Transport Analysis of Dynamics of Plasma Profiles in Helical Devices

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Plasma dynamics and structure are studied using a one-dimensional theoretical model for the anomalous transport diffusivities. In this analysis, the high collisional Pfirsch-Schlüter regime is examined and the anomalous particle diffusivity is employed. The reduction of the anomalous particle diffusivity and a steep gradient in the density profile can be obtained. This prediction may be the theoretical explanation for the internal diffusion barrier observed in super dense core plasmas of large helical device.

Keywords: helical toroidal plasmas, turbulent transport, transport barrier, improvement mode, radial electric field, particle confinement

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1. Introduction

Turbulence-driven transport and transport barriers are the key issues in fusion research. The majority of efforts have been focused in understanding improved confinement modes (such as the H-mode). Among various improved confinement modes, two main types of transport barriers are observed in helical experimental results. At first, because of the radial transition of E_r and the improvement in the electron confinement, an electron internal transport barrier (e-ITB) was observed at the transition point for E_r. Transport analysis with the effect of zonal flows could predict an e-ITB in helical plasmas [1]. A comparison of the analysis results with the experimental results was made [2]. As the second type of transport barriers, an internal diffusion barrier (IDB) was recently observed with the high gradient of the density in a super dense core (SDC) plasma in a large helical device (LHD), when a series of pellets is injected [3]. In the core region, the obtained high density and the temperature are around $5 \times 10^{20}\,\mathrm{m}^{-3}$ and 0.85 keV, respectively. A transport study of a SDC plasma is an urgent and critical issue in helical confinement plasmas. We solve the temporal transport equations for the density, radial electric field, and temperature to study the dynamics of SDC plasmas in a cylindrical configuration. The additional source profile of the density, which corresponds to that in the case of the pellet injection in LHD experiments, is studied in the transport equations. The analytical form used here for the neoclassical particle and heat flux related with the helical ripple trapped particles in the high-collisional regime is given in [4]. The radial electric filed is assumed to be determined by the ambipolar condition, which is constituted with the neoclassical particle flux. The density and temperature profiles are determined from the neoclassical and anomalous transport. A transport model for anomalous diffusivities is adopted to describe the turbulent plasma. A dynamics for a self-consistent solution for plasma profiles is examined, and the barrier of the particle transport at the inner radial point in the density profile can be reproduced. That can explain a SDC plasma, which is accompanied with an IDB in LHD. The profile of the thermal diffusivity is examined when a high-density plasma corresponding to a SDC plasma is obtained as a calculation result.

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2. Model Equations

The model equations used in this study are shown here. The one-dimensional transport model is employed. The cylindrical coordinate is used, and r-axis is taken in the radial cylindrical plasma in this article. The region $0 < \rho < 1$ is considered, where a is the minor radius, and $\rho = r/a$. The expression for the radial neoclassical flux associated with helical-ripple trapped particles is given in [4], which covers the Pfirsch-Schlüter collisional regime, because the plasma state of a SDC corresponds to the high-collisional regime. The total particle flux Γ^{t} is expressed as $\Gamma^{\mathrm{t}} = \Gamma^{\mathrm{na}} - D_{\mathrm{a}} n'$, where Γ^{na} is the neoclassical flux associated with the helical-ripple trapped particles, and the prime denotes the radial derivative. Here, D_{a} is the turbulent (anomalous) particle diffusivity. The energy flux related with the neoclassical ripple transport Q_j^{na} is obtained in the same way as the neoclassical particle flux. The total heat flux Q_j^{t} for the species j is expressed as $Q_j^{\mathrm{t}} = Q_j^{\mathrm{na}} - n\chi_{\mathrm{a}} T_j' - 5D_{\mathrm{a}} n' T_j/2$, where χ_{a} is the anomalous heat diffusivity, and Q_j^{na} is the neoclassical contribution from the Pfirsch-Schlüter regime. A theoretical model for the anomalous heat conductivity χ_{a} is adopted and is explained later. The temporal equation for the density is

$$\frac{\partial n}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\Gamma^{\rm t}) + S_{\rm n} + S_{\rm p}, \tag{1}$$

where the term $S_{\rm n}$ represents the particle source and the parameter $S_{\rm p}$ is the term to simulate the experimental procedure of the pellet injection. The detail about the term $S_{\rm p}$ will be explained later. The equation for the electron temperature is given as

