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(Received 16 July 2008; accepted 22 September 2008; published online 10 October 2008)

The quantum effects on the formation of the negative hydrogen ion (H^-) by the polarization electron capture process are investigated in partially ionized dense hydrogen plasmas. It is shown that the quantum effect strongly suppresses the electron capture radius as well as the cross section for the formation of the negative hydrogen ion. In addition, it has been found that the electron capture position is receded from the center of the projectile in decreasing the quantum effect of the plasma. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2998629]

The electron capture in atomic collision processes has been of great interest since this process is one of the most essential processes in numerous areas of physics, such as astrophysics, atomic physics, condensed matter physics, surface physics, and plasma physics.[1–6] Several classical, semiclassical, and quantum mechanical methods have been explored to investigate the electron capture process depending on physical conditions of the collision system.[2] It has been well known that the Bohr–Lindhard method[2,3] is quite credible for evaluating the electron capture cross section when the relative interaction velocity of the projectile v_P is greater than the ground state orbital velocity $v_Z(=Ze^2/\hbar)$ of the target with nuclear charge Ze. Since the hydrogen atom can be transformed into the negative hydrogen ion (H^-) by capturing an electron due to the polarization interaction, it has been known that the negative hydrogen ion plays an important role in the transport of energy in partially ionized hydrogen plasmas such as the solar atmosphere.[7–9] Hence, it is expected that the formation of the negative hydrogen ion by the electron capture plays an important role in the transport process and distribution of electrons in partially ionized hydrogen plasmas. The formation of the negative hydrogen ion by the electron capture would be different from the formation of the hydrogen atom due to the polarization interaction in partially ionized hydrogen plasmas. When the de Broglie wavelength is comparable to the Bohr radius, the formation of the negative hydrogen ion in dense plasmas would also be quite different from those in free spaces or in weakly coupled plasmas due to the quantum and plasma screening effects on the polarization interactions. Thus, in this letter we investigate the quantum-mechanical and plasma screening effects on the formation of the negative hydrogen ion by the electron capture due to the screened polarization interaction in dense plasmas using the Bohr–Lindhard model[2] with the impact parameter analysis. The screened effective polarization interaction potential,[10] taking into account the quantum-mechanical effects of diffraction and symmetry of particles and plasma screening effects, is employed to describe the interaction between the hydrogen atom and the released electron in partially ionized dense hydrogen plasmas.

Using the Bohr–Lindhard method,[2] the electron capture cross section $\sigma_C(v_P)$ is given by

$$\sigma_C(v_P) = 2\pi \int db\, b P_C(b,v_P), \tag{1}$$

where b is the impact parameter and $P_C(b,v_P)$ is the electron capture probability. This electron capture probability P_C would be represented by the ratio of the collision time to the electron orbital time in the target ion. It has been known that electron capture occurs only when the distance between the projectile and the released electron is smaller than the electron capture radius determined by equating the kinetic energy of the released electron in the frame of the projectile and the binding energy provided by the projectile.[2] The electron capture distance is usually greater than the release distance. The reaction equation for the formation of the negative ion by the interaction between the projectile C and target D by the polarization charge capture can be represented by $C+D \rightarrow C^- + D^+$.

Recently, the Ramazanov–Dzhumagulova–Omarbakiyeva[11] (RDO) model for the effective polarization potential has been given for the interaction between the electron and the atom polarized by the external field generated by the electron in partially ionized dense hydrogen plasmas taking into account the quantum-mechanical effects of diffraction and symmetry of particles and plasma screening effects. In these partially ionized dense hydrogen plasmas,[11] the ranges of the electron number density and the temperature are about $10^{21}-10^{24}$ cm^{-3} and 10^2-10^6 K, respectively. By using the RDO polarization interaction model,[11] the screened polarization potential for the interaction between the hydrogen atom and the electron in partially ionized dense hydrogen plasmas can be represented by

$$V_{\rm RDO}(r) = -\frac{e^2\alpha_p}{2r^4(1-4\lambda^2/r_D^2)}\{e^{-B(\lambda,r_D)r}[1+B(\lambda,r_D)r] - e^{-A(\lambda,r_D)r}[1+A(\lambda,r_D)r]\}^2, \tag{2}$$

