) can be treated as follows :

$$\left[\frac{\partial}{\partial v_y} - ik\, v_y J \frac{\partial}{\partial v_x} \right] \langle U(t) \rangle e^{ik\cdot x(0)} = \frac{\partial (ik\cdot \Delta x)}{\partial v_y} e^{ik\cdot x(0)} \langle e^{ik\cdot \Delta x} \rangle - ikv_y J \frac{\partial (ik\cdot \Delta x)}{\partial v_x} e^{ik\cdot x(0)} \langle e^{ik\cdot \Delta x} \rangle . \quad (42)$$

If we can make the further approximation $\Delta x \simeq -v_x t$, we have

$$\left[\frac{\partial}{\partial v_y} - ikv_y J \frac{\partial}{\partial v_x} \right] \langle U(t) \rangle e^{ik\cdot x(0)} = -k^2 v_y J t \langle U(t) \rangle e^{ik\cdot x(0)} . \quad (43)$$

Using (43), we obtain for the right hand side of (41)

$$i\omega_p^2 \sum_k \frac{|\delta B_k|^2}{4\pi nmc^2 k^2} \left[kv_y \frac{\partial}{\partial v_x} - (\omega_{-k} + kv_x) \frac{\partial}{\partial v_y} \right] \left[\frac{\partial}{\partial v_y} - ikv_y J \frac{\partial}{\partial v_x} \right] \times \langle U(t) \rangle e^{ik\cdot x(0)} = i\omega_p^2 \sum_k \frac{|\delta B_k|^2}{4\pi nmc^2 k^2} \left[k^3 v_y{}^2 t \left(iktJ - \frac{\partial J}{\partial v_x} \right) + k^2 J t (\omega_{-k} + kv_x) \right] \langle U(t) \rangle e^{ik\cdot x(0)} . \quad (44)$$

If we can now make the further approximation in (44) as

$$\frac{\partial J}{\partial v_x} \simeq -iktJ \ , \tag{45}$$

(44) becomes

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} + \Theta^2 t + \Gamma^3 t^2 \right] \langle U(t) \rangle e^{ik \cdot x(0)} = 0 \ , \tag{46}$$

where

$$\Theta^2 = -i\omega_p^2 \sum_k \frac{|\delta B_k|^2}{4\pi n m c^2} (\omega_{-k} + kv_x) J \tag{47}$$

and

$$\Gamma^3 = 2 \ \omega_p^2 \sum_k \frac{|\delta B_k|^2}{4\pi n m c^2} k^2 v_y^2 J \ . \tag{48}$$

From (46) we have

$$\langle U(t) \rangle = \langle U(t) \rangle_0 \ \exp\left(-\frac{\Theta^2}{2} t^2 - \frac{\Gamma^3}{3} t^3\right) \ , \tag{49}$$

where $\langle U(t) \rangle_0$ is defined by

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} \right) \langle U(t) \rangle_0 \ e^{ik \cdot x(0)} = 0 \ . \tag{50}$$

In the linear case, J can be written as follows

$$J = \int_0^\infty e^{i\omega\tau} \langle e^{ik\Delta x} \rangle \, d\tau \simeq \int_0^\infty e^{i\omega\tau} e^{-ikv_x\tau} \, d\tau = \frac{i}{\omega - kv_x} \ . \tag{51}$$

From equations (49) and (51), we get

$$J = \int_0^\infty d\tau \ \exp\left[\ i(\omega - kv_x)\tau - \frac{\Theta^2}{2} \tau^2 - \frac{\Gamma^3}{3} \tau^3 \ \right] . \tag{52}$$

The term Θ^2 represents frequency shift so that we neglect it in (52). Equation (52) can be written as follows :

$$J = \int_0^\infty d\tau \, \exp[\, i(\omega - kv_x)\tau - \frac{\Gamma^3}{3}\, \tau^3 \,] \;, \tag{53}$$

where

$$\Gamma^3 = 2\omega_p^2 \sum_k \frac{|\delta B_k|^2}{4\pi nmc^2}\, k^2 v_y^2 \int_0^\infty d\tau \, \exp[\, i(\omega - kv_x)\tau - \frac{1}{3}\, \Gamma^3\tau^3]. \tag{54}$$

