INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Anomalous Ion Loss by Low Frequency Instabilities

Sanae Inoue*, Kimitaka Itoh*,
Toshio Tange** and Takashi Tuda***

IPPJ -334 June 1978

Further Communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya 464, Japan.

Permanent Address:

* Department of Physics, Faculty of Science, University of Tokyo, Tokyo 113, Japan.

** Faculty of Science, Hiroshima University, Hiroshima 730, Japan.

*** Japan Atomic Energy Research Institute, Tokai-Mura, Ibaraki 319-11, Japan.

Abstract

We present the ambipolarity formula of the cross-field flux of the nonuniform plasma in the presence of the low frequency (ω << ion gyrofrequency) electromagnetic fluctuations. It is shown that owing to the ion polarization drift the ions are pulled out by the diffusing electrons so that the ambipolarity of the flux is maintained independent of the finite ion Larmor radius effect.

The anomalous loss mechanism of the toroidal plasma has been explained by use of the theory of the low frequency instabilities[1]. Such analyses have been developed mainly based on the electrostatic (ES) approximation of the fluctuations. Nowadays, there is an urgent request to understand the plasma flux in the presence of the electromagnetic (EM) drift wave fluctuations[2]. The wave length of the drift wave fluctuation is comparable to the ion gyroradius, one must employ a kinetic approach to obtain a proper understanding of the plasma flux caused by the field fluctuations. When one takes a finite ion gyroradius effect into account, one may directly consider 1) that the ion gyromotion does average out the field fluctuation for ions, therefore ions (contrary to electrons) do not diffuse in the direction of the density gradient and 2) henceforth the dc electric field piles up, stopping the electron particle flux. However, does the finite ion gyromotion truely average out and screen the fluctuations ? Even if the static potential piles up, is it evident that electrons are pulled back by ions instead of pulling ions out ? We here present the ion and the electron fluxes in the presence of EM fluctuations yielding the answer to these questions. The answer we obtain is that the ion polarization drift maintains the ambipolarity of the flux so that ions are pulled out by the diffusing electrons.
We take two main assumptions that the fluctuation has low frequency ($\omega \ll$ ion cycrotron frequency) and that the gradient of the equilibrium distribution is weak enough in comparison with ion gyroradius. We keep the generality of the argument in order that the result is applicable to various low frequency modes of confined plasmas. As an example, we have studied the electron

anomalous flux owing to the collisionless EM drift wave fluctuations, and reported that the anomaly of the particle loss is the same order as that of the heat loss[3).

We consider a collisionless plasma slab having a density inhomogeneity $\nabla n = -\kappa n\hat{x}$. The equilibrium magnetic field is in the z-direction. Note that the following arguments are directly applicable to the case in the presence of the magnetic shear. The basic assumptions used here are as follows: 1) The fluxes across a magnetic field are induced by the inhomogeneity of equilibrium macroscopic quantities, 2) the inhomogeneity is weak and the magnetic field is strong, i.e., $\kappa\rho_i \ll 1$ where ρ_i is the ion gyroradius, and 3) the fluctuations are composed of the low frequency waves ($\omega \ll \omega_{ci}$). We use the notation ~ to indicate the fluctuation, and the quantity without ~ is the static part of each physical quantity.

We have previously derived the "fluid like" equations of the magnetized plasma in which low frequency EM fluctuations are excited. The y-component of the equation of motion (for ions and electrons) is written in the form as[4)

$$n\frac{\partial V_y}{\partial t} + \Gamma_x\frac{\partial V_y}{\partial x} + \frac{1}{m}\frac{\partial P_{xy}}{\partial x} + \frac{qB}{mc}\Gamma_x - \frac{qn}{m}E_y = \frac{q}{m}\langle(\tilde{n}\tilde{\vec{E}}+\frac{1}{c}\tilde{\vec{\Gamma}}\times\tilde{\vec{B}})_y\rangle \tag{1}$$

