§24. Interaction between a Trailing Vortex and External Turbulence

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The interaction between a columnar vortex and surrounding turbulence has been investigated for many years both from the engineering interest because they occur naturally in most sheared turbulent flows and from the theoretical interest to understand motions of swirling fluids. We consider the Lamb-Oseen vortex and q-vortex, and investigate their instabilities in ambient turbulence using direct numerical simulation. As an external field, turbulence produced by the Fourier-spectral method numerically is used. Its Reynolds number based on the Taylor scale is about 120.

The Lamb-Oseen vortex is an exact, stable, and axisymmetric solution of the Navier-Stokes equation;

$$\begin{aligned} v_\theta &= \frac{\Gamma}{2\pi r}\left(1-\exp\left\{-\frac{r^2}{r_0^2}\right\}\right), \\ v_z &= 0, \\ \Gamma &= 40\bar{\omega} r_0^2, \end{aligned}$$

where v_θ is the swirl velocity, v_z is the axial velocity, r is the radial distance from the symmetry axis, Γ is the circulation of the vortex, r_0 is the radius of the vortex, and $\bar{\omega}$ is the root mean square of the initial turbulence.

We can observe spiral structures (deformed worms) around the Lamb-Oseen vortex. A 3D isovortisity plot, Fig. 1, reveals that incoherent vortical structures are swirling azimuthally around the coherent structure.

The growth of the vortex radius are shown in Fig.2. The vortex radii of the numerical data, fitted from the radial profile of the axial vorticity profile, and the exact

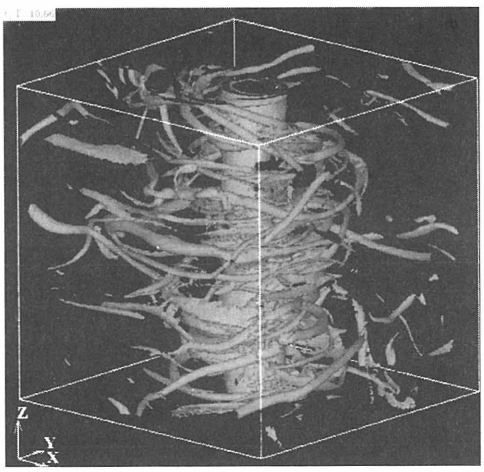

Figure 1: Isovorticity surface.

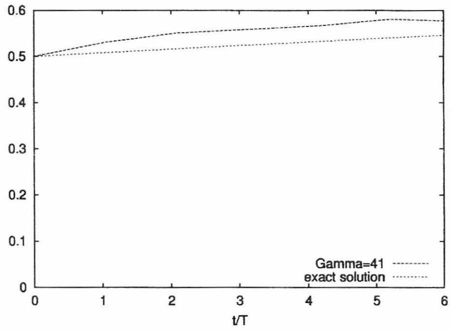

Figure 2: Time evolution of the radius of the Lamb-Oseen vortex.

radius of the Lamb-Oseen vortex are shown. The fitted data grows thicker than the exact radius. This growth is attributed to interactions between the vortex and the turbulence.

Using the two-point energy spectrum tensors, we can capture such phenomena as the Blocking Effect (fluid particles cannot penetrate into the vortex core) and the excitation of bending and axisymmetric vortex waves. As Miyazaki and Hunt[1] predicted, the axisymmetric components grow time-dependency as t squared. They have outstanding magnitude at k_z=10.5.

The q-vortex is the model of the trailing vortex. The vortex has a tangential velocity profile identical to the Lamb-Oseen vortex, and the axial component of the velocity is given by

$$v_z = \frac{\Gamma}{2\pi r_0 q}\exp\left(-\frac{r^2}{r_0^2}\right),$$

where q is the swirl parameter. The linear stability problem for this model has been considered by many authors[2]. A direct numerical simulation was performed with $q = -0.45$. In the initial stages, the turbulent kinetic energy grows rapidly due to the amplification of the linear stability wave. As expected, the growth rate of the perturbation energy agrees well with that of the linear-stability. This corresponds to a formation of vorticity which twist together. When the secondary instability is excited, the columnar vortex collapses and many fine scale vortices appear abruptly.

References

[1] Miyazaki, T. and Hunt, J. C. R.: Journal of Fluid Mechanics **402** (2000) 349.

[2] Mayer, E. W. and Powell, K. G.: Journal of Fluid Mechanics **245** (1992) 91.