The nonlinear susceptibility χ^{NL} is described as follows,

$$\chi^{NL} = \frac{\omega_p^2}{n\omega^2} \int dv[\, \frac{kv_y^2}{i} \, (\int_0^\infty d\tau \, \exp[i(\omega - kv_x)\tau - \frac{\Gamma^3}{3}\, \tau^3] \,) \, \frac{\partial}{\partial v_x} + v_y \frac{\partial}{\partial v_y} \,]\langle f\rangle \;. \tag{55}$$

After Bezzerides and Weinstock's analysis [7], the time integral (55) can be approximated by using a critical velocity v_c given by

$$(\omega - kv_c)^2 \simeq 2\Gamma^2(v_c)\,, \tag{56}$$

and by replacing $\frac{1}{3}\Gamma^3(v_c)\tau^3$ by $\frac{1}{2}\Gamma^2(v_c)\tau^2$. With these approximations, we can evaluate (55) and then solve the real and imaginary parts of (18). At saturation, the real part of χ^{NL} is found to differ from the real part of χ^L by a small amount which can be neglected for the present purpose. Note that $\mathrm{Im}(\chi^{NL})$ nearly equal to $\mathrm{Im}(\chi^L)$ in which one replaces v_{Tx}^{c2} and v_{Tx}^{h2} by $v_{Tx}^{c2} + 2\Gamma^2(v_c)/k^2$ and $v_{Tx}^{h2} + 2\Gamma^2(v_c)/k^2$, respectively. That is

$$\mathrm{Im}(\chi^{NL}) \simeq \mathrm{Im}(\chi^L[v_{Tx}^2 + 2\Gamma^2(v_c)/k^2])\,, \tag{57}$$

$$\mathrm{Re}(\chi^{NL}) \simeq \mathrm{Re}(\chi^L)\,. \tag{58}$$

The nonlinear saturation level can be determined by the fact that the nonlinear growth rate of unstable waves is zero at saturation, i.e.,

$$\mathrm{Im}(1 + \chi^{NL}) = 0. \tag{59}$$

By using (23), (57) and (59), the saturation value of $\Gamma(v_c)$ for mode 1 can be determined as the solution of the following equation,

$$
\begin{aligned}
&(1 + k^2c^2/\omega_p^2)\Gamma^4(v_c) + [(1 + k^2c^2/\omega_p^2)(v_{Tx}^{c2} + v_{Tx}^{h2}) - \alpha v_{Ty}^{h2} \\
&- (1-\alpha)v_{Ty}^{c2}]k^2\Gamma^2(v_c)/2 + [(1 + k^2c^2/\omega_p^2)v_{Tx}^{c2}v_{Tx}^{h2} - \alpha v_{Ty}^{h2}v_{Tx}^{c2} \\
&- (1-\alpha)v_{Ty}^{c2}v_{Tx}^{h2}]k^4/4 = 0 . \qquad (60)
\end{aligned}
$$

For mode 2, we can similarly obtain

$$
\begin{aligned}
&(1 + k^2c^2/\omega_p^2)\Gamma^4(v_c) + [(1 + k^2c^2/\omega_p^2)(v_{Tx}^{h2} + v_{Tx}^{c2}) \\
&- \alpha(v_{Ty}^{h2} + v_d^{h2}) - (1-\alpha)(v_{Ty}^{c2} + v_d^{c2})]k^2\Gamma^2(v_c)/2 + [(1 \\
&+ k^2c^2/\omega_p^2)v_{Tx}^{h2}v_{Tx}^{c2} - \alpha(v_{Ty}^{h2} + v_d^{h2})v_{Tx}^{c2} - (1-\alpha)(v_{Ty}^{c2} + v_d^{c2})v_{Tx}^{h2}] \\
&\times k^4/4 = 0 . \qquad (61)
\end{aligned}
$$

On the other hand, (54) at $v=v_c$ can be simplified as

$$
\Gamma^4(v_c) \simeq 0.27(\pi/2)^{1/2} \frac{e^2k^2v_{Ta}^2}{m^2c^2} \sum_k |\delta B_k|^2 . \qquad (62)
$$

Through the relation (60), (61) and (62), the saturation magnetic field energy $\sum_k|\delta B_k|^2$ can be determined.

