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Theory of pseudoclassical confinement and transition to L mode

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A theory of the self-sustained turbulence is developed for resistive plasma in toroidal devices. Pseudoclassical confinement is obtained in the low-temperature limit. As temperature increases, the current diffusivity prevails upon resistivity, and the turbulence nature changes so as to recover the L-mode transport. Comparison with experimental observation on this transition is made. The Hartmann number is also given.

I. INTRODUCTION

The plasma transport across the magnetic field has been known to be much faster than that expected from the binary collision of particles. This is known as the anomalous transport and efforts to understand it has been one of the main motivations of modern plasma physics. The so-called "Bohm diffusion,"[1] i.e., the thermal conductivity χ (the energy flux per particle divided by the temperature gradient) is given as $\chi_B \equiv T/16eB$ (T is plasma temperature and B is the main magnetic field), was overcome by the concepts of the minimum average B and magnetic shear in toroidal plasmas.[2] By this, the plasma confinement time became longer than Bohm diffusion time τ_B.[3]

For such plasmas, the relation between the confinement time τ and T was studied. It was concluded that $\tau \propto \sqrt{T}/n$, and this nature of plasma confinement was called as the pseudoclassical transport.[4] The form of $\chi \propto \nu_e \rho_{pe}^2$ was proposed (ν_e is electron ion collision frequency and ρ_{pe} is electron gyroradius evaluated by the poloidal magnetic field). This character has been confirmed in internal ring devices, stellarators, and tokamaks in some range of plasma temperature.[4–8] The dependence $\tau \propto \sqrt{T}$ is favorable for thermonuclear fusion research and encouraged constructions of large toroidal devices. The deviation from $\sqrt{T}$ dependence of τ, however, was found;[5] τ again starts to decrease with increments of the temperature. Yoshikawa proposed the form of $\chi \propto T$ and called it neo-Bohm transport.[9] The range of temperature variation has become wider through use of the auxiliary heating on tokamaks. The present database yields the relation $\tau \propto P^{-0.5}$, where P is the heating power.[10,11] This confinement characteristic is called as L mode, but is identical, from the viewpoint of T dependence of χ, to the neo-Bohm confinement. The transition happens at certain temperatures from pseudoclassical transport to L-mode (neo-Bohm) transport.

The origin of the pseudoclassical transport was attributed to resistive instabilities.[4] Much work has been done on linear theory and nonlinear theories.[12–15] Linear theory has predicted that the favorable dependence $\tau \propto \sqrt{T}$ should be replaced by another dependence as $\tau \propto T^{-7/2}$ at the high temperature limit. This would be qualitatively correct. However, this fails to quantitatively explain the L-mode (neo-Bohm) confinement time. The transition point from pseudoclassical to L-mode (neo-Bohm) confinement was not explained. No theory has been successful for the simultaneous explanation of the L-mode and pseudoclassical confinement as well as the transition between them.

We have recently proposed a new theoretical method to analyze the fluctuations in toroidal plasma.[16–18] It is considered that the fluctuation itself has the effects to destabilize the microscopic mode in addition to the stabilization effects. The marginal stability condition for the nonlinear instability was solved, and the anomalous transport coefficient and fluctuation structure were simultaneously obtained. The analysis based on the scale invariance method has also confirmed the results.[19] The analysis was done for high-temperature plasma, for which the resistivity is neglected, and the result was found to explain the L-mode confinement. We apply this method to the low-temperature toroidal plasma with large resistivity. The pseudoclassical transport is recovered in the resistive limit. As the temperature increases, the current diffusivity takes over the destabilization mechanism. The transition from pseudoclassical transport to L-mode (neo-Bohm) transport occurs at a certain temperature. Comparison with the spherator experiment[4] is discussed. By using the formula of the anomalous transport coefficient, the Hartmann number[20] is also obtained.

II. MODEL AND STABILITY ANALYSIS

We study the circular tokamak with the toroidal coordinates (r, θ, ζ). The reduced set of equations[21] is employed. The $E\times B$ nonlinear interactions are renormalized in a form of the thermal conductivity, χ, the ion viscosity, μ, and the current diffusivity, λ. (The detailed derivation is reported in Ref. 18.) We employ Ohm's law $E+v\times B=J/\sigma-\nabla_\perp^2\lambda J$ (σ is the conductivity), the equation of motion

$$n_i m_i\left(\frac{d(\nabla_\perp^2\phi)}{dt}-\mu\nabla_\perp^4\phi\right)=B\nabla_\parallel J+\nabla p\times\nabla\left(\frac{2r\cos\theta}{R}\right)$$

and the energy balance equation $dp/dt=\chi\nabla_\perp^2 p$. Here m_i is the ion mass, n_i is the ion density, ϕ is the streamfunction, B is the main magnetic field, p is the plasma pressure, and J is the current.

