### INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

# RESEARCH REPORT

NAGOYA, JAPAN

A Heating Mechanism of Ions

Due to Large Amplitude Coherent Ion Acoustic Wave

Nobuo YAJIMA, Yoshinobu KAWAI and Ken KOGISO\*\*

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\* Permanent Address: Research Institute for Applied Mechanics

Kyushu University, Fukuoka

\*\* Permanent Address: Department of Physics, Kyushu University,

Fukuoka

#### Synopsis

Ion heating mechanism in a plasma with a coherent ion acoustic wave is studied experimentally and numerically.

Ions are accelerated periodically in the electrostatic potential of the coherent wave and their oscillation energy is converted into the thermal energy of ions through the collision with the neutral atoms in plasma. The Monte Carlo calculation is applied to obtain the ion temperature. The amplitude of the electrostatic potential, the mean number of collisions and the mean life time of ions are treated as parameters in the calculation. The numerical results are compared with the experiments and both of them agree well. It is found that the ion temperature increases as the amplitude of the coherent wave increases and the high energy tail in the distribution function of ions are observed for the case of large wave-amplitude.

#### §1. Introduction

Study of ion acoustic turbulence  $^{1-7)}$  in a plasma has been made from the point of view of heating ions as well as of clarifying nonlinear phenomena. In some cases  $^{8,9)}$ , the observed waves are rather coherent and the coherent waves seem to play an important role in ion heating in the plasma.

We can estimate qualitatively the ion heating caused by such a coherent wave in a weakly ionized plasma where collisions between the ions and neutral atoms are dominant. If a frequency of the coherent waves is of the same order as the collision frequency between the ions and neutral atoms so that the energy which the ions obtain from the coherent wave would be thermalized by the collisions, the heating rate of the ions may be given by  $dE_C/dt = e\phi/\tau_C \simeq \omega\sqrt{W/n_eT_e} T_e$  where  $\phi$  and W are a potential and a wave energy of the coherent wave, respectively, and  $\tau_{c}$  is a collision time which is assumed to be of the same order as  $\omega$ . On the other hand, in case of turbulent waves, the heating of ions would take place statistically  $dE_{t}/dt = M/2(eE/M)^{2}t_{C} \simeq \omega(W/n_{e}T_{e})T_{e}$ , where t<sub>C</sub> and E is a correlation time of the turbulent waves and electric fields, respectively. Since  $W/n_e T_e < 1$  for weak turbulence, we have then  $dE_{c}/dt > dE_{t}/dt$ , which means that we can expect an effective heating of the ions by the coherent waves. This heating would last until the ions are trapped by potential troughs of the coherent waves:  $M/2(\omega/k)^2 = e\phi$ . Once trapping occurs,

an equilibrium between the waves and the ions would be established so that we have  $n_e T_i = W$ . Therefore, substituting the relation  $W = E^2/4\pi k^2 \lambda_D^2 \simeq (e\phi/T_e)^2 n_e T_e$ , we have  $T_i = (\omega/kC_s)^4 T_e/4$ . This is considered to be a maximum temperature which could be achieved.

In this paper, we report that the ion heating is experimentally observed when the ion acoustic waves excited by the current-driven ion acoustic instability<sup>10)</sup> are coherent. In order to explain these experimental results based on the physical model described above, an energy distribution function of ions is calculated by means of a Monte Carlo method. The numerical results indicate a qualitative agreement between the experiment and the calculation.

The experimental arrangement is described in §2 and the results are presented in §3. The theoretical model is qualitatively explained in §4. In §5 the numerical results obtained by the Monte Carlo method are presented and are compared with the experiments. The last section is devoted to summary and discussion.

#### §2. Experimental Arrangement

The experiments were performed using a glass discharge tube of 10 cm in diameter as shown in Fig.1. The central part was constricted in order to increase the electron drift velocity. The constricted part was 4.4 cm in diameter and 20 cm in

length, respectively. A discharge was operated using argon gas at pressure of  $(0.5 - 1.5) \times 10^{-3}$  Torr. A maximum discharge current was 1 A and a discharge voltage (50 - 60 V) was almost constant. The density and temperature of electrons were measured by cylindrical probes made of 0.7 mm-diameter tungsten wire and were found to be homogeneous along the tube except for a hot cathode side of the constriction. Plasma densities were varied in the range from  $9 \times 10^9$  to  $3 \times 10^{10}$  cm<sup>-3</sup> by changing a discharge current  $I_d$ , and the electron temperature was in the range of (3 - 6) eV. The rotatable Langmuir probe was used to obtain the electron drift velocity  $v_d$  and the ratio  $v_d/v_e$  was varied in the range (0.02 - 0.11) by changing  $I_d$ . In our experimental conditions, the ion drift velocity  $3 \times 10^4$  cm/s, which is smaller than the ion acoustic velocity calculated from the electron temperature.