where $\alpha_p\{\equiv[n^6+(7/4)n^4(l^2+l+2)]a_0^3\}$ is the dipole polarizability[10] of the hydrogen atom for the nl-state, n and l are, respectively, the principal and orbital quantum numbers, $a_0(=\hbar^2/me^2)$ is the Bohr radius of the hydrogen atom, m is the electron mass, $\lambda(=\hbar/\sqrt{2\pi m k_B T})$ is the thermal de Broglie wavelength for the electron, k_B is the Boltzmann

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constant, T is the plasma temperature, r_D is the Debye length, $A(\lambda,r_D)\equiv(1+\sqrt{1-4\lambda^2/r_D^2})^{1/2}/(\sqrt{2}\lambda)$, and $B(\lambda,r_D)\equiv(1-\sqrt{1-4\lambda^2/r_D^2})^{1/2}/(\sqrt{2}\lambda)$. The detailed discussions on the physical properties of partially ionized plasmas have also been investigated on the basis of the pseudopotential model.[12–14] When the quantum-mechanical effects are absent in partially ionized hydrogen plasmas, i.e., $\lambda\rightarrow 0$, the effective polarization interaction potential [Eq. (2)] goes over into the screened Buckingham (B) potential[12]

$$V_{\mathrm{RDO}}(r)\rightarrow V_B(r)=-\frac{e^2\alpha_d}{2r^4}\left(1+\frac{r}{r_D}\right)^2 e^{-r/r_D}, \tag{3}$$

since $B\rightarrow r_D$ and $A\rightarrow\infty$ as $\lambda\rightarrow 0$. Then, the electron capture radius R_C by the ground state hydrogen atom in partially ionized dense plasmas can be obtained by the following relation:

$$\frac{9e^2a_0^3}{4R_C^4(1-4\lambda^2/r_D^2)}\{e^{-B(\lambda,r_D)R_C}[1+B(\lambda,r_D)R_C] - e^{-A(\lambda,r_D)R_C}[1+A(\lambda,r_D)R_c]\}^2\cong\tfrac{1}{2}mv_p^2, \tag{4}$$

since the binding energy provided by the polarization interaction should be greater than the kinetic energy of the released electron in the frame of the hydrogen atom. After some mathematical manipulations with the physical conditions for the thermal de Broglie wavelength, Debye length, and the parameters $A(\lambda,r_D)$ and $B(\lambda,r_D)$, i.e., $\lambda\ll r_D$ and $B(\lambda,r_D)<A(\lambda,r_D)$, the scaled electron capture radius $\bar{R}_C(\equiv R_C/a_0)$ is found to be

$$\bar{R}_C(\bar{\lambda},\bar{r}_D,\bar{E}_p)\cong\frac{3}{2}\frac{(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})^{1/2}}{\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2}\bar{\lambda}\bar{E}_p^{1/2}}+\frac{\sqrt{2}\bar{\lambda}}{(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})^{1/2}} \times\text{Product log}\left\{-\frac{3\sqrt{2}(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})}{\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2}\bar{\lambda}^2\bar{E}_p^{1/2}} \times\exp\left[-\frac{3\sqrt{2}(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})}{\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2}\bar{\lambda}^2\bar{E}_p^{1/2}}\right]\right\}, \tag{5}$$

where $\bar{\lambda}(\equiv\lambda/a_0)$ is the scaled thermal de Broglie wavelength, $\bar{r}_D(\equiv r_D/a_0)$ is the scaled Debye length, $\bar{E}_p(\equiv mv_p^2/2\ \mathrm{Ry})$ is the scaled collision energy, and $\mathrm{Ry}(=me^4/2\hbar^2\approx 13.6\ \mathrm{eV})$ is the Rydberg constant, and the Product log $[x]$ (Ref. 15) or the so-called the Lambert W function is the special function for the solution of w in $x=we^w$. According to the Bohr–Lindhard method[2] at intermediate and high velocities $v_P>v_Z$, the electron capture probability is determined by the ratio of the collision time to the electron orbital time. Thus, the probability for the formation of the negative hydrogen ion can be determined by