The ballooning transformation[22] is employed as

$$\phi(r,\theta,\zeta)=\sum_m \exp(-im\theta+in\zeta)\times\int\phi(\eta)\exp(im\eta-inq\eta)d\eta$$

(q is the safety factor), since we are interested in microscopic modes. The linearized equation is reduced to the ordinary differential equation

$$\frac{d}{d\eta}\frac{F}{\hat\gamma+\Xi F+\Lambda F^2}\frac{d\phi}{d\eta}+\frac{\alpha[\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta]\phi}{\hat\gamma+XF}-(\hat\gamma+MF)F\phi=0. \qquad (1)$$

We use the normalization $r/a\to\hat r$, $t/\tau_{Ap}\to\hat t$, $\chi\tau_{Ap}/a^2\to\hat\chi$, $\mu\tau_{Ap}/a^2\to\hat\mu$, $\tau_{Ap}/\mu_0\sigma a^2\to 1/\hat\sigma$, $\lambda\tau_{Ap}/\mu_0 a^4\to\hat\lambda$, $\tau_{Ap}\equiv a\sqrt{\mu_0 m_i n_i}/B_p$, $\gamma\tau_{Ap}\to\hat\gamma$, and notation $\Xi=n^2q^2/\hat\sigma$, $\Lambda=\hat\lambda n^4q^4$, $X=\hat\chi n^2q^2$, $M=\hat\mu n^2q^2$, γ is the growth rate, $s=r(dq/dr)/q$, $F=1+(s\eta-\alpha\sin\eta)^2$, $\kappa\equiv-(r/R)(1-1/q^2)$ (average well), $B_p=Br/qR$, $-\alpha=q^2\beta'/\epsilon$, $\epsilon=r/R$, a and R for the major and minor radii, β is for the pressure divided by the magnetic pressure, and $\beta'\equiv d\beta/d(r/a)$. If we neglect $\hat\lambda$, $\hat\chi$, and $\hat\mu$, Eq. (1) reduces to the resistive ballooning equation. The ideal magnetohydrodynamic (MHD) mode equation is recovered by further taking $1/\hat\sigma=0$.

The stability boundary is derived. Setting $\hat\gamma=0$ in Eq. (1), we have the eigenvalue equation, which determines the relation between $\hat\chi$, $\hat\lambda$, $\hat\mu$, and $\hat\sigma$ for given pressure gradient. We study here the case that the ballooning mode is destabilized by the normal curvature, not by the geodesic curvature, i.e., $1/2+\alpha>s$. For the strongly localized mode, $s^2\eta^2<1$ and $\eta^2<1$, this eigenvalue equation is approximated by the Weber-type equation as

$$\frac{d^2}{d\eta^2}\phi+\frac{\alpha(1/\hat\sigma+\hat\lambda n^2q^2)}{\chi}\times\left[1-\left(\frac{1}{2}+\alpha-s+\frac{s^2}{1+\hat\lambda n^2q^2/\hat\sigma}\right)\eta^2\right]\phi-\hat\mu n^4q^4\times[(1/\hat\sigma+\hat\lambda n^2q^2)+(3\hat\lambda n^2q^2+2/\hat\sigma)s^2\eta^2]\phi=0. \qquad (2)$$

The eigenvalue for the fundamental mode is readily seen as

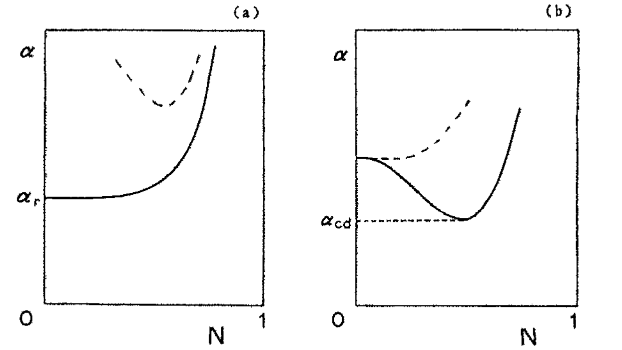

FIG. 1. Marginal stability condition for the nonlinear ballooning instability (schematic). N stands for the normalized mode number. The resistive limit ($\rho\to 0$) and the current diffusive limit ($\rho\gg 1$) correspond to (a) and (b), respectively. Dashed lines indicate limiting expressions: Eqs. (6) in (a) and Eq. (5) in (b), respectively.

