§27. Construction of Generalized Magnetic Coordinates

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Generalized Magnetic Coordinates (GMC) are constructed even in the region without nested magnetic surfaces and outside the outermost magnetic surface. For example, they are applicable to the general toroidal magnetic field involving magnetic islands and the region of chaotic or ergodic magnetic lines of force.

The GMC are curvilinear coordinates (ξ,η,ζ), in which the magnetic field is described by the expression

$$\mathbf{B} = \nabla\Psi(\xi,\eta,\zeta)\times\nabla\zeta + \mathrm{H}^{\zeta}(\xi,\eta)\nabla\xi\times\nabla\eta .$$

Here $\mathrm{H}^{\zeta} = \sqrt{g}B^{\zeta}$ does not depend on ζ, where $\sqrt{g}$ is Jacobian. When the good magnetic surface exists, Ψ becomes independent of ζ and $\Psi(\xi,\eta) = Const.$ expresses the magnetic surface. The ζ-dependent part of Ψ corresponds to the destruction of magnetic surfaces like magnetic islands. The GMC are to be constructed so that Ψ becomes dependent of ζ as little as possible. The function Ψ could be replaced by the covariant ζ component of vector potential A^{ζ} without loss of generality.

The general numerical method to construct a GMC is applied to ABC(Arnol'd-Beltrami-Childress) magnetic field that are periodic in the three directions of Cartesian coordinates (x,y,z).[1] The Cartesian coordinates are expanded into Fourier series in terms of (ξ,η,ζ),

$$x = \xi + \sum_{l=-L}^{L}\sum_{m=-L}^{L}\sum_{n=1}^{L} X_{l,m,n}\exp(2\pi i[l\xi + m\eta + n\zeta]),$$

$$y = \eta + \sum_{l=-L}^{L}\sum_{m=-L}^{L}\sum_{n=1}^{L} Y_{l,m,n}\exp(2\pi i[l\xi + m\eta + n\zeta]),$$

$$z = \zeta .$$

The GMC are applied to the model magnetic field involving clear magnetic islands.[2] Fig.1 shows the GMC meshes at equal intervals on the z=0 plane in the Cartesian coordinates. The Poincaré map of magnetic lines is also overlapped. Fig.2 shows the Poincaré map on the ζ=0 plane in the GMC. In the GMC, the averaged magnetic surfaces are expressed by $\bar{A}^{\zeta}(\xi,\eta) = Const.$, where the bar denotes averaging with respect to ζ. If the breaking of nested magnetic surfaces like magnetic islands doesn't exist, magnetic surfaces are equal to the averaged magnetic surfaces. The distribution of ζ-dependency of A^{ζ} that relates with the breaking of magnetic surfaces is examined.[2] The largest ζ-dependent region of A^{ζ} is located around the outermost magnetic surface.

In the general magnetic configuration of interest, the periodic condition in the three dimensions cannot be used. For the magnetic field with no periodic conditions in the ξ and η directions, the GMC are to be expanded in terms of the B-spline function having local support.

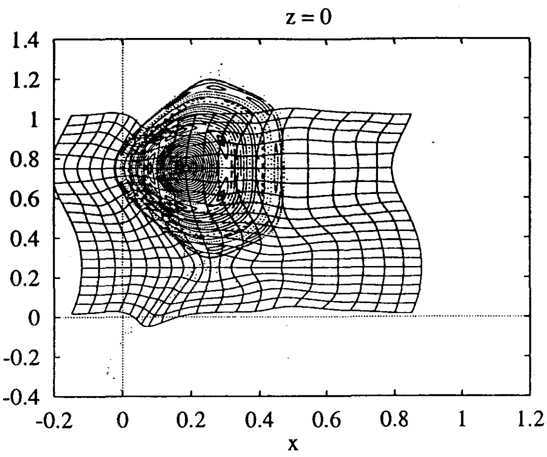

Fig.1. GMC meshes and Poincaré map in the (x,y,z).

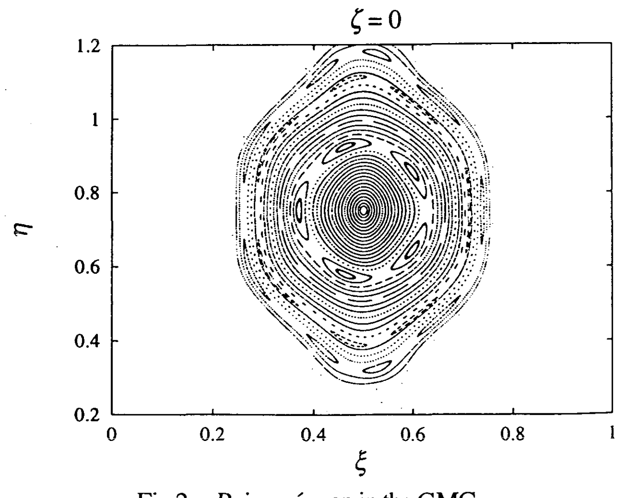

Fig.2. Poincaré map in the GMC.

Reference

1) Kurata, M. and Todoroki, J. : J. Plasma Fusion Res. SERIES, Vol. **1** (1998) 491-494.
2) Kurata, M. and Todoroki, J. : NIFS-PROC-40 (1999) 9-18.