$$P_C(b,v_P)=\frac{1}{\tau}\int_{-t_C}^{t_C}dt, \tag{6}$$

where t_C is the electron capture time and $t=0$ is chosen as the instant at which the projectile positron makes its closest approach to the target. Here, the electron orbital time τ is given by $\tau(=a_n/v_n)=n^3a_0/\alpha c$, where $a_n(=n^2a_0)$ is the nth Bohr radius of the hydrogen atom, $v_n(=\alpha c/n)$ is the electron velocity of the nth Bohr orbit, and $\alpha(=e^2/\hbar c\approx 1/137)$ is the fine-

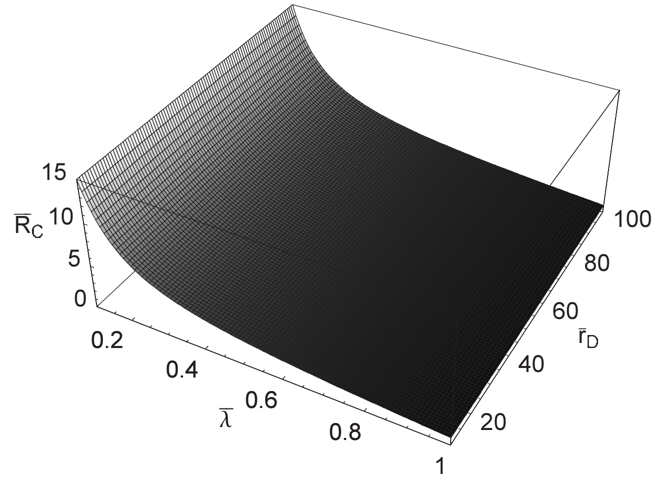

FIG. 1. The three-dimensional plot of the scaled electron capture radius $\bar{R}_C(\bar{\lambda},\bar{r}_D,\bar{E}_p)$ as a function of the scaled thermal de Broglie wavelength $(\bar{\lambda})$ and scaled Debye length $(\bar{r}_D)$ when $\bar{E}_p=2$.

structure constant. Since the straight-line trajectory analysis is reliable for heavy atomic projectiles, the electron capture time t_C would be $2(R_C^2-b^2)^{1/2}/v_P$. The scaled differential cross section $\bar{\sigma}_C[\equiv(d\sigma_C/d\bar{b})/\pi a_0^2]$ in units πa_0^2 for the formation of the negative hydrogen ion by the polarization electron capture in partially ionized dense hydrogen plasmas including the quantum and plasma screening effects is then found to be

$$\bar{\sigma}_C\equiv\frac{d\sigma_C}{d\bar{b}}/\pi a_0^2=\frac{4}{\bar{E}_P^{1/2}}\bar{b}\left\{\left[\frac{3}{2}\frac{(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})^{1/2}}{\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2}\bar{\lambda}\bar{E}_p^{1/2}} +\frac{\sqrt{2}\bar{\lambda}}{(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})^{1/2}} \times\text{Product log}\left[-\frac{3\sqrt{2}(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})}{\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2}\bar{\lambda}^2\bar{E}_p^{1/2}} \times\exp\left[-\frac{3\sqrt{2}(1+\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2})}{\sqrt{1-4\bar{\lambda}^2/\bar{r}_D^2}\bar{\lambda}^2\bar{E}_p^{1/2}}\right]\right]\right]^2-\bar{b}^2\right\}^2, \tag{7}$$

where $\bar{b}(\equiv b/a_0)$ is the scaled impact parameter. For the interaction between the projectile with charge z and the target with charge Z, the electron release radius is known to be proportional to $1+2\sqrt{z/Z}$.[3] Hence, for small values of the de Broglie wavelength, i.e., weakening of the quantum screening effect, the electron capture cross section [Eq. (5)] would be reliable since the electron release distance is smaller than the electron capture radius for intermediate and high projectile energies.