Unstable waves in the plasma were picked up as potential fluctuations by the Langmuir probes, connected to the high impedance amplifier and were analyzed by means of both an oscilloscope and a spectrum analyzer. Density fluctuations of the waves were obtained from electron saturation current fluctuations to the Langmuir probe biased at a potential larger than the plasma potential. Phase velocities of the waves were observed from the phase differences of oscilloscope signals among the cylindrical probes. In order to obtain the ion temperature, the energy distribution function of the ions was

measured by an energy analyzer which consisted of two fine meshed grids and a collector of 5mm in diameter. A resolution of the energy analyzer was less than 0.1 eV.

#### §3. Experimental Results

The ion acoustic waves were excited by the current-driven ion acoustic instability  $^{6-8}$ . Power spectrum measurements show that the waves consist of coherent and continuous components, as is shown in Fig.2. The coherent waves accompanied by many higher harmonics appear in the lower frequency which is of the order of the collision frequency  $v_i$  (10 - 20 kHz) between the ions and the neutral atoms. In the previous paper, larger waves already reported the detailed results of the coherent waves and, here, restrict ourselves to describe the interesting points concerning the ion heating caused by the coherent waves.

When the discharge current  $I_d$  was increased, the amplitude  $\phi$  of the fundamental mode first increased and at the same time the phase velocity decreased compared with the ion acoustic speed  $C_s$  (Details are stated in refs.ll - l2.). When once the discharge current  $I_d$  exceeds a certain critical current  $I_{cr}$ , the spectrum suddenly changes from coherent to continuous one. Furthermore, the coherent waves were found to be modulated by a lower frequency wave, which has been observed elsewhere as well  $^{13,14}$ ). However, the reason for such a sudden transition

of the spectrum has not been understood.

We measured the energy distribution function of the ions for different discharge currents by the energy analyzer, which was aligned perpendicular to a plasma column. Figure 3 is typical V-I characteristics plotted on a semi-logarithmic scale where the experimental values are shown by solid lines. This figure indicates that (1) the observed energy distribution function is considered to be a Maxwellian and (2) the ion temperature increases with increasing wave amplitude and for  $e\phi/T_e>0.24$  ion distribution has a high energy tail, two component Maxwellian. The ion temperature obtained from the V-I characteristics is plotted as a function of  $e\phi/T_e$  by the dotts in Fig.4.

In the present experiment, the measurements of the ion temperature was made carefully in order to distinguish it from an apparent temperature due to potential fluctuations in the plasma. The temperature was almost constant for a high pressure case such that the ions lose the energy obtained from the waves by much collisions, although the wave amplitude increased with increasing the discharge current. This means that the present temperature measurement can be reliable.

Under the present experimental conditions the collision frequencies for both elastic and charge exchange collision can be estimated to be in the range of (10 - 20 kHz) at  $T_i$  = 0.1 eV. On the other hand, the transit time  $t_r$  of the ions

to traverse the diameter of the discharge tube is nearly equal to  $8.2 \times 10^{-5}$  sec and then we get  $t_r v_i \approx 0.8 - 1.6$  so that the ions which interact with the coherent waves can make a collision with the neutral atoms before going to the tube wall. Therefore we can use the model described in §1 in order to explain the experimental results theoretically.

#### §4. Theoretical Model

First, we discuss qualitatively a heating mechanism of ions in plasmas in which a large amplitude coherent ion acoustic wave is excited. The amplitude of the coherent wave is assumed to be constant. This assumption is justified in our experimental situation.

Now consider the motion of ions in the coordinate system which moves with the coherent wave, i.e., the wave frame. The z-axis is taken in the direction of the wave propagation. On this frame, let the electrostatic potential be -  $\phi$  cos(kz). We consider the motion of ion with an initial velocity (  $u_0$ ,  $v_0$ ,  $w_0$ ). The energy conservation law gives the relation

$$(M/2) (dz/dt)^2 - e \phi \cos(kz) = (M/2) w_0^2,$$
 (1)

where M and e are the mass and the electric charge of ion, respectively. The x- and y-components are kept constant values,  $\mathbf{u}_0$  and  $\mathbf{v}_0$ . The initial temperature of ions is suf-

ficiently lower than the electron temparature and, therefore, it may be expected that  $w_0 \simeq -\omega/k$ . The period of ion in the electrostatic potential is given by