$$\frac{\alpha}{\hat\sigma\hat\chi}=g(1+g_1\rho N^2)(1+\rho N^2)^{-2}(1-N^4)^{-2}\times\left(1+\frac{2s^2}{g}\frac{1+\rho N^2}{1+g_1\rho N^2}N^4\right), \qquad (3)$$

where ρ is the ratio

$$\rho=\hat\sigma\hat\lambda(\alpha/\hat\chi\hat\mu)^{1/2}, \qquad (4a)$$

N is the normalized mode number

$$N=nq(\hat\chi\hat\mu/\alpha)^{1/4}, \qquad (4b)$$

and (g,g_1) are coefficients

$$g=1/2+\alpha+s^2-s,\quad g_1=(1/2+\alpha-s)/g. \qquad (4c)$$

The right-hand side of Eq. (3) shows the dependence of the marginal stability condition on the mode number. Figure 1 illustrates the schematic dependence of critical value of α as a function of the normalized mode number. In the resistive limit of $\rho\to 0$ (i.e., the resistive diffusion of the magnetic field is faster than that by current diffusivity), the dependence on the term $(1+\rho N^2)^{-2}$ on the right-hand side (rhs) of Eq. (3) is not important. The limiting equation is reduced from Eq. (3) as

$$\frac{\alpha}{\hat\sigma\hat\chi}=g(1-N^4)^{-2}\left(1+\frac{2s^2}{g}N^4\right). \qquad (5)$$

In this case, the upper bound of α for the stability (for fixed values of transport coefficients) takes place for the low mode number case. Figure 1(a) illustrates this N dependence. If, on the contrary, ρ is greater than unity, the term $(1+\rho N^2)^{-2}$ dictates the minimum of the rhs of Eq. (3). The simplified equation for the limit of $\rho\to\infty$ is reduced from Eq. (3) as

$$\frac{\rho\alpha}{\hat\sigma\hat\chi}=gg_1N^{-2}(1-N^4)^{-2}\left(1+\frac{2s^2}{gg_1}N^4\right). \qquad (6)$$

Figure 1(b) shows the N dependence for the current-diffusive limit. The minimum value of α is given by the intermediate number of N.

In the resistive limit of $\rho \to 0$, the minimum of the rhs of Eq. (3) is given as g. The marginal stability condition for the least stable mode is given as

$$\alpha = \alpha_r \tag{7}$$

with

$$\alpha_r = g\hat{\sigma}\hat{\chi}. \tag{8}$$

In the current-diffusive limit of $\rho \to \infty$, the marginal stability condition for the least stable mode is given as

$$\alpha = \alpha_{\rm cd} \tag{9}$$

with

$$\alpha_{\rm cd} = (\hat{\chi}/\hat{\lambda})^{2/3}(\hat{\chi}\hat{\mu})^{1/3}f(s)^{2/3}, \tag{10}$$

where $f(s) = 2gg_1\sqrt{2+2s^2/gg_1}$.

III. TRANSPORT COEFFICIENT

Based on the stability analysis, we can derive the formula for the anomalous transport coefficient. Equations (7)–(10) dictate the relation between the transport coefficients and the pressure gradient at the stationary state of the nonlinear ballooning mode. This state is thermodynamically stable. When the mode amplitude and the associated transport coefficients are small, Eq. (1) gives the instability. Extra growth of the mode and the resulting enhanced transport coefficients over Eq. (3) lead to damping of the mode.

From Eqs. (7) and (9), $\hat{\chi}$ is expressed in terms of the Prandtl numbers $\hat{\mu}/\hat{\chi}$ and $\hat{\lambda}/\hat{\chi}$. For the resistive plasma, Eq. (7) gives

$$\hat{\chi} = \alpha/\hat{\sigma}g. \tag{11}$$

Using dimensional quantities, Eq. (11) gives

$$\chi = 2\alpha/\mu_0\sigma, \tag{12}$$

which has been known as the transport coefficient of the resistive plasma. Using the relation of $\sigma = ne^2/m_e\nu_e$, Eq. (12) is rewritten as

$$\chi = 4(\epsilon/\hat{L}_p)\nu_e\rho_{pe}^2, \tag{13}$$

where $\hat{L}_p$ is a normalized pressure gradient scale length, $-d\beta/d\hat{r} = \beta/\hat{L}_p$. Apart from a geometrical numerical factor of order unity, Eq. (13) is the pseudoclassical diffusion coefficient.