In order to investigate the quantum-mechanical and plasma screening effects on the formation of the negative hydrogen ion by the polarization electron capture in partially ionized dense hydrogen plasmas, we set $\bar{E}_p>1$ since the Bohr–Lindhard model is known to be valid for intermediate and high collision velocities, i.e., $v_P>v_Z$.[2] Figure 1 shows the three-dimensional plot of the scaled electron capture radius $(\bar{R}_C)$ as a function of the thermal de Broglie wavelength $(\bar{\lambda})$ and Debye length $(\bar{r}_D)$. From this figure, it is shown that

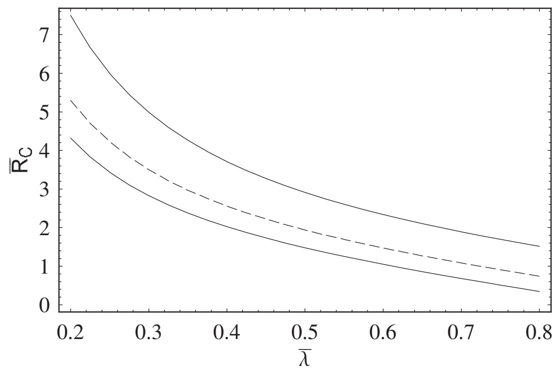

FIG. 2. The plot of the scaled electron capture radius $\bar{R}_C(\bar{\lambda},\bar{r}_D,\bar{E}_p)$ as a function of the scaled thermal de Broglie wavelength when $\bar{r}_D=50$. The solid line is the case of $\bar{E}_p=2$. The dashed line is the case of $\bar{E}_p=4$. The dotted line is the case of $\bar{E}_p=6$.

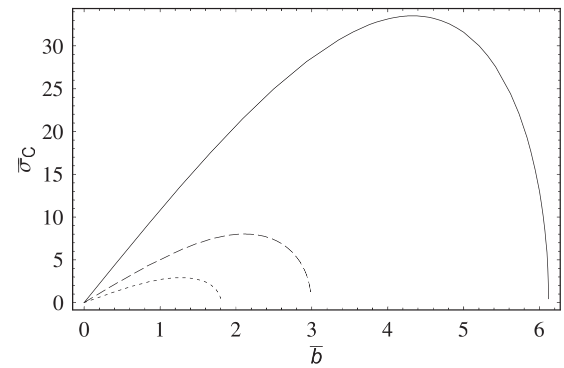

FIG. 4. The plot of the scaled differential cross section $\bar{\sigma}_C[\equiv(d\sigma_C/d\bar{b})/\pi a_0^2]$ in units πa_0^2 as a function of the impact parameter ($\bar{b}$) when $\bar{E}_p=5$ and $\bar{r}_D=50$. The solid line represents the case of $\bar{\lambda}=0.2$. The dashed line represents the case of $\bar{\lambda}=0.4$. The dotted line represents the case of $\bar{\lambda}=0.6$.

the quantum-mechanical effect suppresses the electron capture radius. Figure 2 represents the plot of the scaled electron capture radius ($\bar{R}_C$) as a function of the thermal de Broglie wavelength ($\bar{\lambda}$) for various values of the collision energy ($\bar{E}_p$). As we see in this figure, it is also found that the electron capture radius decreases with increasing collision energy. Figure 3 shows the three-dimensional plot of the scaled cross section ($\bar{\sigma}_C$) for the formation of the negative hydrogen ion by the polarization electron capture as a function of the thermal de Broglie wavelength ($\bar{\lambda}$) and impact parameter ($\bar{b}$). As it is seen in this figure, the differential cross section is strongly suppressed by an increase in the de Broglie wavelength. Thus, it should be noted that the quantum effect strongly diminishes the cross section for the formation of the negative hydrogen ion. In addition, the domain of the differential cross sections significantly decreases with increasing quantum effect. Figure 4 represents the scaled cross section ($\bar{\sigma}_C$) as a function of the scaled impact parameter ($\bar{b}$) for various values of the thermal de Broglie wavelength ($\bar{\lambda}$). It is interesting to note that the electron capture position has been receded from the center of the hydrogen atom with decreasing thermal de Broglie wavelength. Thus, it is shown that the quantum effect plays a very important role in the formation of the negative hydrogen ion by the polarization electron capture in partially ionized dense hydrogen plasmas. These results provide useful information on the quantum-mechanical and plasma screening effects on the polarization charge capture in dense plasmas.