$$t_p = (2\kappa/kc_s) (e\phi/T_e)^{-1/2} K(\kappa^2)$$
  $(\kappa^2 < 1)$ ,

$$t_p = (4/kc_s) (e\phi/T_e)^{-1/2} K(1/\kappa^2)$$
  $(\kappa^2 > 1)$ ,

where

$$\kappa^2 = 4(e\phi/T_e)/[(\omega/kc_s)^2 + 2e\phi/T_e],$$

$$K(\kappa^2) = \int_0^{\pi/2} d\xi/(1 - \kappa^2 \sin^2 \xi)^{1/2},$$

and ions are trapped in the potential troughs of the wave for  $\kappa^2 > 1$  and untrapped for  $\kappa^2 < 1$ . In the present experimental conditions,  $e\phi/T_e$  ranges from 0.1 to 0.3 and then  $\omega_t$  from 6.7 to 15.6. We thus obtain

$$t_r \simeq t_p \simeq \tau_c \simeq 2\pi/\omega$$
. (2)

This means that ions oscillate almost for a period in the troughs of wave potential from a collision to the succeeding collision and the oscillation energy is of the order of  $e\phi$  for the nearly trapped ions.

Next we consider the conversion rate of the energy from the motion along the z-axis to the perpendicular one due to collisions with neutral atoms. For the simplicity, we assume that the ion-neutral scattering is elastic and isotropic in the center of mass system and the target atoms are at rest in the laboratory system. Let  $\mathbf{v}$  and  $\mathbf{v}'$  be velocities of an ion in the laboratory system before and after a collision, respectively. We choose the x-axis so that the vector  $\mathbf{v}$  is on the x-z plane. The vector  $\mathbf{v}$  makes an angle of  $\theta$  with the z-axis. In the center of mass system the direction of the ion velocity is only changed, keeping its magnitude unchanged. Putting the scattering angles in the center of mass system  $(\theta, \Phi)$ , we have

$$\mathbf{v'} = \begin{bmatrix} \mathbf{v'_x} \\ \mathbf{v'_y} \\ \mathbf{v'_z} \end{bmatrix} = |\mathbf{v}|/2 \begin{bmatrix} \sin\theta & \cos\Phi & \cos\theta & + \cos\theta & \sin\theta & + \sin\theta \\ & & \sin\theta & \sin\Phi & \\ & -\sin\theta & \cos\Phi & \sin\theta & + \cos\theta & \cos\theta & + \cos\theta \end{bmatrix}.$$

The averaged perpendicular component of the kinetic energy of the ion after the collision is then

$$\langle M/2(v_x^{1^2} + v_y^{1^2}) \rangle = \frac{5}{12} M v_x^2 / 2 + \frac{1}{6} M v_z^2 / 2$$

where the symbol < · · · > means an average with respect to scat-

tering angles  $(\Theta, \Phi)$ . Since scattering processes are independent of each other, the energy of ion after n+l collisions is

$$\langle E_{\perp}^{n+1} \rangle = \frac{5}{12} \langle E_{\perp}^{n} \rangle + \frac{1}{6} \langle E_{\parallel}^{n} \rangle$$
 (n > 0).

On account of Eq. (2), the energy of motion along the z-axis can be expected to be given by the wave potential,  $\langle E_n^n \rangle = e \phi$ . Finally we obtain

$$\langle E_{\perp}^{n} \rangle = \frac{2}{7} [1 - (5/12)^{n}] e \phi + (5/12)^{n} E_{\perp}^{0}$$

$$\approx \frac{2}{7} [1 - (5/12)^{n}] e \phi, \qquad (3)$$

where the thermal energy of ions is assumed to be small and can be then neglected,  $E_{\perp}^{0} \simeq 0$ .

The probability that the ion experiences n-collisions during time t since it was created is given by

$$p_n(t) = \frac{1}{n!} (t/\tau_c)^n \exp(-t/\tau_c).$$
 (4)

The distribution function of ions with age t is written using the transit time  $t_r$  as

$$f(t) = t_r^{-1} \exp(-t/t_r).$$
 (5)

It follows from Eqs. (4) and (5) that

$$\langle E_{\perp} \rangle = \int_{0}^{\infty} \sum_{n=0}^{\infty} p_{n}(t) \langle E_{\perp}^{n} \rangle f(t) dt$$

$$\simeq \frac{e\phi}{6} \frac{t_{r}/\tau_{c}}{1 + 7t_{r}/(12\tau_{c})}.$$
(6)

If we choose  $t_r/\tau_c$  = 1.5,  $\langle E_{\perp} \rangle$   $\simeq$  (2/15)e $\phi$  and the perpendicular component of the ion temperature is then

$$T_i/T_e = \langle E_{\perp} \rangle / T_e \simeq (2/15) e \phi / T_e$$
 (7)

These values agree well with experimental results in the order of magnitude.