$$\frac{3}{2}\frac{\partial}{\partial t}(nT_{\rm e}) = -\frac{1}{r}\frac{\partial}{\partial r}(rQ_{\rm e}^{\rm t}) - \frac{m_{\rm e}}{m_{\rm i}}\frac{n}{\tau_{\rm e}}(T_{\rm e}-T_{\rm i}) + P_{\rm he}, \tag{2}$$

where the term $\tau_{\rm e}$ denotes the electron collision time. The term $P_{\rm he}$ represents the absorbed power due to the electron cyclotron resonance (ECRH) heating. The temporal equation for the ion temperature is

$$\frac{3}{2}\frac{\partial}{\partial t}(nT_{\rm i}) = -\frac{1}{r}\frac{\partial}{\partial r}(rQ_{\rm i}^{\rm t}) + \frac{m_{\rm e}}{m_{\rm i}}\frac{n}{\tau_{\rm e}}(T_{\rm e}-T_{\rm i}) + P_{\rm hi}. \tag{3}$$

The term $P_{\rm hi}$ represents the absorbed power of ions. The equation for the radial electric field in a nonaxisymmetric system is expressed by the ambipolar condition as

$$\sum_j Z_j \Gamma_j^{\rm na} = 0. \tag{4}$$

In this article, we examine the plasma which consists of electrons and hydrogen ions. Therefore, the ambipolar condition, Eq. (4), can be rewritten as $\Gamma_{\rm i}^{\rm na} = \Gamma_{\rm e}^{\rm na}$. It is well known that the neoclassical transport is dominant when the radial electric field is formed in helical plasmas [5]. The analysis including the temporal equation of $E_{\rm r}$ [6] which includes the effect of the electric field diffusion is left for the future study. The source profiles are selected here as follows. The particle source $S_{\rm n}$ is set as $S_{\rm n} = S_0 \exp((r-a)/L_0)$, where L_0 is set to 0.1 m. This profile represents the peak at the plasma edge of the particle source due to the ionization effect. The intensity S_0 governs the average density, and is taken as a control parameter to specify the density profile in this article. The radial profiles of the electron and ion heating terms, $P_{\rm he}$ and $P_{\rm hi}$, are assumed to be proportional to $\exp(-(r/(0.5a)^2)$ for the sake of an analytic insight.

3. Model for Anomalous Transport Diffusivities and Boundary Conditions

We adopt the model for the turbulent heat diffusivity $\chi_{\rm a}$ based on the theory of self-sustained turbulence due to the ballooning and interchange modes, both driven by the current diffusivity [7, 8]. The anomalous transport coefficient for the temperatures is expressed as $\chi_{\rm a} = \chi_0/(1+G\omega_{E1}^2)$ $(\chi_0 = F(s,\alpha)\alpha^{\frac{3}{2}}c^2 v_{\rm A}/(\omega_{\rm pe}^2 qR))$, where $\omega_{\rm pe}$ is the electron plasma frequency. The factor $F(s,\alpha)$ is the function of the magnetic shear s and the normalized pressure gradient α, defined by $s = rq'/q$ and $\alpha = -q^2R\beta'$. For the ballooning mode turbulence (in the system with a magnetic well), we employ the anomalous thermal conductivity $\chi_{\rm a,BM}$. The details about the coefficients $F(s,\alpha)$, G, and the factor ω_{E1}, which stands for the poloidal $E\times B$ rotation frequency, are given in [8] in the ballooning mode turbulence. In the case of the interchange mode turbulence for the system of the magnetic hill [7], we adopt the anomalous thermal conductivity $\chi_{\rm a,IM}$. The details about the coefficients F, G, and the factor ω_{E1} in the case of the interchange mode were given in [7]. The greater of these two diffusivities is adopted as $\chi_{\rm a} = \max(\chi_{\rm a,BM}, \chi_{\rm a,IM})$. The value for the anomalous diffusivities of the particle is set as $D_{\rm a} = \chi_{\rm a}$ to examine the radial variation in the profile of the particle diffusivity $D_{\rm a}$, when the steep radial gradient in the density profile can be obtained as a calculation result.