where q is the charge and < > denotes the coarse grained average over the time and space. Taking the averaging time t_a and the averaging length ℓ_a, we assume that $\omega^{-1} \ll t_a \ll t_e$ and $k^{-1} \ll \ell_a \ll \kappa^{-1}$, where k is the typical wave number of the fluctuation and t_e is the diffusion time of the equilibrium. The moments n, Γ_i, V_i and P_{ij} are defined by use of the distribution function f as $n = \int f d\vec{v}$, $\Gamma_i = \int f v_i d\vec{v}$, $V_i = \Gamma_i/n$, $P_{ij} = m\int(v_i - V_i)(v_j - V_j) f d\vec{v}$. Note that the derivatives $\partial/\partial t$ and $\partial/\partial x$ operate on the slow variations of the equilibrium, i.e.,

$$\frac{\partial}{\partial t} \sim \frac{1}{t_e} << \omega, \quad \frac{\partial}{\partial x} \sim \kappa << \rho_i^{-1} \tag{2}$$

since the space time average is employed in deriving Eq.(1). The ratio of the 2nd term to the 4th term in the left hand side of Eq.(1) is $\omega_{ci}^{-1}\partial V_y/\partial x \sim (\kappa\rho_i)^2 << 1$. The similar argument is also made to the 3rd term[5], and P_{xy} term is neglected. We thus have

$$\Gamma_x^e = \frac{c}{B} n_e E_y + \frac{1}{B} \langle (c\tilde{n}_e \tilde{\vec{E}} + \tilde{\vec{\Gamma}}_e \times \tilde{\vec{B}})_y \rangle \tag{3}$$

$$\frac{cnm_i}{eB}\frac{\partial V_y^i}{\partial t} + \Gamma_x^i = \frac{c}{B} n_i E_y + \frac{1}{B} \langle (c\tilde{n}_i \tilde{\vec{E}} + \tilde{\vec{\Gamma}}_i \times \tilde{\vec{B}})_y \rangle \tag{4}$$

where the electron mass effect is neglected. Equation (3) shows that the electron particle flux does not depend on the ion gyroradius. Subtracting Eq.(3) from Eq.(4) we have

$$\frac{cnm_i}{eB}\frac{\partial V_y^i}{\partial t} + (\Gamma_x^i - \Gamma_x^e) = \frac{1}{B} \langle \{c(\tilde{n}_i - \tilde{n}_e)\tilde{\vec{E}} + (\tilde{\vec{\Gamma}}_i - \tilde{\vec{\Gamma}}_e)\times\tilde{\vec{B}}\}_y \rangle . \tag{5}$$

The 1st term cannot be neglected unless the plasma density is low enough[6]. The right hand side of Eq.(5) can be represented by the field fluctuations using the Maxwell's equations

$$\tilde{n}_i - \tilde{n}_e = \frac{1}{4\pi e} \nabla\cdot\tilde{\vec{E}} ,$$

$$\tilde{\vec{\Gamma}}_i - \tilde{\vec{\Gamma}}_e = \frac{c}{4\pi e} \nabla\times\tilde{\vec{B}} . \tag{6}$$

When we neglect the ion polarization drift term (the 1st term in Eq. (5)) , the rate of the broken ambipolarity may be appreciable. However, as the plasma slowly diffuses, the equilibrium distribution starts to change causing a polarization drift of ions. We then show that this polarization drift affects the ion particle flux so as to maintain the ambipolarity of the fluxes. The ion drift motion in the y-direction V_y is given by $V_y = -cE_x/B - Tc\kappa/eB$. For the slowly diffusing plasma, the time variation rate of the density gradient is given by use of the continuity equation $\dot{n} + \nabla\cdot\vec{\Gamma} = 0$ as

$$\frac{\partial\kappa}{\partial t} = -\frac{1}{n}\frac{\partial^2 n}{\partial t\partial x} - \frac{\kappa}{n}\frac{\partial n}{\partial t} \cong -\frac{1}{n}\frac{\partial^2 n}{\partial t\partial x} = \frac{1}{n}\frac{\partial^2\Gamma_x}{\partial x^2} .$$

The continuity equation of charge with Poisson's equation gives the relation $\partial E_x/\partial t = -4\pi e(\Gamma_x^i - \Gamma_x^e)$. Combining these relations we have

$$\frac{\partial V_y^i}{\partial t} = \frac{4\pi ec}{B}(\Gamma_x^i - \Gamma_x^e) - \frac{Tc}{neB}\frac{\partial^2\Gamma_x^i}{\partial x^2} . \qquad (7)$$