§5. Numerical simulation of electromagnetic instability

In order to simulate an electromagnetic instability, we carry out a numerical calculation by using an electromagnetic particle in-cell code [8] for electrons. The code is one-dimensional in space (x) and two-dimensional in velocity (v_x and v_y). Ions are fixed in space to form the charge neutralizing background. Induced electric and magnetic fields are respectively indicated by $E=(E_x, E_y, 0)$ and $B=(0, 0, B_z)$. To determine the initial electron velocity distribution function f_0 in (19), we choose the following parameters for mode 1 : $v^c_{Tx}= 0.04c$, $v^c_{Ty}= 0.05c$, $v^h_{Tx}= 0.2c$, $v^h_{Ty}= 0.25c$, $v^c_d= 0.03c$, $v^h_d= 0.15c$ and $\alpha=1/6$.

These parameters satisfy the condition that the electron plasma has neither current nor temperature anisotropy in the steady state as given by the same authers [4]. Therefore we can expect that there is no unstable mode propagating in the y-direction. The simulation is carried out under the conditions that the grid spacing is $0.4c/\omega_p$, the time step is $0.2\omega_p^{-1}$, the length of simulation range is $25.6c/\omega_p$, and the total number of simulation particles is 4096.

Induced magnetic fields at the early stage are plotted in Figure 1(a), from which we can estimate that the maximum growth rate γ_{Max} is about $0.01\omega_p$. This growth rate is consistent with $\gamma\sim 0.004\omega_p$ which is given by the linear theory for $kc/\omega_p= 0.43$. Figure 1(b) shows the intensity of induced magnetic fields. For the fastest growing mode, i.e. $kc/\omega_p= 0.43$, we obtain the intensity in the order of magnitude of $B_{k=0.43\omega_p/c} \sim 4.0\times 10^{-3} \sqrt{4\pi nmc^2}$ at saturation. On the other hand, by using (60) and (62) we compute the value of the field fluctuation as $B_k \sim 4.5\times 10^{-3} \sqrt{4\pi nmc^2}$ at $kc/\omega_p= 0.43$. These values are in good agreement with each other. In Figure 2(a), the thermal and total energies are plotted versus the time, being separated in the x- and y-directions for cold and high-energy electrons. A drastic change in the energy flux Q is observed in Figure 2(b). The energy flux Q decreases exponentially with the time, i.e. $Q \propto \exp(-\nu_Q t)$ with $\nu_Q \sim 3.1\times 10^{-3}\omega_p$. The theory suggests that ν_Q is also of the order of $1.6\times10^{-3}\omega_p$.

For mode 2, we choose the following parameters : $v_{Tx}^c = v_{Ty}^c = 0.05c$, $v_{Tx}^h = v_{Ty}^h = 0.25c$, $v_d^c = 0.04c$, $v_d^h = 0.2c$ and $\alpha=1/6$. In this case we can expect that there is no unstable mode in the y-direction. Induced magnetic fields at the early stage are plotted in Figure 3(a), from which we can also estimate that the maximum growth rate γ_{Max} is about $0.01\omega_p$. The growth rate which is calculated by the linear theory is $0.01\omega_p$ for $kc/\omega_p = 0.54$. The intensity of induced magnetic fields at the early stage is shown in Figure 3(b). The saturated magnetic field is in the order of $B_{k=0.54\omega_p} \sim 4.5\times10^{-3}\sqrt{4\pi nmc^2}$. The theoretical value of the saturation field is about $5.5\times10^{-3}\sqrt{4\pi nmc^2}$ which is in good agreement with the computer experiment value. In Figure 4(a), the thermal and total energies are plotted versus the time. Energies are re-distributed among cold and high energy electrons and the anisotropy of the total energies remains even at the final stage. The energy flux in Figure 4(b) decreases exponentially with $\nu_Q \sim 1.5\times10^{-3}\omega_p$. The theory suggests that ν_Q is also of the order of $\nu_Q \sim 2.5\times10^{-5}\omega_p$.

