§25. On the Bicoherence Analysis of Plasma Turbulence

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The bicoherence of fluctuations [1] in a system of drift waves and zonal flows is discussed. In strong drift-wave turbulence, where broad-band fluctuations are excited, the bicoherence is examined [2]. A Langevin equation formalism of turbulent interactions allows us to relate the bicoherence coefficient to the projection of nonlinear force onto the test mode. The dependence of the summed bicoherence on the amplitude of zonal flows is clarified. The importance of observing biphase is also stressed. The results provide a basis for measurement of nonlinear interaction in a system of drift waves and zonal flow.

The nonlinear dynamical equation may be written in a form

$$\frac{\partial}{\partial t} g + \left(-\gamma + i\mathcal{L}_0\right) g = \sum N g g$$

where γ is a linear growth rate, $\mathcal{L}_0$ represents the linear frequency, and N denotes the coefficient of nonlinear interaction. Considering these experimental situations, we introduce Fourier components as $g(t) = \sum_p g_p \exp(-ipt)$. A response of g_p to the imposition of the nonlinear term g_ω is evaluated as follows. We separate one term $N_{p\omega} g_{p-\omega} g_\omega$ from the total nonlinear terms $\sum N \tilde{g}\tilde{g}$, and express the rest in terms of the nonlinear damping term and fluctuating force as

$$\sum N \tilde{g}\tilde{g} - N_{p\omega} g_{p-\omega} g_\omega e^{-ipt} = -\nu_T' g + \tilde{S}' .$$

We have a solution

$$g_p = \exp\left(-\hat{\nu}_p t\right) \int_{-\infty}^{t} dt' \exp\left(\hat{\nu}_p t'\right) N_{p\omega} g_{p-\omega} g_\omega + \tilde{g}_p$$

and $\tilde{g}_p = \exp\left(-\hat{\nu}_p t\right) \int_{-\infty}^{t} dt' \exp\left(\hat{\nu}_p t'\right) \tilde{S}_p$.

The bispectrum estimator $\hat{B}(\omega, p)$, the squared bicoherence $\hat{b}^2(\omega, p)$, and the summed-bicoherence $\sum \hat{b}^2$ are defined as

$$\hat{B}(\omega, p) = \left\langle g_p^* g_{p-\omega} g_\omega \right\rangle, \quad \hat{b}^2(\omega, p) = \frac{\left|\hat{B}(\omega, p)\right|^2}{\left\langle \left|g_p g_{p-\omega}\right|^2 \right\rangle \left\langle \left|g_\omega\right|^2 \right\rangle},$$

and $\sum \hat{b}^2(\omega) = \sum_p \hat{b}^2(\omega, p)$.

A. Case of broad band turbulence For broad band fluctuations, one has simplified evaluation as

$$\sum \hat{b}^2(\omega) \sim 3\left|\tau_p N_{p,\omega}^*\right|^2 |g|^2,$$

with $|g|^2 = \sum_p \left\langle \left|g_p\right|^2 \right\rangle$. The term $\left|N_{p\omega}\right| |g|$ represents a nonlinear force (in a normalized unit in a dimension of the 'frequency'), and $\tau_p \left|N_{p\omega}\right| |g|$ indicates the competition between this nonlinear force and the effective correlation time τ_p. The nonlinear interaction can be evaluated from experimental data as

$$\left|N_{p\omega}\right| \simeq \frac{1}{\sqrt{3}\tau_p |g|} \sqrt{\sum \hat{b}^2(\omega)} .$$

B. Case of a sharp peak within a broad band fluctuations Here, the suffix ω indicates the mode which belongs to the sharp peak of the spectrum, and $(p, p-\omega)$ denotes the broad band background turbulence. The bicoherence is estimated as

$$\hat{B}(\omega, p) \simeq 2\tau_{p-\omega} N_{p-\omega, p}^* \left|g_p\right|^2 \left|g_{\omega,0}\right|^2 + \tau_{c, p-\omega} N_{\omega, p}^* \left|g_{p-\omega}\right|^2 \left|g_p\right|^2$$

The leading term in the summed bicoherence is

$$\sum \hat{b}^2(\omega) = 4M \tau_{p-\omega}^2 \left|N_{p-\omega, p}\right|^2 \left|g_\omega\right|^2 .$$

C. Application to Hasegawa-Mima Model The response of drift wave-zonal flows system is given by

$$\frac{\partial}{\partial t}\phi_d + \frac{i\omega_*}{1 + k_\perp^2 \rho_s^2}\phi_d - \frac{c_s \rho_s^4}{L_n}\left[\phi_d, \Delta_\perp \phi_d\right] = \frac{c_s}{L_n} \frac{q_x k_y k_\perp^2 \rho_s^4}{1 + k_\perp^2 \rho_s^2} \phi_Z \phi_d$$

The interaction of drift waves has the coupling coefficient

$$N \simeq \frac{c_s}{L_n} \frac{k_x k_y k_\perp^2 \rho_s^4}{1 + k_\perp^2 \rho_s^2},$$

while the interaction between the zonal flow and drift waves has the coefficient

$$N = \frac{c_s}{L_n} \frac{q_x k_y k_\perp^2 \rho_s^4}{1 + k_\perp^2 \rho_s^2}$$

The decorrelation time of drift waves through self-nonlinear interaction has been evaluated as $\tau_p^{-1} \sim h(k_\perp \rho_s)\omega_* \phi$, in the strong turbulence limit. One has the total bicoherence for the mutual nonlinear interaction of drift waves as

$$\sum \hat{b}^2(\omega) \sim 3\left(\frac{1}{h(k_\perp \rho_s)} \frac{k_x k_\perp^2 \rho_s^3}{1 + k_\perp^2 \rho_s^2}\right)^2 .$$

The total bicoherence at the frequency of zonal flows and that at the drift wave range of frequencies are compared as

$$\frac{\sum \hat{b}^2(\mathrm{GAMs})}{\sum \hat{b}^2(\mathrm{drift})} \sim \frac{4M}{3}\left(\frac{q_x}{k_x}\right)^2 \frac{\phi_Z^2}{\phi_d^2} + \frac{4}{3} .$$

The importance of observing the biphase was also demonstrated. When ω is chosen at the frequency of zonal flows, the phase of the bispectrum estimator $\hat{B}(\omega, p)$ has a weak dependence on p. In contrast, the biphase of $\hat{B}$ spreads over the range of 0 and 2π for the interaction of drift waves. These properties will be used in the experimental study of turbulence.

References

[1] Y. Nagashima, et al., Phys. Rev. Lett. **95** (2005) 095002

[2] K. Itoh, Y. Nagashima, S-I Itoh, P. H. Diamond, A. Fujisawa, M. Yagi, A. Fukuyama
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