On the other hand, thermal conductivity in the current diffusive limit was given as

$$\hat{\chi} = \alpha^{3/2}(\hat{\lambda}/\hat{\chi})\sqrt{\hat{\chi}/\hat{\mu}}/f(s). \tag{14}$$

The relations $\hat{\lambda}/\hat{\chi} \simeq \delta^2/a^2$ and $\hat{\mu}/\hat{\chi} \simeq 1$ hold and the explicit form of χ was given as ($v_{\rm A}$ is the Alfvén velocity)

$$\chi = f(s)^{-1}q^2(R\beta'/r)^{3/2}\delta^2 v_{\rm A}/R. \tag{15}$$

Compared to Yoshikawa's formula for the neo-Bohm transport, this form of χ has a slightly stronger temperature dependence. Equation (15) also suggests that the poloidal magnetic field, not the toroidal field, is important in determining the anomalous transport. This fact was discovered in the multipole devices as well. The theoretical prediction Eq. (15) is consistent with experimental results known for the L mode as was discussed in detail.[18] It is noted that Eq. (15) is related to the Ohkawa model of χ:[23] the geometrical factor is correctly kept in Eq. (15).

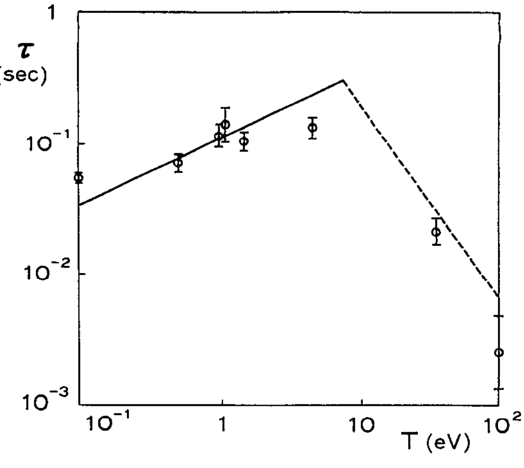

FIG. 2. Confinement time as a function of the temperature. Formulas (17) and (18) are fitted to the spherator plasma. Data points are quoted from Ref. 4. The solid line is pseudoclassical law (17) and the dotted line is for the neo-Bohm (L-mode) law (18). For the parameter of interest, the turnover temperature is predicted as 8 eV. Numerical coefficients of χ are adjusted to reproduce the original line of Ref. 5 in the low-temperature limit.

The change from the pseudoclassical transport to the L-mode transport occurs at the condition $\alpha_r \sim \alpha_{\rm cd}$. This condition is written as $\hat{\sigma} \simeq \alpha^{-1/2}(a/\delta)^2$, or in a dimensional form as

$$\nu_e\tau_{Ap} \sim \alpha^{1/2} \tag{16a}$$

or

$$\nu_e \sim v_{Ti}/\sqrt{\hat{L}_p R}, \tag{16b}$$

where v_{Ti} is the ion thermal velocity. Using the normalized collision frequency ν_* (ratio of ν_e to the bounce frequency $\sqrt{\epsilon}v_{Te}/qR$), Eq. (14) is rewritten as $\nu_* \sim q\sqrt{m_e/m_i}\epsilon\hat{L}_p$. The transition from the pseudoclassical confinement to the L-mode confinement is predicted to occur in the banana regime of electrons, if the parameter $\hat{L}_p$ is of order unity.