§6. Summary and conclusions

We have investigated thermal-flux reduction in the electron plasma by electromagnetic instabilities. Because of the anisotropic distribution function with electron thermal flux, electromagnetic fields thus generated reduce the electron thermal flux.

From the obtained relation between the temperature gradient and thermal flux

$$Q = - K_{eff}\frac{\partial T}{\partial x}, \tag{63}$$

where $K_{eff} = nT/m\nu_{eff}$, one can simply estimate the anomalous heat-flux reduction. Denoting $\partial/\partial x$ by $1/L$ and k by ω_p/c, we have

$$Q = -n\varepsilon v_T \eta \tag{64}$$

where η is the reduction of the thermal flux from its free streaming value, v_T and ε, respectively, denote the averaged thermal velocity and thermal energy.

1) For mode 1, we obtain

$$\eta \sim \varepsilon/[\ (18\pi)^{1/2} m\omega_p cL(\ \sum_k |\delta B_k|^2/4\pi mnc^2)\ ]\ \sim\ 0.008, \tag{65}$$

assuming $\varepsilon = 1$ kev, $n = 10^{22} cm^{-3}$, $L = 100\mu$ and $\sum_k |\delta B_k|^2/4\pi nmc^2 = 10^{-4}$.

2) For mode 2, we obtain

$$\eta \sim \varepsilon v_{Txa}/[\ (\pi(2\pi)^{1/2}/12)^{1/2} m\omega_p c^2 L(\ \sum_k |\delta B_k|^2/4\pi nmc^2)^{3/2}\ ]$$
$$\sim 0.80\ , \tag{66}$$

with the same parameters as those given for 1).

These value are consistent with the reduction of heat flow deduced from recent laser-plasma experiments [1]. Our theory with respect to the reductive rate of thermal flux with time and the saturated intensity of induced magnetic fields are in good agreement with the results by computer simulation.

Acknowledgement

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Figure Captions

Fig.1 : Energies of induced magnetic fields, (a) at the linear stage and (b) at the nonlinear stage with initial conditions ; $v_{Tx}^{c} = 0.04c$, $v_{Ty}^{c} = 0.05c$, $v_{Tx}^{h} = 0.2c$, $v_{Ty}^{h} = 0.25c$, $v_{d}^{c} = 0.03c$ and $v_{d}^{h} = 0.15c$.

Fig.2 : (a) Thermal and total energies versus the time and (b) energy flux versus the time with the same conditions as those in Fig.1.

Fig.3 : Energies of induced magnetic fields, (a) at the linear stage and (b) at the nonlinear stage with initial conditions ; $v_{Tx}^{c} = v_{Ty}^{c} = 0.05c$, $v_{Tx}^{h} = v_{Ty}^{h} = 0.25c$, $v_{d}^{c} = 0.04c$ and $v_{T}^{h} = 0.2c$.

Fig.4 : (a) Thermal and total energies versus the time and (b) energy flux versus the time with the same conditions as those in Fig.3.