The equations of density, temperature and electric field (1)-(4) are solved with the prescribed source profiles, under the appropriate boundary conditions. We fix the boundary condition at the center of the plasma ($\rho = 0$), such that $n' = T_{\rm e}' = T_{\rm i}' = 0$. The boundary conditions at the edge ($\rho = 1$), with respect to the density and temperature, are given by specifying the gradient scale lengths. We employ these conditions, $-n/n' = 0.1$ m and $-T_{\rm e}/T_{\rm e}' = -T_{\rm i}/T_{\rm i}' = 0.1$ m, in this article. The machine parameters that are similar to those of LHD are set to be $R = 3.6$ m, $a = 0.6$ m, $B = 3$ T, $\ell = 2$, and $m = 10$. In this case, we set the safety factor and the helical ripple coefficient as $q = 1/(0.4 + 1.2\rho^2)$ and $\epsilon_{\rm h} = 2\sqrt{1-(2/(mq(0))-1}I_2(mr/R)$, respectively. Here, $q(0)$ is the value of the safety factor at $\rho = 0$ and I_2 is the second-order modified Bessel function.

4. Dynamical Response after Pellet Injection

Using these parameters and boundary conditions, the one-dimensional transport analysis for the LHD-like plasma has been performed, and the profiles of n, $T_{\rm e}$, $T_{\rm i}$, and $E_{\rm r}$ are solved using equations (1), (2), (3), and (4). We adopt a theoretical model for the anomalous transport diffusivities driven by the current diffusivity as a candidate. An example is taken from the plasma, which is sustained by the ECRH. At first, we obtain self-consistent steady profiles of n, $T_{\rm e}$, $T_{\rm i}$, and $E_{\rm r}$ for the given source profile. To set the line-averaged density to be approximately $\bar{n} \simeq 1\times10^{20}\,{\rm m}^{-3}$, the line-averaged temperature of electrons to be approximately $\bar{T}_{\rm e} \simeq 0.4$ keV, and the line-averaged temperature of ions to be approximately $\bar{T}_{\rm i} \simeq 0.4$ keV, the absorbed power of electrons is set to be 1 MW and the coefficient S_0 is taken as $7\times10^{20}\,{\rm m}^{-3}{\rm s}^{-1}$, to obtain the above-mentioned anomalous transport coefficients, where the absorbed power of ions is taken as 0 kW. The radial electric field takes a negative value in the entire radial region. Next, we use these obtained profiles of n, $T_{\rm e}$, $T_{\rm i}$, and $E_{\rm r}$ as an initial condition, i.e., we begin a new calculation from the profile of the density as $\bar{n} \simeq 1\times10^{20}\,{\rm m}^{-3}{\rm s}^{-1}$ and the negative $E_{\rm r}$ in the entire region. In this parameter region ex-

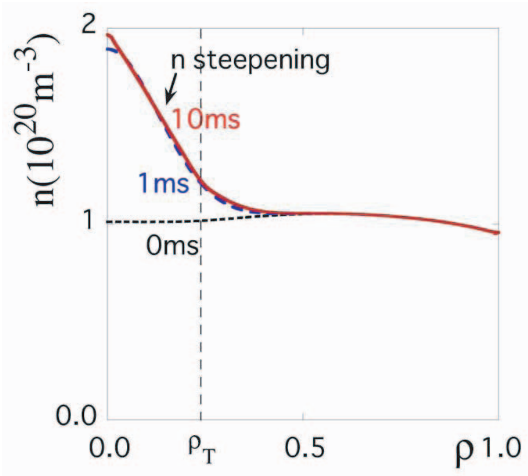

Fig. 1 The dynamics of the density radial profiles. The dotted line, the dashed line and the solid line show the states at the times 0 ms, 1 ms and 10 ms, respectively.