We neglect $\rho_i^2\partial^2/\partial x^2$ in comparison with unity. From Eqs. (5), (6) and (7), we finally obtain the general formula

$$\Gamma_x^i - \Gamma_x^e + \frac{\rho_i^2}{\varepsilon_\perp}\frac{\partial^2\Gamma_x^e}{\partial x^2} = \frac{1}{\varepsilon_\perp}\frac{c}{4\pi eB}\{\langle\nabla\cdot\tilde{\vec{E}}\tilde{E}_y\rangle + \langle[\nabla\times\tilde{\vec{B}}\times\tilde{\vec{B}}]_y\rangle\} \qquad (8)$$

with Eq. (3) and

$$\varepsilon_\perp = 1 + \frac{4\pi n_i m_i c^2}{B^2} .$$

If one Fourier transform $\tilde{\vec{E}}$ and $\tilde{\vec{B}}$ as $\tilde{\vec{E}} = \Sigma \tilde{\vec{E}}_{k\omega}(x)\exp[i(ky+k_{/\!/}z-\omega t)]$ ($k >> k_{/\!/}$), we have

$$\Gamma_x^i - \Gamma_x^e + \frac{\rho_i^2}{\varepsilon_\perp}\frac{\partial^2\Gamma_x^e}{\partial x^2} = \frac{c}{\varepsilon_\perp 4\pi eB}\Sigma\,\mathrm{Re}\left[\frac{\partial E_{k\omega x}}{\partial x}E^*_{k\omega y} - \frac{i}{k}\frac{\partial^2 B_{k\omega x}}{\partial x^2}B^*_{k\omega x}\right]. \quad (8')$$

Comparing Eq.(8) with Eq.(5), one can easily understand that the deviation of $\varepsilon_\perp$ from 1 is owing to the ion polarization drift.

Let us now approximately evaluate $\varepsilon_\perp$ for the thermonuclear plasmas. Measuring n in $10^{13}/cm^3$ and B in 10^4G, $\varepsilon_\perp$ is written as $\varepsilon_\perp = 1 + 2\times10^3 n/B^2$. Thus the ion polarization drift term reduces the broken ambipolarity by the factor 10^{-3}. It should be also noted that the right hand side of Eqs.(3) and (8) is expressed only by the field fluctuation spectrum, and the nonambipolar flux $\Gamma_x^i - \Gamma_x^e$ does not drastically depend on the ratio of ion gyroradius to the wave length, $\rho_i k$. This is our answer to the question in the beginning of this letter; ions are pulled out according to the electron flux. The gyroradius effect on the ambipolarity comes from the fluctuation spectrum.

We then estimate the rate of the broken ambipolarity ($\Gamma_x^i/\Gamma_x^e - 1$) for typical low frequency fluctuations.

Firstly, we consider the quasi-electrostatic fluctuations like EM drift wave fluctuations. In this case, the relation $B_z << B_x \sim O(\beta)E_y$ holds ($\beta = 4\pi nT/B^2 \ll 1$), so that $\tilde{\vec{\Gamma}}\times\tilde{\vec{B}}$ term in Eqs.(3) and (4) are merely small corrections[2]. Estimating $\partial E_x/\partial x \sim kE_y$ and $\partial^2\Gamma_x^e/\partial x^2 \sim \kappa^2\Gamma_x^e$, we have

$$|\ \Gamma_x^i/\Gamma_x^e - 1\ | \cong (k\rho_i)^2\left(\frac{\lambda_D}{\rho_i}\right)^4 + \lambda_D^2\kappa^2 \quad (9)$$

where λ_D is the Debye length defined by $\lambda_D^2 = T_e/4\pi n_e e^2$. Writing n in $10^{13}/cm^3$ and B in 10^4G, the value $(\lambda_D/\rho_i)^4$ is given by $3\times10^{-7} B^4/n^2$, and the 2nd term is also negligibly small. Therefore the ambipolarity of the flux in this case is almost complete. In addition to it, the ion finite gyroradius effect does not prevent ions from diffusing as fast as electrons. The fact $(\Gamma_x^i/\Gamma_x^e - 1) \propto \rho_i^{-2}$ implies that the larger the ion gyroradius is, the more accurately the ambipolarity holds.