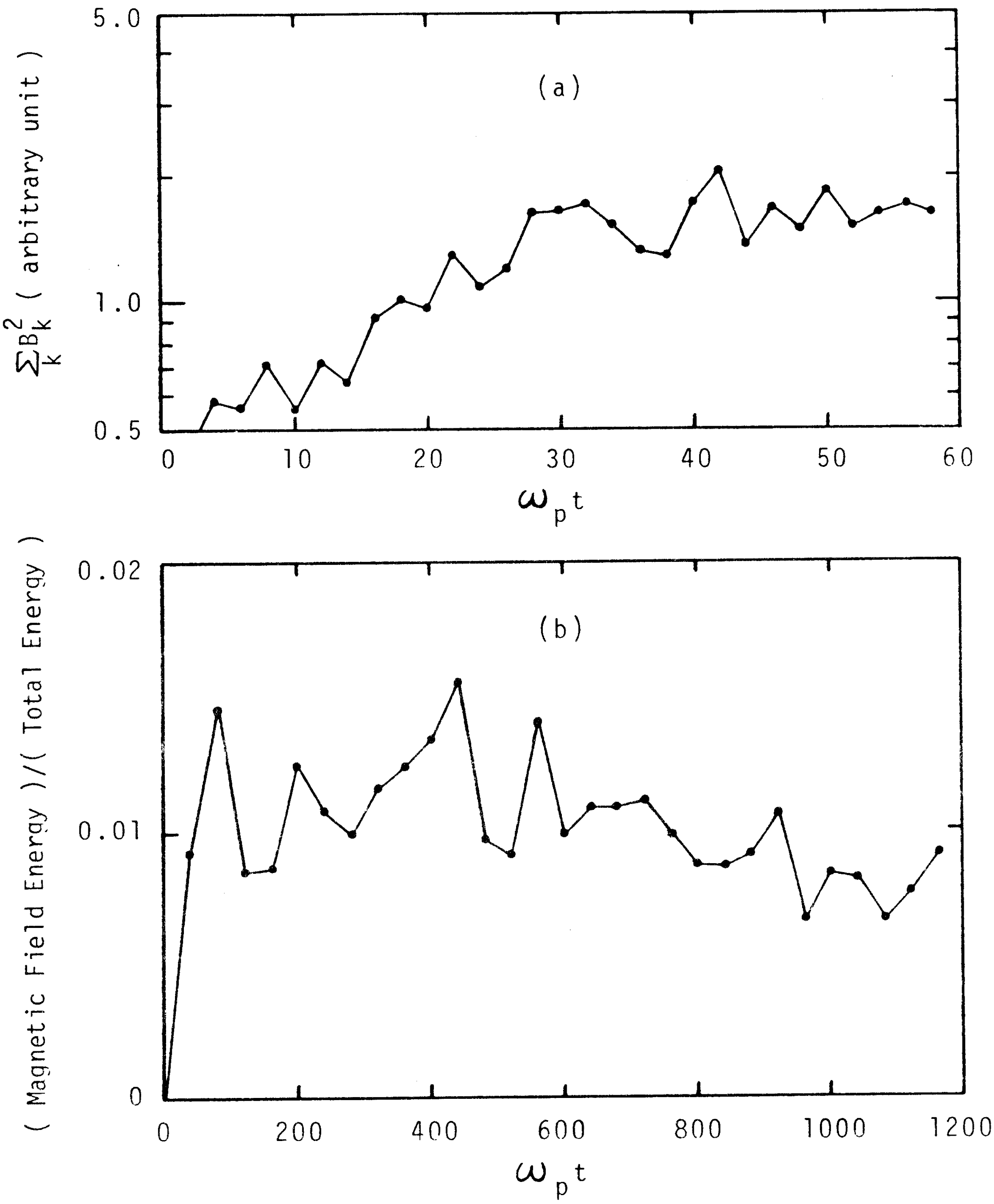