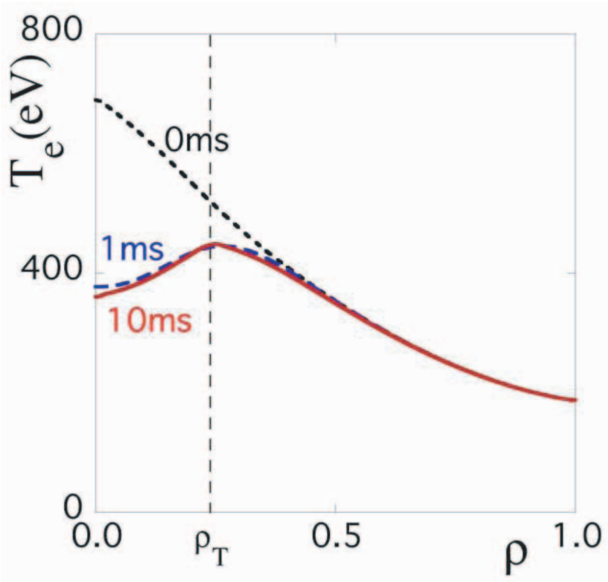

Fig. 2 The profiles of the electron temperature at the times 0 ms, 1 ms and 10 ms. Because the density rapidly increases in the core region, the electron temperature is found to decrease in that region ($\rho < \rho_{\mathrm{T}}$).

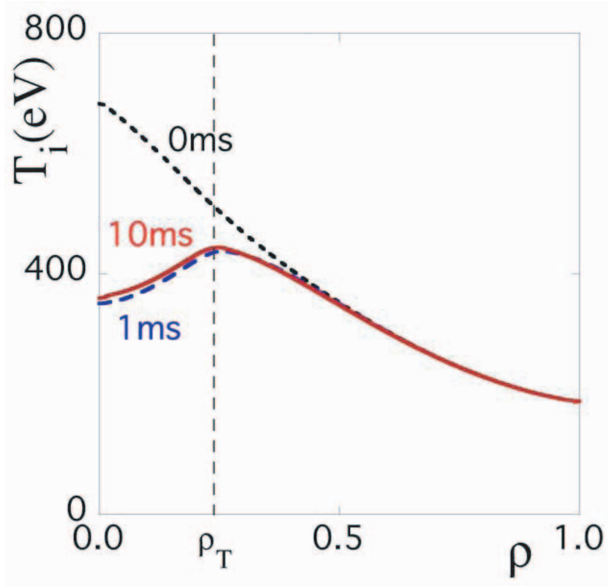

Fig. 3 The profiles of the ion temperature at the times 0 ms, 1 ms and 10 ms.

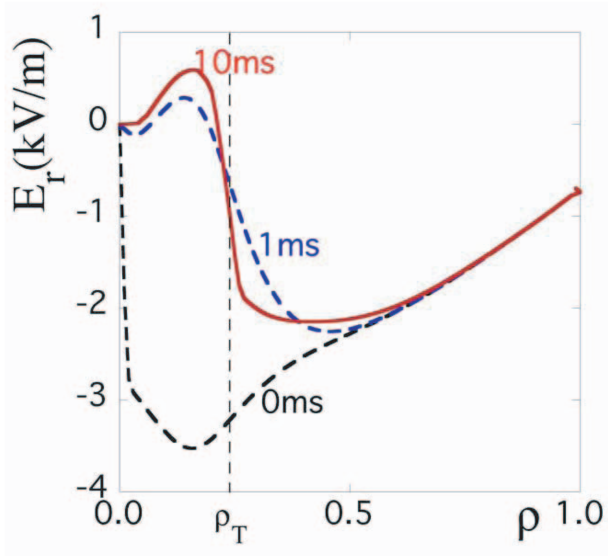

Fig. 4 The dynamics of the radial electric field. At the time 10 ms, the radial transition of E_{r} from the positive value to the negative value can be obtained.

