Geodesic Acoustic Eigenmodes

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The eigenmode of a geodesic acoustic mode in the presence of a temperature gradient is discussed. Eigenmodes are obtained and the characteristic wavelength scales as $\rho_{\rm i}^{2/3}L_{\rm T}^{1/3}$ ($\rho_{\rm i}$: ion gyroradius, $L_{\rm T}$: temperature gradient scale length). The direction of propagation is discussed.

Keywords: geodesic acoustic mode, zonal flow, eigenmode, eigenfrequency, temperature gradient

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Zonal flows have attracted attention owing to their essential role in the turbulent transport of magnetically confined plasmas [1]. The geodesic acoustic mode (GAM) is a kind of zonal flow, which has finite real frequency owing to the geodesic curvature of a toroidal magnetic field [2], and is driven by microscopic turbulence [3, 4]. Measurements of GAMs have been recently reported [5–11]. It has been known that the GAMs have real frequency $\omega_{\rm G} = \sqrt{2}c_{\rm s}/R$ in tokamaks ($c_{\rm s}$: ion sound velocity, R: majour radius). [The coefficient $\sqrt{2}$ depends on the model of plasma dynamics [1], but this is not an issue addressed in the present article.] In tokamaks and other toroidal plasmas, the plasma temperature changes in the radial direction, so that the dispersion relation $\omega = \omega_{\rm G}$, which is provided by the local theory, predicts different frequencies at different radii. In contrast, fluctuations with a common frequency are observed within a region which has a substantial width in radial direction [10, 11]. This indicates that the GAM oscillation appears as an eigenmode. In this article, we discuss the eigenmode of GAM oscillation in the presence of a temperature gradient. Due to the finite ion gyroradius, local oscillations on different magnetic surfaces interfere with one another so as to constitute a radial eigenmode. The characteristic wavelength is found to scale as $\rho_{\rm i}^{2/3}L_{\rm T}^{1/3}$ ($\rho_{\rm i}$: ion gyroradius, $L_{\rm T}$: temperature gradient scale length) and propagates outward if the temperature decreases towards the edge.

The dispersion relation of GAMs, $\omega = \omega_{\rm G}$, is derived by balancing the cross-field current $\tilde{J}_{{\rm D},r}$ (due to the magnetic field curvature) and the ion polarization current $\tilde{J}_{{\rm p},r}$ under the imposition of an electrostatic perturbation that has a form $\tilde{\phi}\exp({\rm i}kr - {\rm i}\omega t)$ in the leading order [12–15]. In order to study the radial eigenmode with analytic transparency, we take a simple collisionless limit with $T_{\rm e} \gg T_{\rm i}$ and $k\rho_{\rm i} \ll 1$. In the limit of $T_{\rm e} \gg T_{\rm i}$, the relation $v_{\rm th,i}/R \ll \omega$ holds for $\omega \sim \omega_{\rm G}$, and $\tilde{J}_{{\rm D},r}$ is dominated by the electron response ($v_{\rm th,i}$: ion thermal velocity) [14]. Therefore, $\tilde{J}_{{\rm D},r}$ is not significantly influenced by the finite gyroradius effect. In contrast, the ion polarization current, which is in proportion to ω, is screened by the factor $1 - k^2\rho_{\rm i}^2$ owing to the finite gyroradius effect. Thus, the relation $\tilde{J}_{{\rm p},r} + \tilde{J}_{{\rm D},r} = 0$ provides

$$(1 - k^2\rho_{\rm i}^2)\omega^2 = \omega_{\rm G}^2, \tag{1}$$

where the lowest order finite-gyroradius correction is included (see [12–16] for a more detailed derivation). We consider the case in which the temperature decreases in radius, and choose the radius r_0 where $\omega^2 = \omega_{\rm G}^2(r_0)$ holds. Taking the radial gradient of temperature into account, we write $\omega_{\rm G}^2(r) = \omega_{\rm G}^2(r_0)\left[1 - (r - r_0)L_{\rm T}^{-1}\right]$. The dispersion relation (1) can be rewritten as an eigenmode equation

$$\rho_{\rm i}^2\frac{{\rm d}^2}{{\rm d}r^2}\phi(r) + \frac{r - r_0}{L_{\rm T}}\phi(r) = 0, \tag{2}$$

by the replacement $k^2\rho_{\rm i}^2 \to -\rho_{\rm i}^2{\rm d}^2/{\rm d}r^2$. Equation (2) has a characteristic scale length,

$$\lambda = \rho_{\rm i}^{2/3}L_{\rm T}^{1/3}, \tag{3}$$

and is normalized as

$$\frac{{\rm d}^2}{{\rm d}x^2}\phi(x) + x\phi(x) = 0, \tag{4}$$

where $x = (r - r_0)\lambda^{-1}$. Equation (4) is readily solved by employing the Airy function:

$$\phi(x) = {\rm Ai}(-x). \tag{5}$$

The result seen in Eq. (5) shows that the eigenmode peaks near the region $x \simeq 0$, propagates in the lower-temperature region ($x > 0$), and is evanescent in the higher

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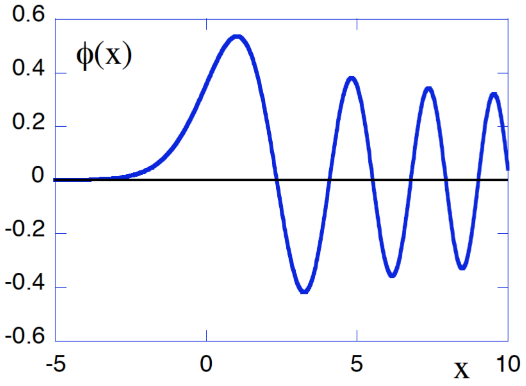

Fig. 1 GAMs radial eigenmode. Horizontal axis is normalized as $x = (r - r_0)\lambda^{-1}$.

temperature region ($x < 0$). Figure 1 illustrates the radial eigenfunction. The wave length is a few times λ. For this solution (5), the finite gyroradius correction has the order of magnitude $k^2\rho_i^2 \sim \rho_i^{2/3}L_T^{-2/3}$, and is much smaller than unity if $\rho_i \ll L_T$ holds. The assumption $k\rho_i \ll 1$ is verified *a posteriori*. We note that, in the limit of $\rho_i \to 0$, an eigenmode is localized to a magnetic surface.

In summary, the GAM oscillation was found to exist in a form of radial eigenmode when the temperature is inhomogeneous. This is consistent with the observation that GAM oscillations are observed as radial eigenmodes [11]. The radial wavelength has a dependence of $\rho_i^{2/3}L_T^{1/3}$, showing that GAMs are mesoscale fluctuations.

One can extend this analysis in a couple of ways. The extension to a more general profile of temperature $T(r)$ is possible. When T_e comes closer to T_i, the screening owing to the finite-gyroradius effect also appears in $\tilde{J}_{D,r}$ as was explained in [12–15], so that the coefficient to $k^2\rho_i^2$ in Eq. (1) becomes smaller (i.e., the radial wavelength becomes shorter). As was pointed in [17], the finite ion gyroradius effect can lead to the collisionless ion damping even in the limit of $k_{\|}v_{th,i} \ll \omega$, such collisionless damping having recently been studied theoretically [15]. When a small but finite damping rate is introduced, the eigenfunction shows an oscillation in the region of $x < 0$. Details of these investigations are left for future research.

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