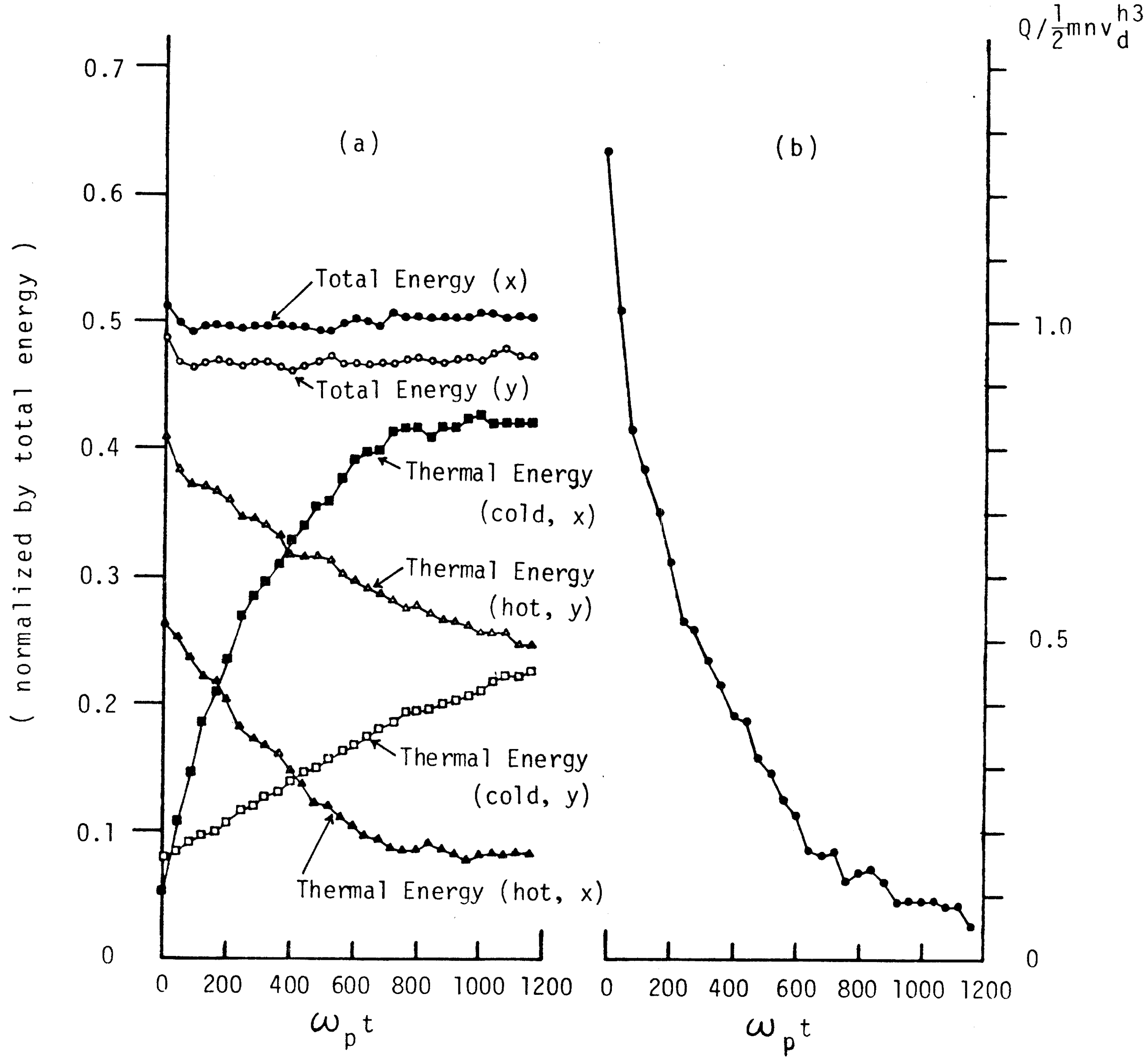