amined here, both typical energy and particle confinement times are approximately 1 s. To simulate the experimental procedure of the pellet injection, we include the parameter S_{p} of the particle source in Eq. (1). This parameter S_{p} has a distribution as $S_{\mathrm{p}} = S_{\mathrm{p0}} \exp(-(r/r_{\mathrm{p}})^2)$, which is set to have a value from the initial time $t = 0$ to 1 ms. We set the half width of the profile S_{p} as $r_{\mathrm{p}} = 0.2a$. In other words, we set $S_{\mathrm{p0}} = 1 \times 10^{23}\ \mathrm{m^{-3}s^{-1}}$ for $0 < t \leq 1$ ms and $S_{\mathrm{p0}} = 0\ \mathrm{m^{-3}s^{-1}}$ for $t > 1$ ms. We show the dynamics of the plasma radial profiles n (Fig. 1), T_{e} (Fig. 2), T_{i} (Fig. 3) and E_{r} (Fig. 4) at the times 0 ms, 1 ms and 10 ms. The profiles labelled by 0 ms represent the initial conditions used in the calculation in this article (with the dotted lines). We use the radial mesh $\mathrm{d}x = a/100$ for evaluating the equations (1)-(3). If we use smaller radial and temporal meshes, we can reproduce these calculation results shown in all figures of this article. As the time goes on, the density increases and the positive electric field appears. In Fig. 1, the rapid change of the gradient in the density profile at $t = 10$ ms can be found at $\rho = \rho_{\mathrm{T}} (\simeq 0.2)$. The parameter ρ_{T} represents the location of the transition from the positive E_{r} to the negative E_{r} at 10 ms in Fig. 4. Also in this figure, we can show the steep gradient of the radial electric field at the transition point ρ_{T}. (The profile of the radial electric field is determined by the ambipolar radial electric field, and the single solution

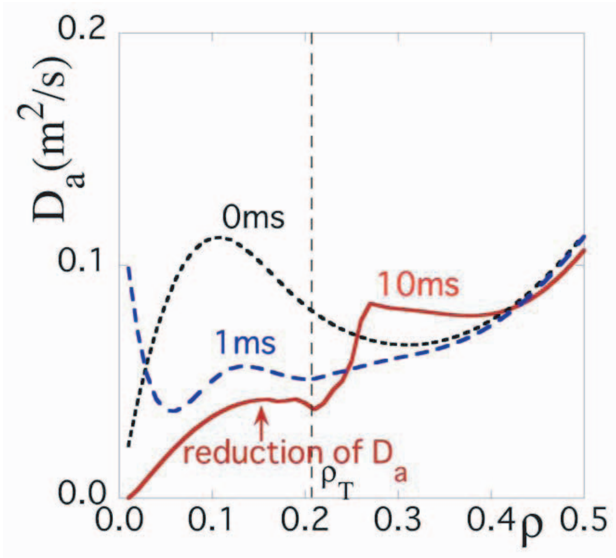

Fig. 5 The radial profiles of the anomalous particle diffusivity D_a in the region $0 < \rho < 0.5$ at the times 0 ms (with the dotted line), 1 ms (with the dashed line) and 10 ms (with the solid line). We can show the clear reduction of the anomalous particle diffusivity at the time 10 ms in the region $\rho < \rho_T$.

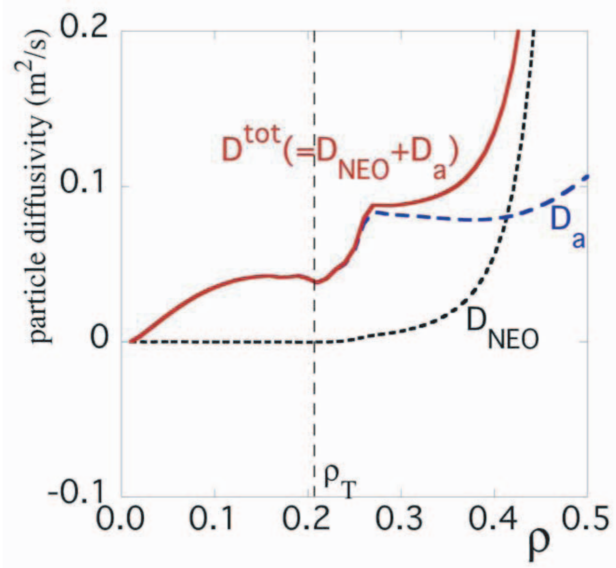

Fig. 6 The radial profiles of the particle diffusivity in the region $0 < \rho < 0.5$ at the times 10 ms. We can show the strong reduction in the sum of the neoclassical particle diffusivity and the anomalous particle diffusivity at the time 10 ms in the region $\rho < \rho_T$ with the solid line.

of this field for the ambipolar condition can be obtained at a radial point.) The gradient of the radial electric field is sufficiently strong to suppress the anomalous transport $|E_r'| \simeq 1 \times 10^5\,\mathrm{Vm^{-2}}$ at $\rho = \rho_T$. Therefore, the improvement near the transition point can be achieved. In Figs. 2 and 3, the temperatures are found to decrease toward the plasma center when the value of the density rapidly increases. The rapid increase in the density between the times 0 and 1 ms in the core region mainly depends on the pellet input. After the time 1 ms, the density in the core region continues to increase because there is an inward neoclassical transport due to the positive gradient of the temperature profiles. If the time passes the confinement time, the plasma profiles takes the initial values of the parameters. At $t = 10$ ms, the clear barrier with respect to particle transport in the density profile can be obtained in Fig. 1.

