## § 18. Stochastic Transition in Multiple-Scale Turbulence

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To derive the hierarchical model for multiple-scale turbulence, the effect of the semi-micro mode to the micro mode is taken into account. The local pressure steepening and the **E**x**B** shearing induced by semi-micro fluctuations are incorporated into the model. The former has the destabilizing influence and the latter has suppressing effect on micro mode. The change of local magnetic shear by the semi-micro mode is not kept in this model, since the semi-micro mode is assumed to be electrostatic mode. ITG mode and current diffusive interchange mode (CDIM) are considered as semi-micro and micro modes, respectively. A detailed derivation of the Langevin equation and the deduction of the statistical average are explained in [1]. Langevin equations for semi-micro mode and micro mode are given by

$$\frac{dx_l}{d(\omega_l t)} + \frac{1}{2}\left(\frac{\sqrt{x_h}}{r} + \sqrt{\frac{x_h}{r^2} + 4x_l} - 2\right)x_l - \frac{\varepsilon}{r^3}x_h^{3/2}$$
$$+ \frac{1}{2}\left(\frac{\sqrt{x_h}}{r} + \sqrt{\frac{x_h}{r^2} + 4x_l}\right)x_l w_l + \frac{\varepsilon}{r^3}x_h^{3/2}w_h = 0$$

$$\frac{dx_h}{d(\omega_h t)} - x_h^{3/2}w_h + \varepsilon^{1/2}w_l +$$
$$\left[\sqrt{x_h} - \sqrt{1 + \frac{\sqrt{x_l}}{\sqrt{x_h}/r + \sqrt{x_h/r^2 + 4x_l}}} \,/\,(1 + pr^2 x_l)\right]x_h = 0$$

where superscripts $l$ and $h$ denote the semi-micro and micro modes[4]. $x_l = I_l / D_l^2$ and $x_h = I_h / D_h^2$ denote the spectral intensities of semi-macro mode, $I_l = \sum_{k_l} \langle \phi_{k_l}^* \phi_{k_l} \rangle$ and micro mode, $I_h = \sum_{k_h} \langle \phi_{k_h}^* \phi_{k_h} \rangle$ normalized by driving sources, $D_l, D_h$ , which are defined by

$$D_l = \frac{2}{2 - C_l}\frac{\gamma_l}{k_l^2}\frac{1}{1 + \left(\omega_{E0}/\omega_{Ec}^l\right)^2} \quad \text{and}$$

$$D_h = \frac{2}{2 - C_h}\frac{\gamma_h}{k_h^2}\frac{1}{1 + \left(\omega_{E0}/\omega_{Ec}^h\right)^2} .$$

$\gamma_{l,h}$ and $k_{l,h}$ are the maximum growth rate and corresponding wave number, $C_{l,h}$ , numerical coefficients of order unity, $\omega_{E0}$ , the equilibrium electric field shear, $\omega_{Ec}^{l,h}$ , the critical value of the inhomogeneous radial electric field for the suppression of modes. Explicit expressions of these valuables for ITG and CDIM are given in literature. $\omega_{l,h} = k_{l,h}^2 D_{l,k} \sim \gamma_{l,k}$ represents typical growth rate for these modes and $w_l, w_h, w_t$ are Gaussian white noise with mean zero and deviation unity. Other parameters are given by $r = D_l / D_h$ (the driving ratio of semi-micro and micro modes), $\varepsilon = \frac{C_h}{2 - C_l}(k_l / k_h)^2$ (the coupling coefficient for the drive of the semi-micro mode by the nonlinear noise of the micro mode) and $p = D_h^2 / I_{eff}$ with $I_{eff} = (1 + (\omega_{E0}/\omega_{Ec}^h)^2)(\omega_{Ec}^h)^2 / k_l^4$ .

To investigate the noise effect on multiple-scale turbulence, the Fokker-Planck equation is derived for the semi-macro mode. We use the adiabatic approximation for the micro mode. Figure 1 shows the steady state distribution function of the semi-micro mode. The solid curve represents the case with $r = 0.7, p = 30$ and the dashed curve represents the case with $r = 1.5, p = 30$ . In the case with $r = 0.7$ , Two peaks are observed, however, the well between two peaks is very shallow, which is due to the noise effect. Figure 2 shows the most probable state for $x_l$ as a function of $r$ . The results from deterministic model are also shown by the dashed curve.

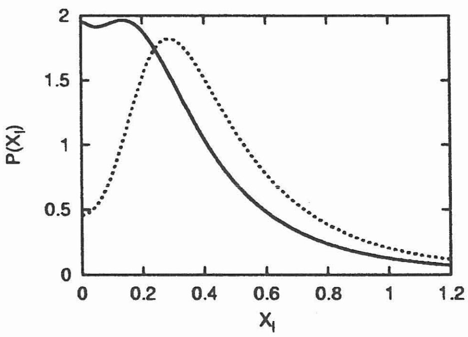


*FIG.1 The steady state distribution function of the semi-micro mode.*

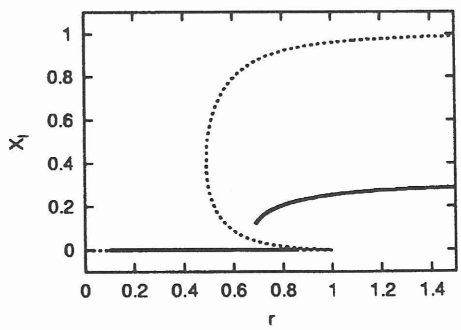


*FIG.2 The most probable state for $x_l$ as a function of $r$ .*

**Reference**
Itoh S.-I. and Itoh K., Plasma Phys. Control. Fusion **43** (2001) 1055