§5. Results of the Monte Carlo Calculation and Comparison with Experimental Results

On the basis of the model, which is qualitatively discussed in the previous section, the heating rate of ions is calculated using the Monte Carlo method. In this section, we assumed the initial temperature of ions and the temperature of neutral atoms are both 0.01  $_{\rm e}$ . The velocity distribution of ions in the direction perpendicular to the wave propagation was obtained by pursuing the orbits of 3000 ions for each set of parameters. Details of the calculation are described in Appendix.

We calculated numerically the V-I characteristics of the ions for different amplitude. Figure 3 gives a typical example, where (a)  $e\phi/T_e = 0.1$  and (b)  $e\phi/T_e = 0.24$ . case,  $\omega/kc_s$  = 0.6 and  $t_r/\tau_c$  = 1.5 were chosen so as to fit the experimental results. It has been found that (1) the energy distribution function can be regarded as being a Maxwellian for the case of small amplitude waves and (2) the high energy tail begins to appear in the distribution with increasing wave-amplitude. In order to make the appearance of the high energy tail clear, we show the energy distribution function dividing into two parts, one is composed of the never-trapped ions and the other the ions which have been at least once trapped, as is shown (a) and (b) in Fig.5. trapped ion has a large relative velocity to the target neutral atom compared with the untrapped ions, so that the perpendicular component of the velocity of the trapped ion becomes It follows from this that the high energy tail large enough. mainly corresponds to the trapped ions and the bulk mainly to the untrapped ions.

In Fig.6, the averaged perpendicular energy of ions,  $E_{\dot{1}}$ , is shown as a function of the wave amplitude  $\phi$ , where  $E_{\dot{1}}$  is defined using perpendicular velocity distribution function  $f(\mathbf{v}_{\perp})$  by

$$E_{i} = \int (M/2) \mathbf{v}_{\perp}^{2} f(\mathbf{v}_{\perp}) d\mathbf{v}_{\perp}.$$

The phase velocity of the wave,  $\omega/k$ , was taken as a parameter, i.e.,  $\omega/kC_s=0.6$ , 0.7 and 0.85, and the ratio  $t_r/\tau_c=1.5$  was chosen. It is seen that  $E_i$  increases as  $\phi$  is increased and has a tendency toward a saturation for large  $\phi$ . This is understood from the fact that the averaged energy  $E_i$  strongly depends on the number of trapped ions which is increased with the amplitude and saturates near  $e\phi=M/2$  ( $\omega/k$ )<sup>2</sup>. Figure 7 assures this interpretation, in which the ratio of the trapped ions to the total ions at the time when the ions are produced is shown.

Figure 8 illustrates the dependence of  $E_i^-\phi$  curve on the number of collisions, where the ratio  $t_r/\tau_c=0.5$ , 1.5 and 2.0 is taken. The collision of ions with neutral atoms is effective to the thermalization of the ion energy, so that  $E_i$  increases with the parameter  $t_r/\tau_c$ . When the collision becomes frequent, the thermalization takes place before ions gain sufficient energy from the wave and the increase of  $E_i$ , therefore, saturates for large  $t_r/\tau_c$ .

In Fig.4, dependence of ion temperature  $T_i$  on the wave potential  $\phi$  is shown for the case  $\omega/kC_s=0.6$  and  $t_r/\tau_c=1.5$ . The ion temperature  $T_i$  was determined from the slope of the V-I characteristics for the numerical experiments as well as for the laboratory experiments. The temperature so obtained has an error resulting from a reading of the slope of the V-I curve. As is seen from Fig.4, the high energy tail begins to appear around  $e\phi/T_e \simeq 0.15$ . The calculated temperature of high

energetic ions and that of bulk ions agree with the experimental ones. The appearance of the high energy tail can be interpreted in terms of the ion-trapping in the potential trough of coherent wave.