A profile in the region $\rho < 0.5$ of the anomalous transport diffusivity is shown in Fig. 5. In the region $\rho < \rho_T$, the reduction of the anomalous particle diffusivity can be observed, because the value of the anomalous transport diffusivity is proportional to the square of the safety factor q, and the temperatures of electrons and ions decrease toward the plasma center at 10 ms. A clear reduction of the anomalous particle diffusivity is observed at the transition point $\rho = \rho_T$, due to the strong gradient of E_r compared with the region $\rho > \rho_T$. In the region $\rho < 0.3$ at the time 10 ms, the neoclassical particle diffusivity $D_{\mathrm{NEO}}(= -\Gamma^{\mathrm{na}}/n')$ is found to be much smaller than the absolute value of the effective anomalous particle diffusivity and is $|D_{\mathrm{NEO}}| < 0.01\,\mathrm{m^2s^{-1}}$ represented as the dotted line. In the core region, the slightly negative value for the effective neoclassical particle diffusivity D_{NEO} and the particle neoclassical pinch are obtained at the time 10 ms. In Fig. 6, the dashed line represents the radial profile of the anomalous particle diffusivity D_a, and the solid line shows the radial profile of the sum of neoclassical and anomalous particle diffusivity $D_{\mathrm{tot}}(= D_{\mathrm{NEO}} + D_a)$ at the time 10 ms. The clear reduction can be obtained in the sum of the neoclassical and anomalous transport. Therefore, we can obtain the clear barrier with respect to the particle transport in Fig. 1 (n profile at $t = 10$ ms). It is emphasized that the barrier formation starts to occur after the end of the particle fuelling. It takes a few ms for the establishment of a barrier seen in Fig. 5. It is found that the anomalous particle transport is more important in the improvement of the particle confinement in the plasma core region than in the neoclassical particle transport in the parameter region examined here. If we set much smaller value of S_{p0}, i.e., $S_{\mathrm{p0}} = 1 \times 10^{22}\,\mathrm{m^{-3}s^{-1}}$ than that ($S_{\mathrm{p0}} = 1 \times 10^{23}\,\mathrm{m^{-3}s^{-1}}$) in this calculation, we can not determine the steepening in the density gradient, because the gradient of E_r after the pellet fuelling (1 ms) is not sufficiently strong. It is shown that there is a threshold value for S_{p} to form the barrier with respect to the particle confinement in the density radial profile.

5. Summary and Discussions

We have studied the strong reduction of particle transport when the dynamics and the radial structure of profiles of the density, the temperatures and the electric field are examined in toroidal helical plasmas. The analysis is performed using the one-dimensional transport equations. These transport equations include the contributions from the neoclassical transport and the anomalous transport driven by the current diffusivity. The neoclassical theory for the particle and heat flux in the the Pfirsch-Schlüter regime is adopted. The radial profile of the electric field is determined by the ambipolar condition. The clear change of the gradient in the density profile and the reduction of the anomalous particle transport in the core region after the particle fuelling, if the value of the particle fuelling (the pellet size in the experiment) exceeds the threshold, can be shown when the temporal evolution of the plasma profiles are examined. This theoretical prediction may explain the internal diffusion barrier (IDB) observed in LHD plasmas. It is predicted that the additional particle source $S_{\rm p}$ has a threshold value to obtain the reduction of the anomalous transport diffusivity. Furthermore, this threshold value of $S_{\rm p0}$ strongly depends on the shape of the distribution, e.g., the parameter $r_{\rm p}$ which determining the half-width length of the additional particle source. To study the threshold value of $S_{\rm p0}$ for obtaining the clear barrier of the particle transport, the parameter survey of the calculations results is required. These are left for future studies.

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