We can imagine another limiting case where the magnetic fluctuation is dominant and $\tilde{B}_x \gg \tilde{B}_z, |\tilde{E}|$ holds. We here do not discuss the existence of such micromagnetic modes and only assume it. From the linearized Vlasov equation, we have $\tilde{\Gamma}_z^e = i\tilde{B}_x en\omega \times (\omega-\omega_* - k_{/\!/}u)[1+\xi Z(\xi)]/T_e k^2 ck$ holds. Considering the case $\omega \cong k_{/\!/}v_A$ (v_A : the Alfvén velocity), $\Gamma_x^e \sim env_A^2|\tilde{B}_x|^2/T_e Bck$ while $|\Gamma_x^i - \Gamma_x^e| \sim c|\tilde{B}_x|^2/4\pi eB\ell^2\varepsilon_\perp k$ (ℓ: the localization length of the wave in the x-direction, Z is the plasma dispersion function and $\xi=\omega/\sqrt{2}k_{/\!/} v_e$). Thus we have

$$| \Gamma_x^i - \Gamma_x^e | \cong \frac{\lambda_D^2}{\ell^2} \Gamma_x^e \quad . \tag{10}$$

Even in this extraordinary case, the broken ambipolarity is negligibly small. It is evident that the difference $\Gamma_x^i - \Gamma_x^e$ does not depend on the ion gyroradius but on the structure of the wave. When $\mathrm{Re}(i\partial^2 B_x/\partial x^2/B_x) < 0$ holds (i.e., the fluctuation is convective damping type in the x-direction)[3], Γ_x^i can exceed Γ_x^e ! Thus the plasma flux is determined by the spectrum and the spatial structure of the waves which compose the fluctuations.

In conclusion, the cross field particle fluxes for electrons and ions are obtained in terms of the field fluctuations by Eqs. (3) and (8). It is found that by low frequency fluctuations the electrons diffuse out independent of the ion gyroradius effect. The anomaly of the electron particle loss is of the same order as that of the electron heat loss.[3] The piling up of the dc electric field is prevented by the ion polarization drift so that ions diffuse out as fast as electrons do. The broken ambipolarity of the flux is almost completely negligible. If we retain $\Gamma_x \partial V_y/\partial x$ term in Eq.(1) for ions, it is also found that the value $|\Gamma_x^i/\Gamma_x^e - 1|$ decays with the e-folding time $|\Gamma_x \kappa/n|^{-1}$.

Finally, it should be noted that the formula of ambipolarity, Eq.(8), indicates the balance between the ion particle flux and the electron particle flux. However the energy distribution of the diffusing ions is not specified. In order to understand the ion heat flux, there remains an important open question whether the higher energy ions ($v^2 > T_i/m_i$) contribute to the ion particle flux or the lower energy ions do.

The authors wish to thank Profs. S.Yoshikawa and K.Nishikawa for helpful discussions and encouragements. Work partially supported by Grant in Aids for Scientific Research of the Ministry of Education, Science and Culture in Japan.

References

1] See, for example, S.Yoshikawa, Phys. Rev. Letters 25 (1970) 353.

2] J.D.Callen, Phys. Rev. Letters 39 (1977) 1540, and A.B.Rechester and M.N.Rosenbluth, Phys. Rev. Letters 40 (1978) 38.

3] S.Inoue, K.Itoh and T.Tange, IPPJ-Report 308 (1977, Nagoya).

4] T.Tange, S.Inoue, K.Itoh and K.Nishikawa, in preparation

5] From the assumption 1), the spatial asymmetry around z-axis is originated from the inhomogeneity. This implies that P_{xy} is proportional to κ^2, since P_{xy} vanishes in the limit $\kappa \to 0$, and P_{xy} does not change the sign by the transformation $\kappa \to -\kappa$. Thus $\partial P_{xy}/\partial x \propto \kappa^3$ while the 4th term $\omega_c \Gamma_x \propto \kappa$.

6] F.F.Chen and C.Etievant, Phys. Fluids 13 (1970) 687.