#### §6. Summary and Discussion

In this paper, we have pointed out that the ions are heated effectively by the coherent ion acoustic wave, which is excited in a turbulent plasma. A possibility of the excitation of such a coherent ion acoustic wave has been already shown by Kono and one of the authors (N.Y.): The sound speed of a turbulent plasma can be lower than the ionic sound velocity  $C_s = (T_e/M)^{1/2} \text{ for the larger wavelength region, so that the resonant interaction is allowed between three ion acoustic waves and the long coherent wave can be then excited in a turbulent plasma due to this decay instability. Ions oscillate in this wave potential getting the energies during the motion from its top to trough and this oscillation energy is thermalized by the collision with neutral atoms.$ 

The collisions between ions and neutral atoms are very important in our theoretical model. When the electron temperature becomes sufficiently high, the collisional effect can not be expected. The heating mechanism through the effect of long coherent wave may seem to become less effective for such high temperature plasmas, but the turbulent waves play a role of

scatterer in place of neutral atoms so that our model, even in the high temperature plasma, is as valid as in the present case. The effective collision frequency of ions in the plasma with turbulent ion acoustic waves is estimated using the velocity diffusion coefficient,

$$\tau_{\text{turb}} \simeq (D/v_{\text{T}}^2)^{-1}$$

where  $v_T^2 = (T_i/M)$  and D is estimated by the quasilinear theory,

$$D = \pi (e/M)^{2} \sum_{\mathbf{k}} |\mathbf{E}(\mathbf{k})|^{2} \delta(\omega_{\mathbf{k}} - \mathbf{k} \cdot \mathbf{v}) \simeq (\tilde{\omega}/nM) W,$$

$$W = \sum_{\mathbf{k}} |\mathbf{E}(\mathbf{k})|^2 / 4\pi k^2 \lambda_{\mathbf{D}}^2,$$

in which  $\mathbf{E}(\mathbf{k})$  is a Fourier component of the turbulent field and  $\overset{\sim}{\omega}$  is a characteristic frequency of the turbulence. We then have

$$(\tau_{\text{turb}})^{-1} \simeq \tilde{\omega} (W/nT_e) T_e/T_i$$
.

Letting the amplitude of the coherent wave be  $\ \phi$ , we obtain the heating rate of ions as

$$dE/dt \simeq e\phi/\tau_{turb} \simeq (e\phi/T_i)^{\circ}_{\omega}(W/nT_e)T_e.$$

On the other hand, the rate of stochastic heating of ions in the turbulent plasma is

$$(dE/dt)_{stoch.} \simeq T_i/\tau_{turb} \simeq \overset{\circ}{\omega} (W/nT_e)T_e.$$

Therefore, we can say that the heating mechanism through the coherent wave is more effective than the stochastic heating because of the existence of the large factor  $(e\phi/T_{\bf i})$ .

#### References

- 1) S.Watanabe: J.Phys.Soc.Japan 35 (1973) 600.
- 2) T.Hara, T.Honzawa, Y.Kawai, S.Watanabe and J.Fujita: Phys. Letteres 43A (1974) 203.
- 3) M.Yamada and M.Raether: Phys.Rev.Letters 32 (1974) 99.
- 4) A.Mase and T.Tsukishima: Phys.Fluids 18 (1975) 464.
- 5) D.B.Ilic: Phys.Rev.Letters 34 (1975) 464.
- 6) Y.Kawai, M.Hasegawa and H.Nakashima: J.Phys.Soc.Japan 38 (1975) 1227.
- 7) Y.Kawai and M.Hasegawa: J.Phys.Soc.Japan 40 (1976) 607.
- 8) Y.Kawai and M.Tanaka: J.Phys.Soc.Japan 41 (1976) 1029.
- 9) Y.Nakagawa, M.Tanikawa, K.Watanabe, K.Adati, H.Iguchi, K.Ishii, Y.Ito, J.Jacquinot, T.Kawabe, T.Kawamura, K.Muraoka, T.Oda and R.Sugihara: Proc.5th Int.Conf. on Plasma and Control.Nucl.Fusion Res. (Tokyo 1974) No. IAEA-CN-361 C3-3.
- 10) S.Ichimaru: Ann. Physics 20 (1962) 78.
- 11) M.Kono and N.Yajima: J.Phys.Soc.Japan 41 (1976) 272.
- 12) Y.Kawai, preparing for publication.
- 13) S.Watanabe and H.Tanaca: Nucl.Fusion 12 (1972) 593.
- 14) A.Mase and T.Tsukishima: Phys.Letters 57A (1976) 140.
- 15) S.C.Brown: Basic Data of Plasma Physics (M.I.T. Press, Cambridge, 1966).