(a)
$\sum_k B_k^2$ (arbitrary unit)
5.0
1.0
0.5
0
10
20
30
40
50
60
$\omega_p t$
(b)
(Magnetic Field Energy)/(Total Energy)
0.02
0.01
0
0
200
400
600
800
1000
1200
$\omega_p t$

Fig. 1



(a)
(b)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
(normalized by total energy)
Total Energy (x)
Total Energy (y)
Thermal Energy (cold, x)
Thermal Energy (hot, y)
Thermal Energy (cold, y)
Thermal Energy (hot, x)
0 200 400 600 800 1000 1200
$\omega_p t$
$Q/\frac{1}{2}mnv_d^{h3}$
1.0
0.5
0
0 200 400 600 800 1000 1200
$\omega_p t$

Fig. 2

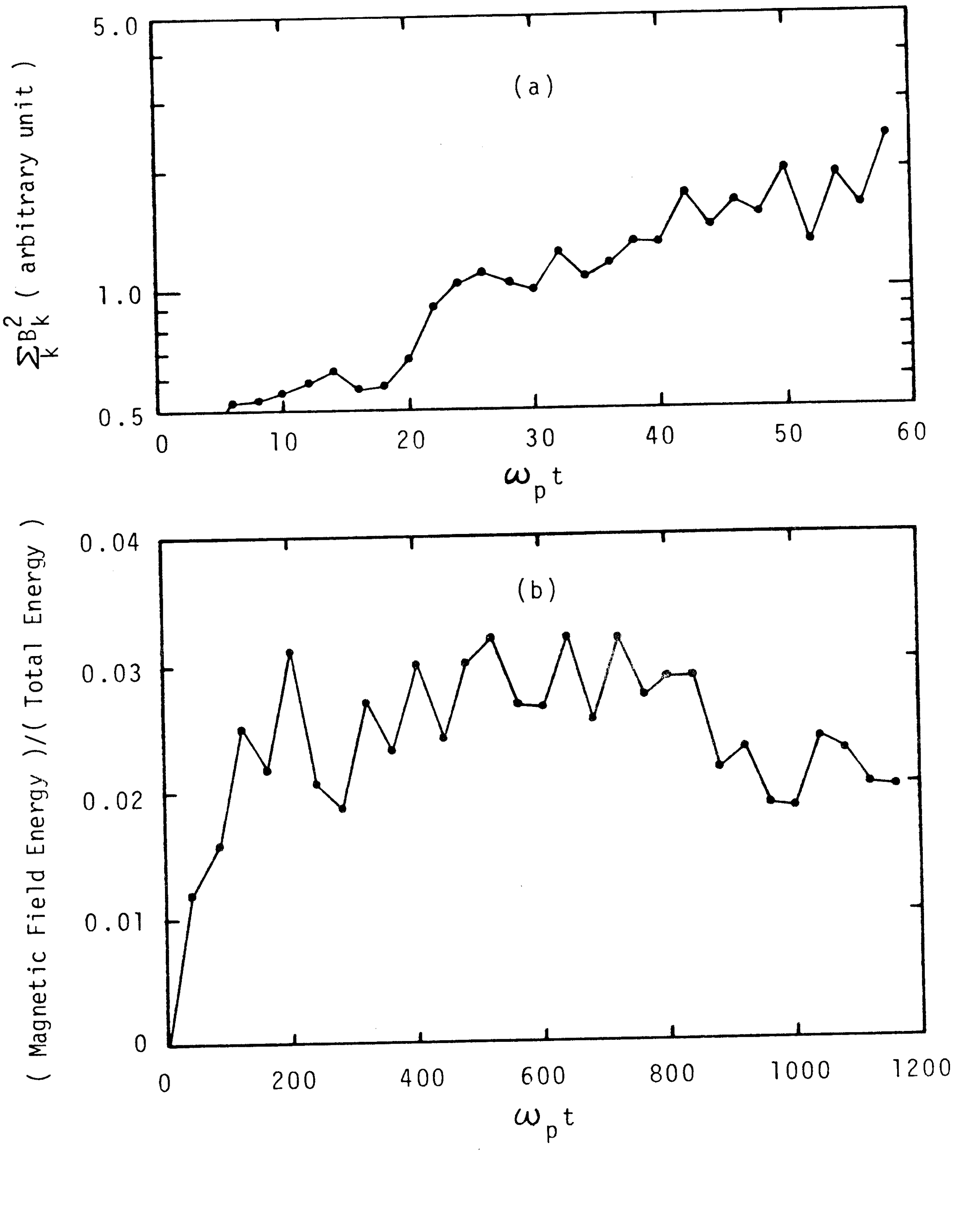

(a)
5.0
1.0
0.5
$\sum_k B_k^2$ (arbitrary unit)
0
10
20
30
40
50
60
$\omega_p t$
(b)
0.04
0.03
0.02
0.01
0
(Magnetic Field Energy)/(Total Energy)
0
200
400
600
800
1000
1200
$\omega_p t$

