§28. Development of a Neoclassical Moment Equation Solver II (An Application of Generalized Taguchi's Formulation as a Validation)

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The traditional moment equation approach for neoclassical transport [1] is constructed by using the parallel force balance equations as $\int \mathbf{v}_{\|} v^{2n} d^3\mathbf{v}$ moments of the Landau equation. This sequence of the moments naturally leads us to well-known Laguerre (Sonine) polynomial expansion of the distribution function and the gyro-phase-averaged kinetic equation. A required knowledge on the spherical harmonic order l=1 of the Coulomb collision operator is only the friction coefficient obtained by Braginskii's direct Laguerre expansion method for the operator given in the Landau's expression [2]. An important advantage of this framework is that the collisional momentum conservation and the self-adjoint property of the operator [1-4] are retained even when the polynomial expansion of the order l=1 is truncated to finite terms for converting the kinetic problem into a linear algebraic equation. However, a complicated non-diagonal coupling between the Laguerre orders may be a disadvantage. In Refs.[3,4] for e.g., the numerous non-diagonal parallel viscosity coefficients $M^{a}_{j+1,\,k+1}$ and the friction coefficients $l^{ab}_{j+1,\,k+1}$ with $j \neq k$ describe this coupling effect. Because of it, the obtained $\int \mathbf{v}_{\|} v^{2n} f_a d^3\mathbf{v}$ moments generally depend on the choice of maximum order of the expansion $j_{\max}$, and this number can be determined only by experiences on the convergence [3,4].

There is another weighting scheme for constructing the moment equations that may reduce the non-diagonal terms $j \neq k$ before the truncation. This second idea was originally proposed by Taguchi [5] and was generalized later in Refs.[3,6]. Even though the momentum conservation and the self-adjoint property are not guaranteed completely there, this formulation can be a useful validation method for the traditional approach. However, in spite of a fact that this second method does not require the $\int \mathbf{v}_{\|} v^{2n} d^3\mathbf{v}$ sequence, all of Refs.[3,5,6] used the conventional Laguerre expansion of the Landau operator with only few terms. This is an inappropriate shortcut that should be corrected. In the present study [8], the spherical harmonic expanded RMJ (Rosenbluth-MacDonald-Judd) operator [7] without the Laguerre expansion is directly implemented in the generalized Taguchi's procedure shown in Ref.[3]. Figure 1 shows calculation examples of parallel flow moments $\langle B u_{\| aj} \rangle$ of Laguerre orders $0 \leq j \leq 3$ given by this method in a e^{-}+H^{+} plasma at a radial position of r/a=0.5 in a magnetic configuration with the magnetic axis position R_{ax}=3.6m in the LHD. These assumed conditions and parameters $n_e(r)$, $T_e(r)$, $T_i(r)$ are identical to those in Fig.2 in Ref.[4]. The result indicates that the generalized Taguchi's formulation [3] with the direct implementation of the RMJ operator (without the Sonine polynomial expansion) gives a good agreement with the results given by the traditional moment equation method in Ref.[4].

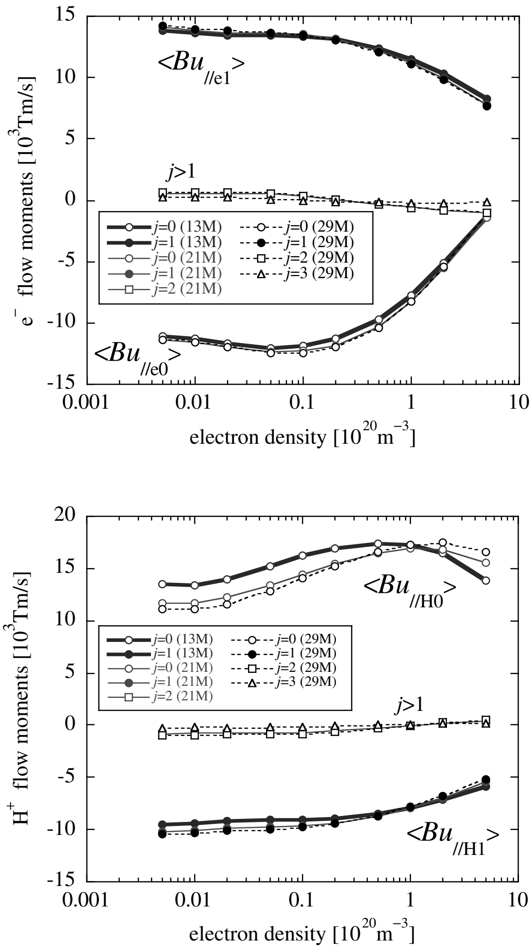

Fig.1 The $0 \leq j \leq 3$ Laguerre moments of the flux-surface-averaged parallel flow given by the 13M, 21M, and 29M approximations for a e^{-}+H^{+} plasma in a LHD configuration with R_{ax}=3.6m, B=2.45T (at r/a=0.5).

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