#### Appendix

Here, we explain the details of the Monte Carlo calculation applying to the present model. Consider the motion of an ion which is initially at the position  $\mathbf{z}_0$  with the velocity  $\mathbf{v}_0$ , oscillates under the effect of the wave potential, collides with a neutral atom and, repeating such processes, transits the discharge tube to be annihilated at the wall. The initial velocity distributions of ions and neutral atoms are assumed to be Maxwellian with the same temperature  $\mathbf{T}_i^0 = 0.01 \; \mathbf{T}_0$ ; i.e.,

$$f(v) dv = (M/2\pi T_i^0)^{3/2} \exp(-Mv^2/2T_i^0) dv.$$
 (A-1)

The velocities of the ion and the neutral atom before the collision,  ${\bf v}_0=(u_0,\,v_0,\,w_0)$  and  ${\bf V}=(\,{\tt U},\,{\tt V},\,{\tt W}\,\,)$  respectively, are given by the random number obeying the normal distribution (A-1). The initial position of the ion  ${\tt Z}_0$  is given by the uniform random number. The life time of the ion  ${\tt L}_D$  is given using a uniform random number  ${\tt E}$  in the interval (0,1);

$$t_{D} = t_{r} \ln (1/\xi)$$
. (A-2)

This means that the distribution of  $t_D$  is governed by

$$g(t_{D}) dt_{D} = t_{r}^{-1} exp(-t_{D}/t_{r}) dt_{D}.$$
 (A-3)

The time  $t_1$  when the first collision between the ion and the neutral atom takes place is similarly given by

$$t_1 = \tau_C \ln (1/\eta_1),$$
 (A-4)

where  $\eta_1$  is a uniform random number in the interval (0,1). The time of the n-th collision is then given by

$$t_n = \sum_{j=1}^{n-1} t_j + \tau_c \ln (1/\eta_n).$$
 (A-5)

The number of collisions, N, is determined by the condition  $\mathbf{t_{N+1}} \ ^{>} \ \mathbf{t_{D}} \ ^{>} \ \mathbf{t_{N}} \ .$ 

The motion of ion between two successive collisions, t  $_{j}$  < t < t  $_{j+1}$  , is obtained from the conservation law

$$\frac{M}{2} \left(\frac{dz}{dt}\right)^2 - e\phi \cos(kz) = \frac{M}{2} \left(w_{j}^{!}\right)^2 - e\phi \cos(kz_{j}), \quad (A-6)$$

where  $w_j^{\boldsymbol{t}}$  is the z-component of the velocity just after the n-th collision and  $z_j^{\boldsymbol{t}}$  is the position where the j-th collision occurs. The velocity  $\boldsymbol{v}_j^{\boldsymbol{t}}$  is given by assuming the isotropic and elastic scattering in the center of mass system;

$$\mathbf{v}_{j}^{!} = (1/2) [|\mathbf{v}_{j} - \mathbf{v}| \mathbf{n} + (\mathbf{v}_{j} + \mathbf{v})],$$

where  $\mathbf{v}_j$  is the velocity of the ion just before the j-th collision and  $\mathbf{v}$  that of the neutral atom. The unit vector  $\mathbf{n} = (\mathbf{n}_x, \mathbf{n}_y, \mathbf{n}_z)$  is in the direction of the ion motion in the center of mass system after the collision;

$$n_{x} = (1 - n_{z}^{2})^{1/2} \cos \phi$$
 ,  $n_{y} = (1 - n_{z}^{2})^{1/2} \sin \phi$  ,  $n_{z} = \cos \theta$  ,

and is distributed isotropically.

The velocity of the ion at an arbitrary time can be thus obtained. The perpendicular component of the ion velocity is changed only by the collisions and takes a constant values between them. The distribution function of the perpendicular component of ion velocity is calculated by assigning the statistical weight proportional to the time interval  $t_{j+1} - t_{j}$  to this constant velocity values.

#### Figure Captions

- Fig.1: Schematic diagram of the experimental apparatus.
- Fig.2: Typical frequency spectrum, where  $I_d = 200 \text{ mA}$ .

  Ordinate is the wave amplitude in logarithmic units.
- Fig.3: Typical example of V-I characteristics, where (1)  $e\phi/T_e$  = 0.1 and (2)  $e\phi/T_e$  = 0.24. Solid lines are experimental values. Dotts are calculated values by the Monte Carlo method, where  $\omega/kC_s$  = 0.6 and  $t_r/\tau_c$  = 1.5.
- Fig.4: Ion temperature as a function of wave amplitude. Dotts and triangles are measured temperature of the bulk and the high energetic ions, respectively. Calculated temperatures are shown by shaded regions. For  $e\phi/T_e > 0.15$ , high energy tail appears.
- Fig.5: Example of V-I characteristics of (a) never-trapped ions and (b) ions which have been at least once trapped, where  $\omega/kC_s$  = 0.6,  $t_r/\tau_c$  = 1.5 and  $e\phi/T_e$  = 0.15.
- Fig.6 : Dependence of  $\phi$ -E characteristics on phase velocity, where t<sub>r</sub>/ $\tau_c$  = 1.5.