Fig. 3

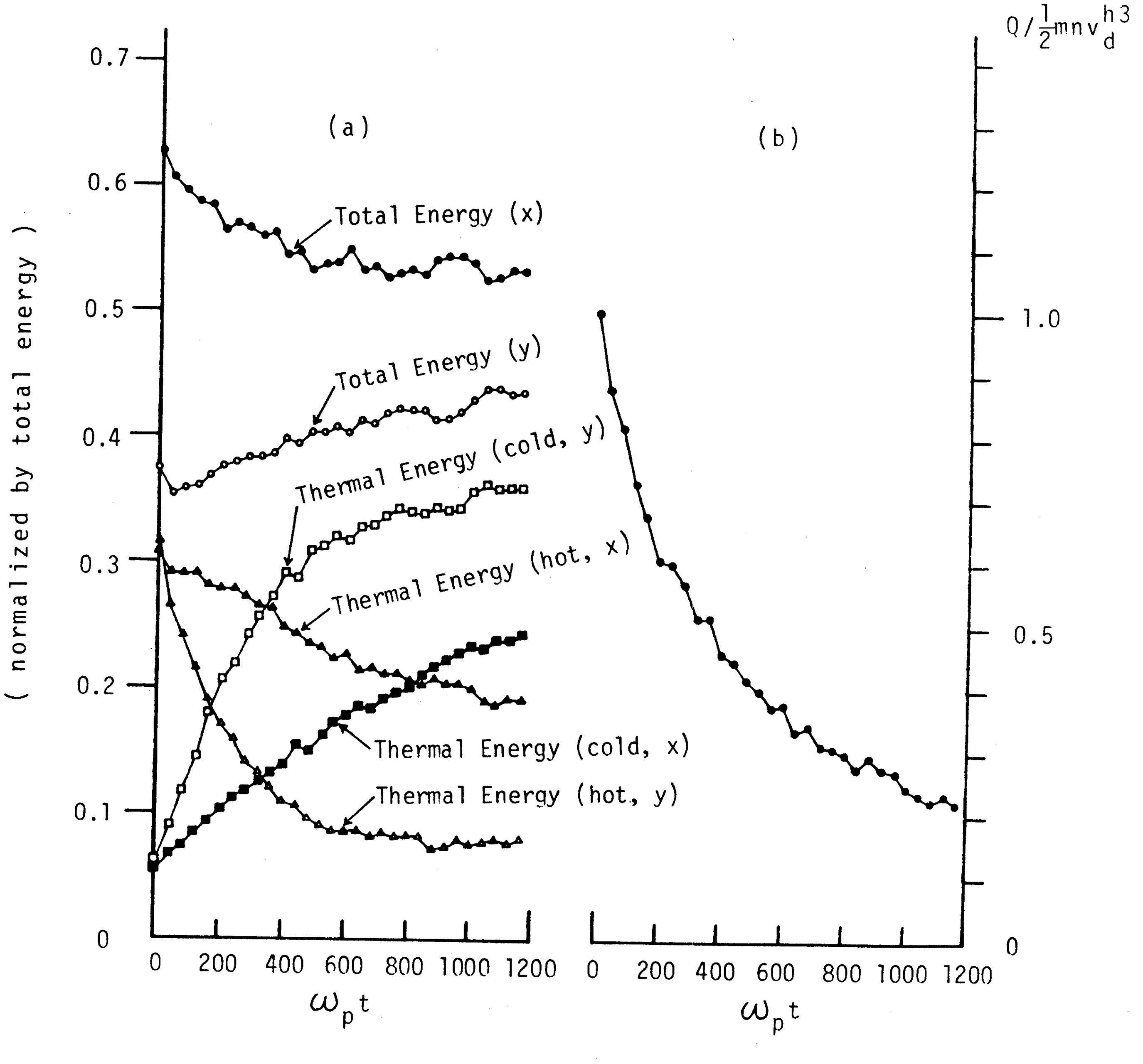

(a)
(b)
(normalized by total energy)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Total Energy (x)
Total Energy (y)
Thermal Energy (cold, y)
Thermal Energy (hot, x)
Thermal Energy (cold, x)
Thermal Energy (hot, y)
0 200 400 600 800 1000 1200
ωpt
Q/½mnv d h3
1.0
0.5
0
0 200 400 600 800 1000 1200
ωpt

Fig. 4