§14. Statistical Properties of the Radial Particle Diffusion in a Tokamak Equilibrium with Irregular Field

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The statistical properties of the radial particle diffusion in a tokamak equilibrium with an irregular magnetic field are examined by numerically evaluating the cumulant, diffusion, and autocorrelation coefficients. The same procedure as that in the companion paper of this annual report [1] is used except for the magnetic field. As the magnetic field,

$$\vec{B}_t = \vec{B} + \delta\vec{B}, \quad \delta\vec{B} = \nabla \times (\alpha \vec{B}) \qquad (1)$$

is used where $\vec{B}$ and $\delta\vec{B}$ are equilibrium and perturbed fields, respectively, so that the drift velocity $\vec{v}$ of Eq.(1) in [1] is expressed as

$$\vec{v} = v_{\parallel} \frac{\vec{B} + \nabla \times \left[\left(\rho_{\parallel} + \alpha \right) \vec{B} \right]}{B + \hat{n} \cdot \nabla \times \left[\left(\rho_{\parallel} + \alpha \right) \vec{B} \right]}, \quad \hat{n} = \frac{\vec{B}}{B}, \qquad (2)$$

with $v_{\parallel} = \vec{v} \cdot \hat{n}$, $\rho_{\parallel} = v_{\parallel}/\Omega$.

In the tokamak MHD equilibrium with an irregular magnetic field, two types of stochastic origin coexist. One is due to the Coulomb collision, and the other is due to the irregular magnetic field. The former is created in the velocity space by the pitch angle scattering (energy scattering is neglected), and the latter appears in the configuration space. In order to examine the stochastic properties of the radial particle diffusion in such a situation, extensive numerical calculations are performed in the two-dimensional parameter space (ν, s_b), where ν is the deflection frequency of the pitch and s_b indicates the relative strength of the irregular magnetic field, respectively. In the Boozer coordinates (ψ, θ, ζ), α is assumed as

$$\alpha(\psi, \theta, \zeta) = \sum_{m,n} \alpha_{m,n}(\psi) \cos(n\zeta - m\theta), \qquad (3)$$

$$\alpha_{m,n}(\psi) = s \exp\left[-\frac{(\psi - \psi_{m,n})^2}{\Delta\psi^2} \right], \qquad (4)$$

where $\psi_{m,n}$ denotes a resonance surface with $q = m/n$, $\Delta\psi/\psi_{edge} = 0.1$, and three Fourier harmonics $m/n = 10/7$, $3/2$, and $11/7$ are used. Note that this type of perturbation makes bounded irregular field in the radial coordinates, and that the relative magnitude of the perturbation at the resonant surface with $q = 3/2$, to which initially particles are loaded, is approximately given by $s_b \equiv \max|\delta\vec{B} \cdot \hat{r}|/B \sim 4.8s$.

The below table indicates type of the radial particle diffusion, where the horizontal and vertical axes denote strength of the perturbed field normalized by the value of the island overlapping and the collision frequency normalized by the transit frequency, respectively.

$(\nu/\nu_t)\backslash(s_b/s_{bc})$	0.0	0.23	0.91	2.3	23.
0.0			S_{osc}	S_{st}	U
0.45 (plateau)	W	S	S	S	S
4.5 (P-S)	W	W	S	S	S
45. (P-S)	W	$W^{\dagger}$	W	W	S

The symbol W means a Wiener-like process where the diffusion is normal diffusive ($D =$ const), Gaussian, statistically non-stationary with $A(t,t') = \sqrt{t/t'}(t' \geq t)$, and Markov process [1], and $W^{\dagger}$ is similar to the Wiener except for the slower decorrelation of the autocorrelation coefficient. The symbol U indicates a uniform mixing-like process with non-diffusivity ($D = 0$), uniform distribution, statistically stationarity with $A(t,t') = \exp[-(t'-t)/\tau]$, and Markovianity after the correlation time τ, which comes from almost uniformly perturbed magnetic field ($s_b/s_{bc} \gg 1$) and fast parallel motion $|v_{\parallel}| \gg |\vec{v}_d|$. The symbol S means a strange diffusive process with subdiffusivity ($dD/dt < 0$), non-Gaussianity and non-uniformity, and statistically non-stationarity with power law-like $A(t,t')$ [Markovianity depends on the situation]. Both processes labeled by S_{osc} and S_{st} have similar properties to S except for oscillatory behaviors in time and $A(t,t') \sim A(t'-t)$, respectively. The former may come from the effects of regular field structure for $s_b \sim s_{bc}$, and the latter may be related to the transition to the uniform mixing process.

As s_b/s_{bc} increases, the collisionless fast free streaming along a perturbed field line leads to fast radial transport. However, the collision interupts the fast free streaming, so that the correlation of particle motion is recovered in some extent, leading to subdiffusivity. The higher the collision frequency and the smaller the perturbation are, the nearer to Wiener the process is, thus usual transport analyses based on the local Fick's law are available. However, in the opposite situation, the process becomes subdiffusive, which may be related to the non-locality of the particle motion, so that the particle flux instead of the diffusion coefficient must be evaluated in a global analysis with including an appropriate particle source term.

[1] Statistical properties of the neoclassical radial particle diffusion in a tokamak equilibrium