- Fig.7: Ratios  $P(\phi)$  of the trapped ions to the total ions are plotted as a function of wave amplitude, where the ion distribution is assumed to be a Maxwellian with temperature  $0.01T_{\rm e}$  and the phase velocity of the wave is 0.6, 0.7, 0.85 and 1.0, normalized by  $C_{\rm s}$ .
- Fig.8 : Dependence of  $\phi$ -E characteristics on the number of collisions.

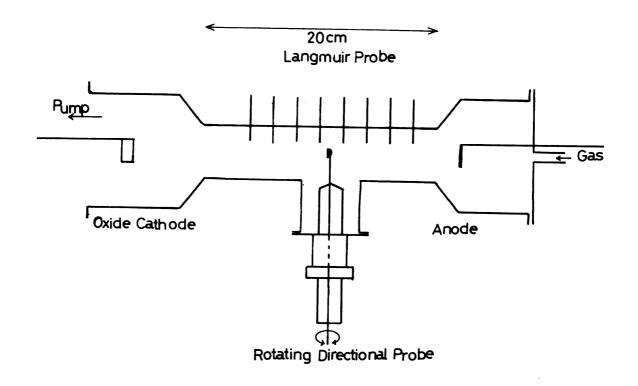
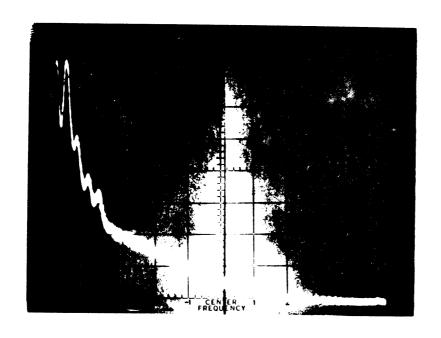
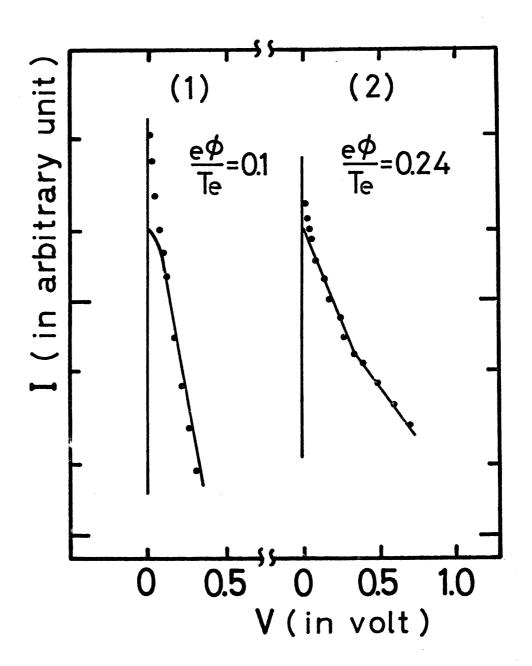


