§17. Image Reconstruction by Hopfield Neural Network for Bolometer Tomography of LHD Plasma

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Hopfield neural network is a tool of regularizing the ill-behaved least-squares solutions of linear equations. With mutually interconnected neurons, the Hopfield system has such a dynamical behavior as an energy function tends to decrease with time when the system has a monotone increasing input-output function of neuron with symmetry of the interconnection weights. This dynamics can be used for the tomographic image reconstruction from projections when the Lagrangian function of Tikhonov-Phillips type [1] to be minimized is analogously regarded as the energy function.

In the formulation of image reconstruction $H\boldsymbol{f}=\boldsymbol{g}$, where an unknown K-dimensional image vector $\boldsymbol{f}$ is operated by a projection matrix H to produce the M-dimensional data vector $\boldsymbol{g}$, the analogy leads to the following design of a Hopfield model having neurons as many as the number of pixels K:

the weight matrix $W=-2(H^TH+M\gamma C^TC)$;

the bias vector $\boldsymbol{\theta}=(2/M)H^T\boldsymbol{g}$.

Here, C is the K-dimensional Laplacian operator. By this design, the matrix W is symmetric as required with the (i, j) element w_{ij} (interconnection weight from the j-th neuron to the i-th neuron) such that $w_{ij}=w_{ji}$.. To fit the neuron outputs to the projection data, the term H^TH leads to negative weights interconnecting the neurons that are located on each line of sight. The term $M\gamma C^TC$ gives to each neuron a negative self-connection weight w_{ii} and positive weights w_{ij} from 4 neighboring neurons; these weights increase in magnitude with the regularization parameter γ and assume a role of statistical stabilization and spatial smoothing of the neuron outputs f_i, that is, the reconstructed image. On the other hand, the bias $\boldsymbol{\theta}$ is the back-projection, which is well-known in computerized tomography, and involves the projection data $\boldsymbol{g}$, which is time-variant as the M-channel detector signals. When the sigmoid function is used as the input-output function of neuron, the Tikhonov-Phillips solution will be nonlinearly modified so that the pixel values f_i are guaranteed to be positive.

This Hopfield model design has been made on the system of two 20-channel fan-beam cameras (M=40) with AXUVD silicon photodiodes that has been installed in a semi-tangential cross section using 3.5-U and 4-O ports of LHD [1,2]. The temporal change of the neural net was simulated by computer simply in Euler approximation of the differential equation system of Hopfield. The square region of imaging that covers the triangular region of magnetic surface was divided into K=32x32 pixels. Both the time interval Δt in Euler approximation and a parameter u_0 relating to the slope of the sigmoid function were appropriately chosen. The amplitude scaling of detector signal data was made to avoid the stagnation and saturation of the neuron outputs due to the lower and upper limits of sigmoid function. On an observation of the asymmetric radiative collapse of NBI heated plasma (Shot 28961, 2.162 sec), a result is shown in Figs. 1 and 2. With an initial uniform image nearly zero over the square region of imaging, the energy function was monotonically decreased with an evolution of plasma image. The image obtained in convergence is similar to that of the maximum entropy method (MEM) [1]. Instead of the preblur technique in MEM, the nonlinear method of Hopfield has a feature of simple adoption of the smoothing operator C. Also, using the Euclid norm in place of the Kullback-Leibler infor-mation, the model image can be easily adopted by modifying only the bias vector.

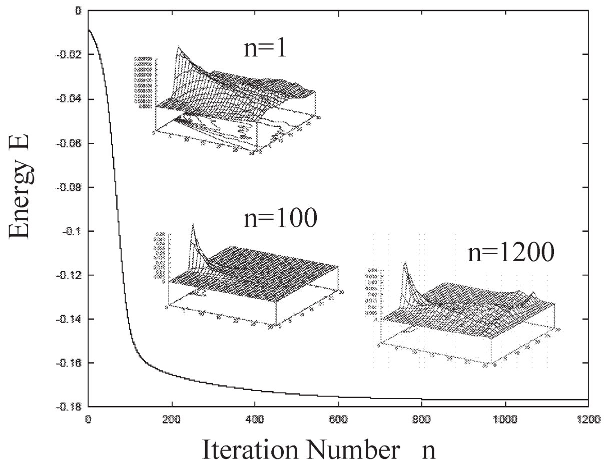

Fig. 1. Changes of the energy function E and the plasma image; $\gamma=1.0\times10^{-4}$, $\Delta t=0.01$, $u_0=1.0$.

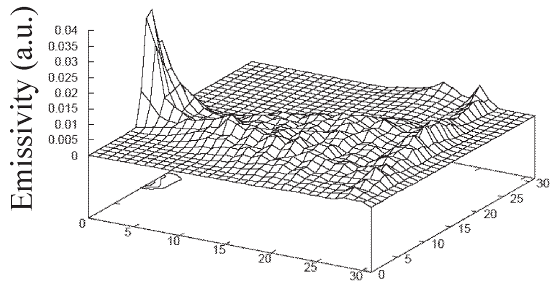

Fig. 2. Reconstructed plasma image at the iteration number n=1200 of changing the whole image.

References

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3) Iwama, N., Hosoda, Y. and Peterson, B.J., Ann. Rep. NIFS (2002-2003) 162; ibid. (2003-2004) 167.