In the limit $\nu_e\tau_{Ap} \gg \sqrt{\alpha}$, the relation between the confinement time τ and temperature

$$\tau \propto \sqrt{T} \tag{17}$$

holds (assuming that other parameters are fixed). In the other limit, we have

$$\tau \propto T^{-3/2}. \tag{18}$$

Figure 2 compares the theoretical predictions with experiments on the spherator.[5] (Typical parameters are used: $n_e = 10^{17}$ m^{-3}, $R = 0.4$ m, $R/a = 6$, $\hat{L}_p = 0.4$.) For the set of parameters, the turnover from pseudoclassical to neo-Bohm transition occurs at around 8 eV. In Fig. 2, the coefficients of order unity are adjusted to recover the original line of the pseudoclassical law of Yoshikawa (solid line of Fig. 2) in the low-temperature limit. Formula (18) takes over at the connection point of 8 eV. Since the plasma profile is not compared, only the semiquantitative comparison is, at most, possible. We emphasize that Eq. (15) depends on the density profile as well as on the temperature profile. The larger χ value is predicted at the edge region due to the large collisionless skin depth (i.e., low density). Though the theoretical prediction is based on the very simplified point model argument, the theoretical results Eqs. (16)–(18) may explain the pseudoclassical transport and transition to neo-Bohm transport in the spherator experiments.

Using the results on the anomalous transport coefficient, we calculate the Hartmann number.[20] The Hartmann number is an important parameter for the global instabilities.[24] The other important parameter is the magnetic Prandtl number, $P_M = \mu_0 \sigma \mu$, which may have a key role in the dynamo mechanism.[25] Hartmann number $\mathbf{M}$ is defined as $\mathbf{M} = BL\sqrt{\sigma/m_i n_i \mu}$, where L is the typical scale length. Substituting plasma minor radius into L, we have the relation $\mathbf{M}^2 = (qR/a)^2(\hat{\sigma}/\hat{\mu})$. The Prandtl number μ/χ remains of order unity.[16,17] Using the formula of Eq. (11), we have

$$\mathbf{M} = (qR/a)\hat{\sigma}\sqrt{g/\alpha} \tag{19}$$

for the plasmas which satisfy pseudoclassical scaling law. This result shows that the Hartmann number increases in proportion to the magnetic Reynolds number. The linear dependence of $\mathbf{M}$ on the plasma temperature is also found. The geometrical factor is explicitly included in Eq. (19).

As the transition from the pseudoclassical confinement to L-mode confinement takes place, the Hartmann number changes its dependence on the temperature. Using Eq. (15), we have

$$\mathbf{M} = (qR/\delta)\hat{\sigma}^{1/2}\alpha^{-3/4} \tag{20}$$

for the L-mode plasma. In an explicit form, Eq. (20) can be rewritten as

$$\mathbf{M} \propto B^2 q^{-1} \epsilon^{1/4} n^{-1/2} a^{3/2}. \tag{21}$$

The Hartmann number no longer depends on the plasma temperature. The plasma with the lower density has the higher Hartmann number.

The magnetic Prandtl number is rewritten as $P_M = \hat{\mu}\hat{\sigma}$. In the resistive limit (pseudoclassical limit), we have

$$P_M = \alpha/g; \tag{22}$$

P_M is an increasing function of the plasma beta value. In the current-diffusive limit (L-mode limit), P_M depends more strongly on the plasma temperature, as $P_M \propto T^3$. For the parameters of present-day experiments, P_M can exceed unity.

IV. SUMMARY AND DISCUSSION

The theory of the anomalous transport and self-sustained turbulence was applied to the resistive plasmas. The system with magnetic well and shear, such as in a tokamak, was investigated. The pseudoclassical transport coefficient was obtained in the low-temperature limit. Using the formula of the transport coefficients, the Hartmann number and magnetic Prandtl number are also obtained.

From this analysis, the pseudoclassical transport is found to be connected to the L-mode (neo-Bohm) transport at a certain temperature. The pseudoclassical transport and L-mode transport are now expressed in terms of our unified anomalous transport theory, which is obtained for the self-sustained ballooning mode turbulence. Equations (12) and (15) are the generic expressions for the transport coefficients in toroidal plasmas. It is noted that the Yoshikawa formula on pseudoclassical scaling and the Ohkawa formula keeping $\delta^2 v_A/R$ dependence represent the two limiting features of the anomalous transport in toroidal plasmas. "The possibility that anomalous plasma diffusion depends on geometric factor," which was posed in Ref. 4, is demonstrated and formulated in Eqs. (12) and (15).

It is a straightforward extension to apply this method to the system with the magnetic hill. The interchange mode is analyzed instead of the ballooning mode. With the introduction of the additional coefficients which reflect the magnetic hill, a similar formula was obtained. This also explains the change of the confinement in stellarators from the pseudoclassical scaling to the L-mode-type scaling.

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