FIG.1



## 100 kHz/div



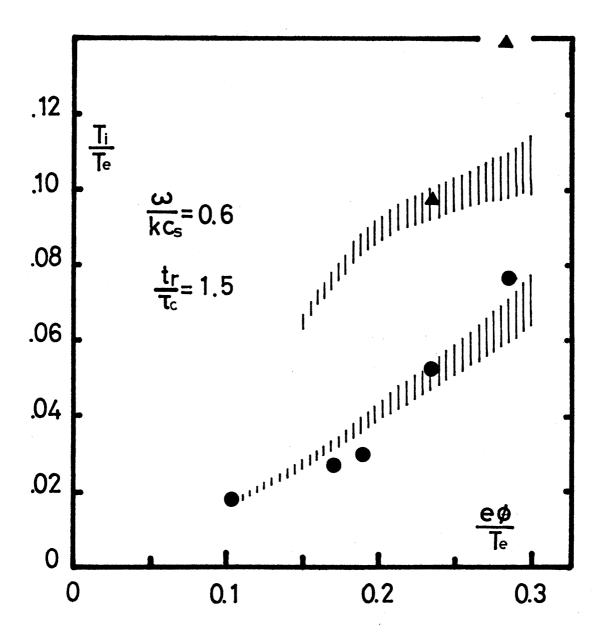


FIG.4

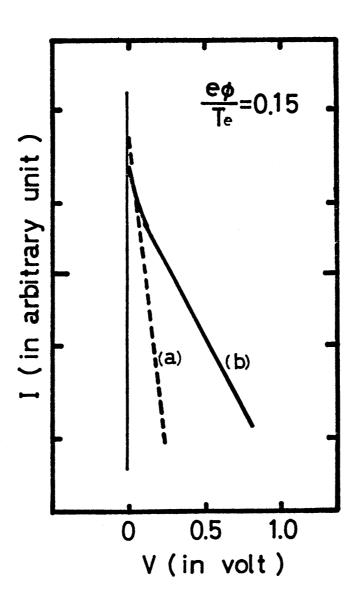


FIG.5

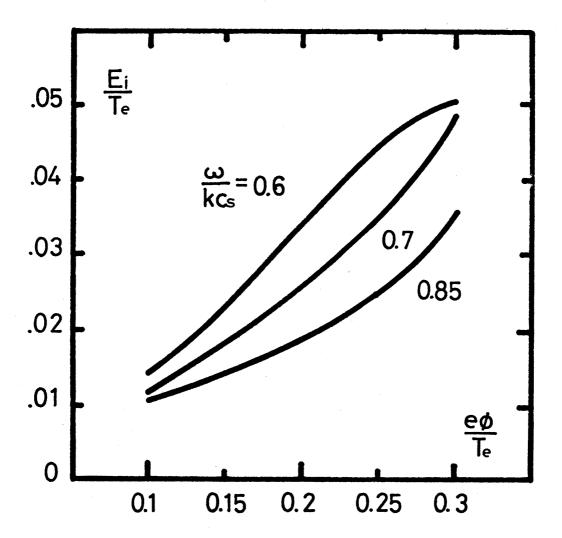
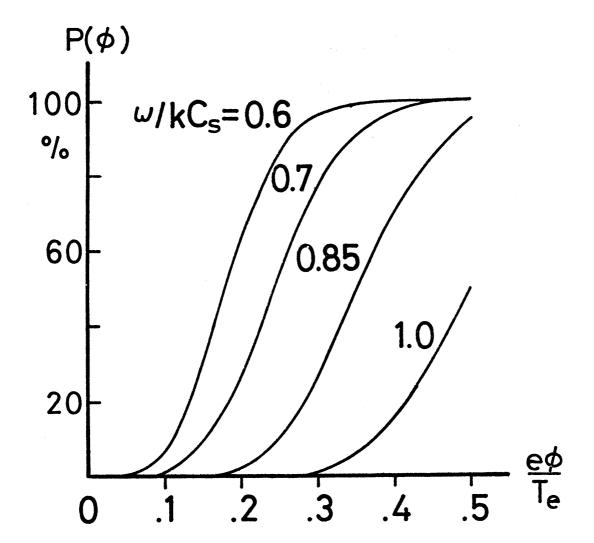


FIG.6



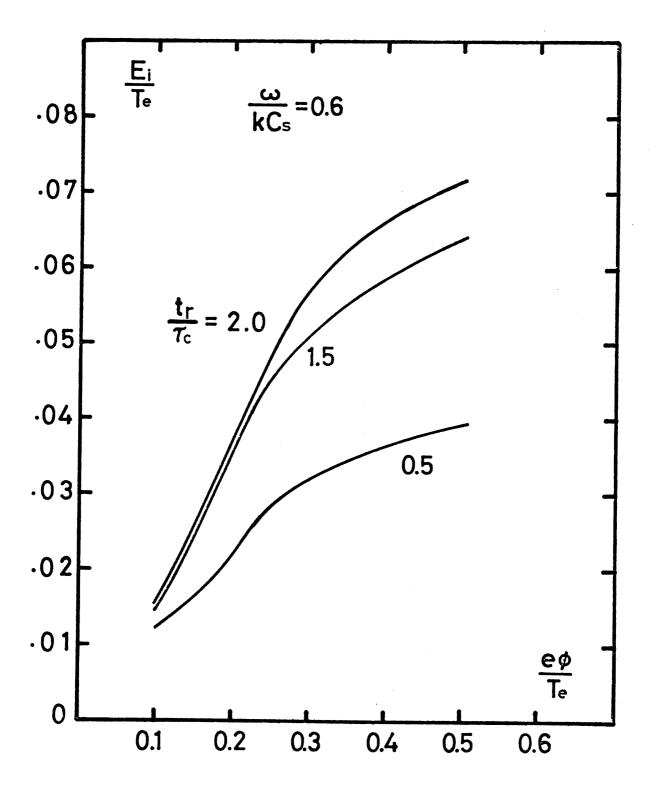


FIG.8