§15. Identification of Tubular Vortices in Turbulence

Hideaki Miura

ABSTRACT- A new method to extract axes of tubular vortices in turbulence is proposed. Loci of centers of swirling motions are detected by using a nature that the pressure tends to be lower in the center of a swirling motion than surrounding region. This method is applied to a homogeneous isotropic turbulence to demonstrate that swirling motions actually exist around these axes.

1 Introduction

Identification of swirling vortices is an important topic of both plasma physics and fluid mechanics. Swirling vortices play an important role in transport process of plasmas and fluids. Furthermore, large numerical simulations have revealed existence of coherent structures in turbulent flows. Since coherent structures were discovered in turbulence, relation between coherent structures and swirling vortices has been discussed. However, it is not an easy task to define swirling vortices[1], leaving notion of coherent structures ambiguous. Although many methods have been proposed to identify swirling vortices these two or three decades, none of them seems to have given a sufficient result.

Recently we have developed a new method which identifies central axis of swirling motions.[2] This method, called sectionally local minimum method, finds a plane on which fluid is swirling by using eigenvalues of hessian of the pressure. In the next section, we see how to identify swirling axes and show that there are actually swirling motions around the axes.

2 Sectionally local minimum method

In this section, we first look for sectional local minima of pressures. We expand the pressure around each grid point (X_1, X_2, X_3) up to the second order as

$$P(x_1, x_2, x_3) = P^{(0)} + P_i^{(1)}(x_i - X_i) + \tfrac{1}{2}P_{ij}^{(2)}(x_i - X_i)(x_j - X_j), \quad (1)$$

where $P^{(0)} = P(X_1, X_2, X_3), P_i^{(1)} = (\partial/\partial X_i)\times P(X_1, X_2, X_3)$ and $P_{ij}^{(2)} = (\partial^2/\partial X_i \partial X_j)\times P(X_1, X_2, X_3)$. Next, by rotating the coordinate system around the origin from (x_1, x_2, x_3) to (x'_1, x'_2, x'_3), we have a normal form

$$P = P_{\min} + \tfrac{1}{2}\lambda^{(i)}(x'_i - c_i)^2 \quad (2)$$

where $\lambda^{(i)}$ $(i = 1, 2, 3)$ are the eigenvalues of pressure hessian $\{P_{ij}^{(2)}\}$, being real numbers because $\{P_{ij}^{(2)}\}$ is a real symmetric tensor. Here we assume, without loss of generality, that $\lambda^{(1)} \geq \lambda^{(2)} \geq \lambda^{(3)}$. The x'_i-axis is parallel to the eigenvector associated with $\lambda^{(i)}$. Then we have two condition to find a local minimum of the pressure. First condition is given by $\lambda^{(2)} > 0$. Next, consider a straight line l which is parallel to the x'_3-axis passing through point $\boldsymbol{C}(c_1, c_2, c_3)$. Second condition is that a foot point $\boldsymbol{C}'(c'_1, c'_2, c'_3)$ of a line l from the grid point $\boldsymbol{X}$ should locate within a single grid volume centered at $\boldsymbol{X}$, i.e. $|X_i - c'_i| < \frac{1}{2}\Delta$ $(i = 1, 2, 3)$, Δ being the grid width. Each point $\boldsymbol{C}'$ selected by above criterion is then connected with its nearest-neighbors within two mesh-sizes in the three orthogonal directions to construct the axial lines of tubular vortices. This method has been applied to a simulation of homogeneous and isotropic turbulence, to give a swirling motion around vortex axes. (See Fig.1)

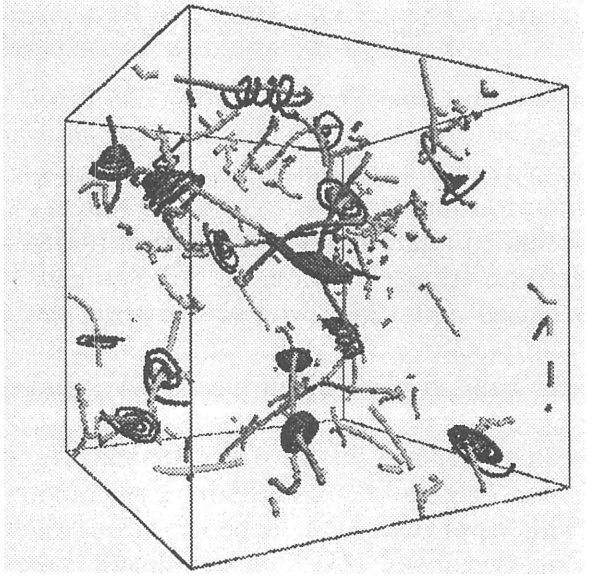

Figure 1: Swirling streamlines around vortex axes

References

[1] Jeong, J. and Hussain, F. On the identification of a vortex. *J. Fluid Mech.*, 285:69–94, 1995.

[2] H. Miura and S. Kida. Identification of tubular vortices in complex flows. *J. Phys. Soc. Jpn.*, pages 1331–1334, 1997.