§15. Challenge to Image Reconstruction of Large Size from Standpoint of Numerical Analysis

Iwama, N. (Daido Univ.),
Abe, K. (Gifu Shotoku Gakuen Univ.),
Hosoda, Y. (Univ. Fukui),
Teranishi, M. (Hiroshima Inst. of Tech.),
Nagayama, Y., Peterson, B.J., Ohdachi, S.

Investigating numerical techniques for the image reconstruction of large size is needed in various scientific measurements such as three-dimensional (3D) tomographic imaging in nuclear fusion research, electron microscopy and the adaptive optics for large telescope. Advanced study on the missing observation problem is needed on the basis of the fusion research activity of 2D tomography with limited angular data.

Since the missing observation requires us to take the approach of finding a solution (image reconstruction) that fits to available data, we have to meet linear equation systems that are large, sparse and ill-conditioned, especially when we take model-free algebraic approach. As the systems become larger, the Tikhonov-like direct solvers using the singular value decomposition etc. are anticipated to become disadvantageous. Rather, high interest is taken in the recently developed iterative solvers.

The fast iterative solver ART and the least-squares one SIRT, which are well known in fusion research, are "stationary" in the sense that the operator for solution update is kept constant throughout the iteration process. The Hopfield neural network [1] is also stationary under a framework involving a strong means of regularizing the solution. Meanwhile, one says that the "unstationary" solvers using the Krylov subspace [2] can be faster than the stationary solvers, and that they might be weak for ill-conditioned equation systems. Experimental study is a current subject.

The "missing wedge" problem in electron tomography is worth taking as a priority target. After the small image reconstruction of one virus particle by the Hopfield methods [3], some methods of Krylov subspace have been examined on 2D numerical phantoms in relation to the multi-slice scan of the whole specimen. A result is shown in Figs. 1 and 2. With respect to the original image of specimen (Fig. 2), the projection matrix, that is, the coefficient matrix A of a linear equation $A\boldsymbol{x}=\boldsymbol{b}$ to be solved, has a size of 202,124 x 2,097,152 with a low density of nonzero element 0.04%. A sequence of 1D projections $\boldsymbol{b}$ was numerically generated, with additive Gaussian noises of 1%, at rotation angles that are equally spaced with a period of 2º in the interval [-70º, 70º].

As seen in Fig. 1, in comparison with the well-known method of conjugate gradient, a modified method of BiCGSTAB is improved in the stability of convergence, and the method of Orthomin is excellent also in speed. The relative residual is defined as $\| \boldsymbol{r}_k \|^2 / \| \boldsymbol{r}_0 \|^2$ for the k-th iteration, and the results of solving $AA^T\boldsymbol{y}=\boldsymbol{b}$ with an initial random-number solution $\boldsymbol{y}_0$ are exhibited here. In the reconstructed image (Fig. 2), the effect of missing wedge is recognized as artifacts that are elongated in the horizontal direction where the projection data are missed. These methods were comparable with ART in reconstructed images and computing times.

The solved linear equation system is large enough in size for the 3D tomography of LHD using infrared imaging video bolometers (IRVB). To obtain the efficiency of image reconstruction superior to ART and SIRT, the next step of research is the application of the new methods to the least-squares regularization of Tikhonov type, that is, the minimization of penalty functions under explicit constraints.

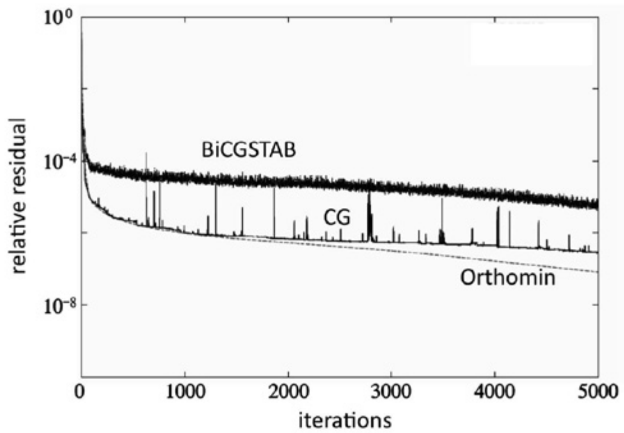

Fig. 1: Convergence histories in image reconstruction by the Krylov subspace methods of CG (Conjugate Gradient), BiCGSTAB (BiConjugate Gradient STABilized), and Orthomin(100).

Fig. 2: A result of image reconstruction; (upper) the original image with 512x4096 pixels, (lower) the image reconstructed after 5,000 iterations by the method of Orthomin(100).

Thanks are expressed to Profs. K. Morikuni and K. Hayami of the National Institute of Informatics on numerical analysis, and to Profs. K. Murata and K. Nagayama of the National Institute of Physiological Sciences on electron tomography for their useful advices and suggestions. The research was supported by the budget NIFS10KBAP003.

1) Iwama, N. et al.: J. Plasma Fusion Res. SERIES **8** (2009) 97
2) Saad, Y.: *Iterative Methods for Sparse Linear Systems*, SIAM (Philadelphia, 2003).
3) Iwama, N. et al.: Ann. Rep. NIFS 2011, p. 437.