§3. Development of Multi-Scale MHD Simulation Scheme

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In order to investigate the potential mechanism for the plasma stabilization, we have developed a nonlinear MHD code, NORM, based on the reduced MHD equations[1)]. To consistently understand the plasma dynamics, we need to examine a continuous evolution of the plasma as the beta increases. Therefore, we have to evolve not only the perturbation but also the equilibrium to include the beta increase effect. However, equilibrium evolution takes place over times of the order of 10msec and the basic time scale linked to the instabilities is 0.5μsec. Therefore, the evolutions involve a change of time scales of the order of $10^5 - 10^6$. Thus, we develop a numerical scheme to treat this multi-scale problem. We consider an iterative scheme with the VMEC[2)] and the NORM codes.

We consider the numerical scheme for the multi-scale problem in the beta range of $\beta_{min} \leq \beta \leq \beta_{max}$. Figure 1 shows the flow chart of the scheme.

At first, we divide the whole range of β into N intervals. We focus on the interval of $\beta^i \leq \beta \leq \beta^{i+1}$ and consider the equilibrium evolution from β^i to β^{i+1} ($0 \leq i \leq N$, $\beta^0 = \beta_{min}$, $\beta^N = \beta_{max}$). In the end of the nonlinear calculation for $\beta^{i-1} \leq \beta \leq \beta^i$, both the equilibrium and the perturbed parts of the pressure at β^i are already obtained as shown later, which are denoted by P^i_{eq} and $\tilde{P}^i$, respectively. Then, the average pressure profile at β^i can be calculated, which is defined by $\langle P \rangle^i = P^i_{eq} + \tilde{P}^i_{0,0}$. Here $\tilde{P}^i_{0,0}$ is the $(m, n) = (0, 0)$ component of $\tilde{P}^i$, where m and n are the poloidal and the toroidal mode numbers, respectively. We assume that the equilibrium pressure at β^{i+1} is given by $P^{i+1}_{eq} = (\beta^{i+1}/\beta^i)\langle P \rangle^i$. By employing this pressure, we calculate the equilibrium at β^{i+1} with the VMEC code. Then we obtain the equilibrium quantity for β^{i+1}, which is denoted by Q^{i+1}_{eq} generically. This part of the scheme gives the evolution in the time scale of $\sim$10msec.

Next, we consider the calculation of the nonlinear dynamics with the NORM code for the interval of $\beta^i \leq \beta \leq \beta^{i+1}$. We can calculate the nonlinear dynamics only separately for different beta values. Thus, in order to keep a smooth continuity of the perturbation, we divide the nonlinear calculation of this interval into $L + 1$ steps. The beta value and the equilibrium quantity are updated every step by utilizing a linear interpolation. The beta value of the j-th step $\beta^{i,j}$ is given by $\beta^{i,j} = \beta^i + j\delta\beta^i$ for $0 \leq j \leq L$, where $\delta\beta^i = (\beta^{i+1} - \beta^i)/L$. The equilibrium quantity of the j-th step $Q^{i,j}_{eq}$ is given by $Q^{i,j}_{eq} = Q^i_{eq} + j\delta Q^i_{eq}$, where $\delta Q^i_{eq} = (Q^{i+1}_{eq} - Q^i_{eq})/L$. Note that $\beta^{i,L} = \beta^{i+1,0} = \beta^{i+1}$ and $Q^{i,L}_{eq} = Q^{i+1,0}_{eq} = Q^{i+1}_{eq}$. Then, the nonlinear dynamics of the perturbation $\tilde{Q}^{i,j}$ is calculated for $\beta^{i,j}$ and $Q^{i,j}_{eq}$ at the j-th step. The time range of $\delta t_j(\tau_A)$ is assigned to each step. Here τ_A denotes the poloidal Alfvén time, which corresponds to 0.5μsec in the LHD plasma. Therefore, we can examine the nonlinear evolution in the time scale of μsec in this part.

In the end of the interval of $\beta^i \leq \beta \leq \beta^{i+1}$, we set $\tilde{P}^{i+1}_{0,0} = \tilde{P}^{i,L}_{0,0}$. This quantity is used in the VMEC calculation for the next interval. By employing appropriate initial value of $\langle P \rangle^0$ at $\beta = \beta^0$ and $t = T_{min}$ and continuing this iteration scheme until β reaches β_{max}, we can obtain the solution for the whole beta region. The solution is given by the sequence of the total quantity $Q^{i,j}_T = Q^{i,j}_{eq} + \tilde{Q}^{i,j}$.

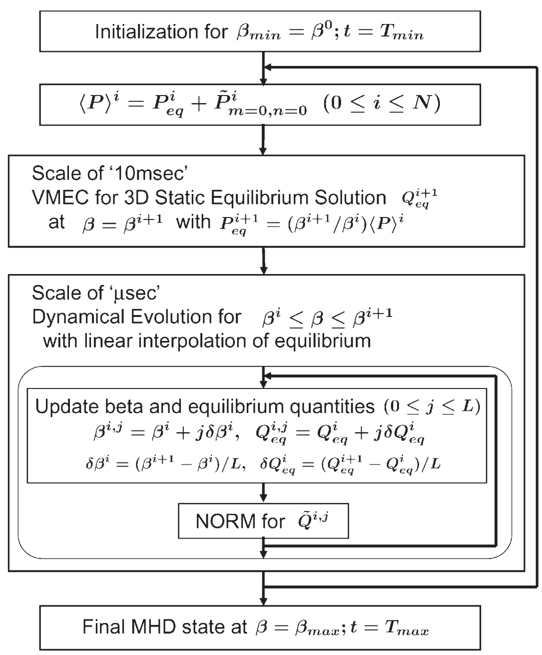

Fig.1 Flow chart of numerical scheme.

References

1) Ichiguchi, K., et al.: Nucl. Fusion 43 (2003)1101.

2) Hirshman H.P. et al. : Comput.Phys.Commun. 43(1986)143.