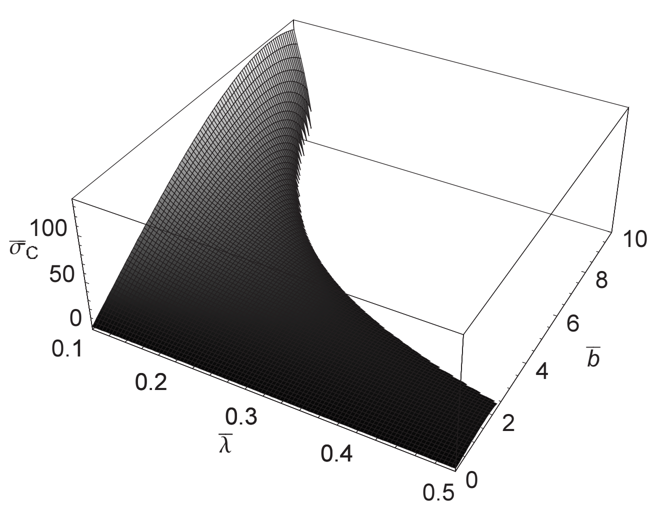

FIG. 3. The three-dimensional plot of the scaled differential cross section $\bar{\sigma}_C[\equiv(d\sigma_C/d\bar{b})/\pi a_0^2]$ in units πa_0^2 for the formation of the negative hydrogen ion as a function of the scaled thermal de Broglie wavelength ($\bar{\lambda}$) and scaled impact parameter ($\bar{b}$) when $\bar{E}_p=5$ and $\bar{r}_D=50$.

Y.-D.J. gratefully acknowledges the Director-General Professor O. Motojima, Director Professor M. Sato, Director Professor Y. Hirooka, and Professor I. Murakami for their warm hospitality while visiting the National Institute for Fusion Science (NIFS) in Japan as a long-term visiting professor. The authors are also grateful to NIFS for supporting the research. This work was done while Y.-D.J. visited NIFS.

[1]H. Ryufuku and T. Watanabe, Phys. Rev. A **20**, 1828 (1979).
[2]D. Brandt, Nucl. Instrum. Methods Phys. Res. **214**, 93 (1983).
[3]B. H. Bransden and M. R. C. McDowell, *Charge Exchange and the Theory of Ion-Atom Collisions* (Oxford University Press, Oxford, 1992).
[4]I. Ben-Itzhak, A. Jaint, and O. L. Weaver, J. Phys. B **26**, 1711 (1993).
[5]V. Shevelko and H. Tawara, *Atomic Multielectron Processes* (Springer, Berlin, 1998).
[6]C.-G. Kim and Y.-D. Jung, Plasma Phys. Controlled Fusion **46**, 1493 (2004).
[7]H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer, Berlin, 1957).
[8]S. Chandrasekhar, Astrophys. J. **128**, 114 (1958).
[9]M. Harwit, *Astrophysical Concepts*, 2nd ed. (Springer, New York, 1988).
[10]V. P. Shevelko, *Atoms and Their Spectroscopic Properties* (Springer, Berlin, 1997).
[11]T. S. Ramazanov, K. N. Dzhumagulova, and Y. A. Omarbakiyeva, Phys. Plasmas **12**, 092702 (2005).
[12]T. Ramazanov, K. Galiyev, K. N. Dzhumagulova, G. Röpke, and R. Redmer, Contrib. Plasma Phys. **43**, 39 (2003).
[13]T. S. Ramazanov, K. Zh. Galiyev, K. N. Dzhumagulova, G. Röpke, and R. Redmer, J. Phys. B **36**, 6173 (2003).
[14]T. S. Ramazanov and K. N. Turekhanova, Phys. Plasmas **12**, 102502 (2005).
[15]R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, Adv. Comput. Math. **5**, 329 (1996).